Charge-Symmetry Breaking in the Nucleon-Nucleon Interaction

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New calculations of charge-symmetry breaking in the NN interaction lead to radically altered predictions for the difference between nn and pp scattering lengths. Inclusion of the η' meson and use of realistic magnetic couplings for the vector meson changes the sign as well as the magnitude of earlier results. This sensitivity arises in part from cancellations within the pseudoscalar and vector contributions separately, making precise calculations difficult. Meson mixing is determined from quark mass differences, and the finite width of the ρ meson is included.

New calculations of charge-symmetry breaking in the nucleon-nucleon interaction due to quark mass differences have led to several surprising results. The charge-symmetry breaking is taken to be due to SU(2)-invariance violation intrinsic to the strong interaction, and neglects electromagnetic and kinematic effects. At low energies, π-η' mixing, heretofore neglected, is shown to be more important and of opposite sign to π-η mixing. This leads to a total pseudoscalar mixing which is repulsive between neutrons in the two-body system, unlike the π-η contribution alone, while remaining attractive in the three-body system. The vector-meson contribution is very sensitive to the magnetic couplings of the ρ and ω mesons to nucleons, so sensitive that use of realistic rather than vector-dominance values changes the sign of this contribution. In addition, this treatment includes the effects of the finite ρ width. For values of the magnetic couplings near the vector-dominance estimates the finite ρ width has a substantial effect on the computed scattering length. Finally, we stress that there are large cancellations in each term and hence isospin is an even better quantum number for the low-energy nucleon-nucleon system than would be predicted by the small quark-mass ratios.

Quark-mass ratios are determined by relating SU(2)- and SU(3)-symmetry breaking, as described in Ref. 1. By use of the pseudoscalar meson masses, and the kaon and baryon mass spectra, the following results are obtained:

$$m_u/m_d = 0.38 \pm 0.13,$$

$$m_d/m_s = 0.045 \pm 0.011,$$

$$\delta = \frac{m_d - m_u}{2m_s - (m_d + m_u)} = 0.0144 \pm 0.003, \qquad (1)$$

where δ is the ratio of SU(2)- to SU(3)-symmetry breaking. Constraining the up (or down) quark to be massless gives a value of $\delta = \pm 0.032$, the use of which would change these results by a factor of ($\pm$) 2.22.

The leading mechanism whereby the quark mass mass differences lead to charge-symmetry breaking is due to induced mixing among the physical $T=0$ and $T=1$ mesons. To check Eq. (1), one computes the $\eta \to 3\pi$ decay[1] and ρ-ω mixing implied by photopion and electropion production.[2] The results are in good agreement. Quark mass differences now provide a theoretical framework for the use of phenomenological meson mixing as in previous calculations. For a summary, and a complete reference list, see Henley and Miller.[3]

This determines the charge-symmetry-breaking part of the interaction, which for low energies we take to be the nonrelativistic reduction of the meson exchange diagrams. For the pseudoscalar mesons in the $^1S_0$ channel to $O(\mu^2/M^2)$ this gives

$$V^{\pi\eta} = (\tau_{1z} I_2 + \tau_{2z} I_1)\frac{g_\pi g_\eta}{4\pi}\frac{-\mu_{\pi\eta}{}^2}{\mu_\eta{}^2 - \mu_\pi{}^2} \times [V_{\rm ps}(\mu_\pi, r) - V_{\rm ps}(\mu_\eta, r)], \qquad (2)$$

where

$$V_{\rm ps}(\mu, r) = \hbar c\,\frac{e^{-\mu r}}{r}\,\frac{\mu^2}{4M^2};$$

i_i is the identity operator in isospin space for particle i, and η stands for either η or η'. The mixing angles are

$$\frac{-\mu_{\pi\eta}{}^2}{\mu_\eta{}^2 - \mu_\pi{}^2} = \frac{2}{\sqrt{3}}\,\frac{\mu_K{}^2 - \mu_\pi{}^2}{\mu_\eta{}^2 - \mu_\pi{}^2}\,\delta = 0.93\delta, \qquad (3)$$

$$\frac{-\mu_{\pi\eta'}{}^2}{\mu_{\eta'}{}^2 - \mu_\pi{}^2} = 2\sqrt{\tfrac{2}{3}}\,\frac{\mu_K{}^2 - \mu_\pi{}^2}{\mu_{\eta'}{}^2 - \mu_\pi{}^2}\,\delta = 0.414\delta. \qquad (4)$$

The final results will clearly depend on the coupling constants. We used values $g_\pi{}^2/4\pi = 13.40$, $g_\eta{}^2/4\pi = 3.947$, $g_{\eta'}{}^2/4\pi = 7.894$ which are intermediate between the SU(6) limit and fits to baryon-baryon interactions.[4]

The values for $\mu_{\pi\eta'}{}^2$ and $g_{\eta'}$ are obtained by the quark-model assumption that the $s\bar{s}$ state decouples from nucleons and does not mix with the π^0. η-η' mixing is ignored because it is higher order in symmetry breaking. Its inclusion could change the strength of the η and η' potentials by about 20%.

Proceeding in similar fashion, the charge-symmetry–breaking potential in the ${}^1S_0$ channel due to mixing of zero-width vector mesons is

$$V^{\rho\omega}=(\tau_{1z}I_2+\tau_{2z}I_1)\frac{g_\rho g_\omega}{4\pi}\frac{-\mu_{\rho\omega}{}^2}{\mu_\omega{}^2-\mu_\rho{}^2}\left[V_{\rm v}(\mu_\rho,r)-V_{\rm v}(\mu_\omega,r)\right], \tag{5}$$

where

$$V_{\rm v}(\mu,r)=\hbar c\frac{e^{-\mu r}}{r}\left\{1-\frac{3}{8}\frac{\mu^2}{M^2}-\frac{5}{128}\frac{\mu^4}{M^4}-K_\Sigma\frac{\mu^2}{4M^2}\left(1+\frac{\mu^2}{8M^2}\right)-K_\pi\frac{\mu^2}{2M^2}\right\}, \tag{6}$$

with $K_\Sigma=K_\rho+K_\omega$ and $K_\pi=K_\rho K_\omega$. K_ρ (K_ω) is the $\rho\bar{N}N$ ($\omega\bar{N}N$) magnetic coupling. We take $-\mu_{\rho\omega}{}^2=(0.388\ \text{GeV}^2)\delta\approx 0.0056\ \text{GeV}^2$. Inclusion of the electromagnetic contribution would reduce this by about 13%.

Because of the large value of K_ρ, the magnetic terms may change the sign of the potential. Results are therefore presented as a function of K_ρ and K_ω. Rather than vary g_ρ and g_ω as well we have taken the product $g_\rho g_\omega/4\pi=2.0$, which is in rough agreement with most sets of coupling constants.

A spectral function based on the propagator of Ref. 4 was used to weight the effect of the finite ρ width by computing $\Delta a=|a_{nn}|-|a_{pp}|$ as a function of the ρ mass, and integrating. This allows a general result to be expressed in terms of K_Σ and K_π by assuming linearity of Δa with the strength of the potential, an excellent approximation here.

Given the charge-symmetry–breaking potential, its effect on the low-energy nucleon-nucleon system is evaluated by using the phenomenological soft-core (RSC) and hard-core (RHC) potentials of Reid[5] to describe the proton-proton interaction. The neutron-neutron interaction was then determined with use of Eqs. 2 or 5. In the results reported here nothing else was changed between the pp and nn calculations. All kinematic effects deriving from nucleon mass differences are neglected. The results presented are due only to the intrinsic symmetry breaking in the strong interaction.

For the pseudoscalar mesons the charge-symmetry–breaking potential $V_{T3}=V_{nn}-V_{pp}$ is repulsive at long range and attractive at short range. In Table I the values of Δa are given for $\pi\eta$ and $\pi\eta'$ mixing separately. The conventions are chosen so that negative signs signify repulsion in the nn relative to the pp channel. The results indicate stronger repulsion for the η' than attraction for the η. The π-η mixing result for the RSC potential is consistent with McNamee, Scadron, and Coon[6] up to differences in coupling constants. These results are illustrated by the curves in Fig. 1. The rapidly falling (solid) curve shows the change in scattering length for a charge-sym-

TABLE I. A change in scattering length Δa due to π-η and π-η' mixing; all values in fermis.

	π-η	π-η'	Total	$\Delta a\, g_\pi/g_\eta$	$\Delta a\, g_\pi/g_{\eta'}$
RSC	0.13	−0.34	−0.21	0.24	−0.44
RHC	0.05	−0.38	−0.33	0.08	−0.50

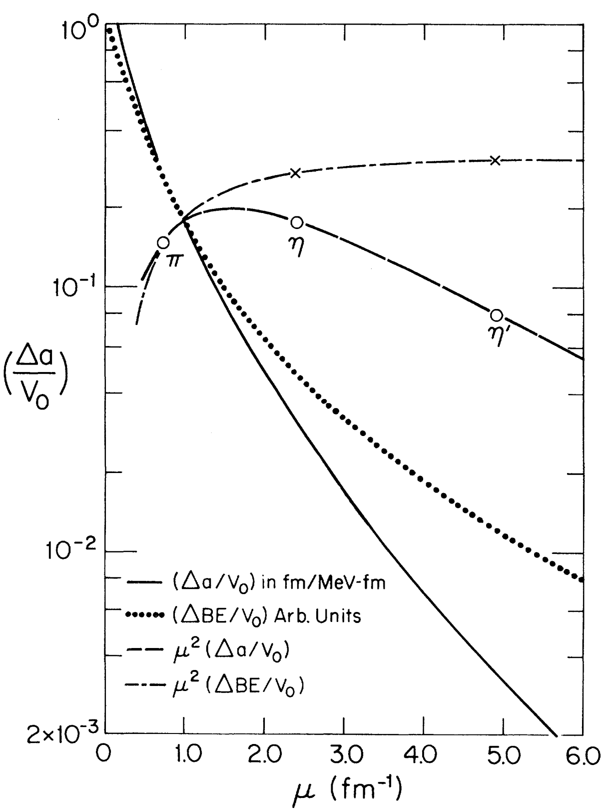

FIG. 1. Effects of pseudoscalar mixing on scattering length and mass-3 binding.

metry–breaking potential of the form

$$V_{T3}=V_0 e^{-\mu r}/r \qquad (7)$$

if the phenomenological potential is the Reid soft-core. The actual form of the potential is the difference of two such terms, weighted with the mass squared. Also plotted (dashed curve) is $\mu^2 V_{T3}$ with the positions of the physical mesons indicated by the open circles. We see that in the π-η mixing, the pionlike contribution largely cancancels the η-like part, whereas for π-η' the pion contribution is much larger. The difference in scattering length is therefore dominated by the relatively weak but long-range (pionlike) contribution. As the energy increases the short-range potential becomes more important. By 20 MeV there is net attraction, and at higher energies both contributions are attractive. At low energies amenable to experiment, large cancellations exist.

To contrast the zero-energy two-body system with the three-body system, we follow a suggestion by Brandenburg, Coon, and Sauer[7] for estimating the effect of a local charge-dependent potential on the binding energies of ^{3}He and ^{3}H. A simple and illustrative estimate of the change in binding energy as a function of mass may be made by neglecting the q dependence of the proton and neutron form factors and assuming the ^{3}He and ^{3}H densities to have identical Gaussian form factors. Then, a charge-symmetry–breaking potential of the form (7) gives a change in binding energy,

$$\Delta E_B = 2(\pi\beta)^{-1/2} V_0 [1-\pi^{1/2} x \exp(x^2)\,\mathrm{erfc}(x)], \qquad (8)$$

where $\beta=\frac{1}{2}\langle r^2\rangle$ and $x^2=\beta\mu^2$, and erfc is the complementary error function. ΔE_B is shown by the dotted line, as well as $\mu^2(\Delta E_B)$ (dash-dotted line). Both curves have been arbitrarily normalized to intersect the corresponding two-body curves at the pion mass. The much slower falloff of ΔE_B compared with Δa is apparent. In fact, $\mu^2 \Delta E_B$ increases asymptotically to $V_0/\beta(\pi\beta)^{1/2}$ and so the (attractive) η- and η'-like parts of the potential will dominate. Although crude, this simple model duplicates the μ dependence of the detailed treatment in Ref. 7 and illustrates how repulsion in the two-body system and attraction in the three-body system could come about, a possibility first pointed out by Gibson and Stephenson.[8]

In the consideration of the vector mesons we have near degeneracy instead of large differences in particle masses. To first order, the potential in Eq. (6) is independent of the difference in ρ and ω masses, depending only on the average.[9,10] Nevertheless, the effect of ρ-ω mixing was calculated with use of the physical values ($\mu_\rho=776$ MeV, $\mu_\omega=783$ MeV). For potentials of the form (7) we find that for equal V_0 the effect of the ρ meson will be about 3% larger than that of the ω. However, in the presence of large magnetic couplings the situation is going to be much more delicate. To take two specific examples, the parameters used in Ref. 6 ($K_\rho=3.7$, $K_\omega=0.12$, obtained from vector dominance) lead to near cancellations of the direct by the magnetic terms in both the ρ-like and ω-like parts of the potential. The large values of $K_\rho=8.11$, $K_\omega=0.694$ obtained in Ref. 4 lead to domination of the charge-symmetry–breaking potential by the magnetic terms. However, the ω-like piece of the potential is deeper, by almost 3% and so there are again large cancellations in the effect of the charge-symmetry–breaking potential at zero energy. Use of the phenomenologically determined coupling constants, unlike the vector-dominance values, leads to net repulsion in the nn channel.

Results are presented in Table II for the change in scattering length as a function of K_Σ and K_π, according to

$$\Delta a \equiv |a_{nn}| - |a_{pp}| = A - BK_\Sigma - CK_\pi, \qquad (9)$$

with specific numbers given for two sets of parameters. We see drastic effects due to the choice of magnetic couplings. The effect of finite ρ width is also substantial, between 15% and 35%, depending on the magnetic coupling. Modest (10%) effects are noticeable due to change in the phenomenological potential as well.

The effects of K_ρ, K_ω, and ρ width are illustrated more completely in Fig. 2 where contours of constant Δa are plotted against K_ρ and K_ω for both the zero-width (dashed curve) and finite (solid curve) cases. The largest sensitivity to the ρ width is for negative K_ω and small K_ρ, the region suggested by vector dominance.

TABLE II. The dependence of Δa on magnetic couplings for fixed $g_\rho g_\omega/4\pi=2.0$.

	A	B	C	Δa[a]	Δa[b]
RSC, $\Gamma_\rho=0$	1.58	0.092	0.255	1.36	−0.66
RSC, $\Gamma_\rho=155$	1.28	0.101	0.210	1.01	−0.78
RHC, $\Gamma_\rho=155$	1.18	0.104	0.212	0.90	−0.92

[a]Ref. 6.
[b]Ref. 4.

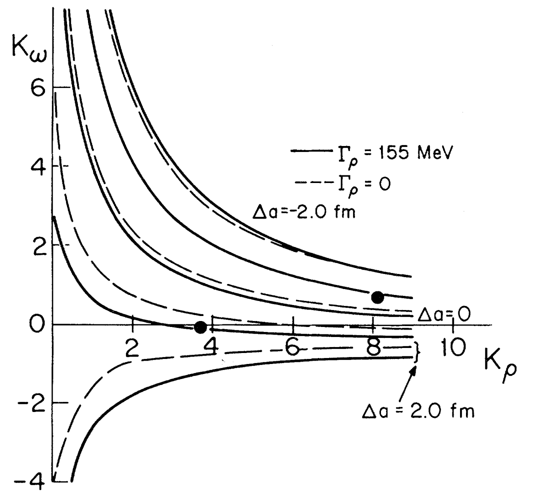

FIG. 2. Dependence of scattering length on magnetic couplings and ρ width. The magnetic couplings from Refs. 4 and 6 are indicated by the points.

Because of the uncertainties due to coupling constants, it seems inappropriate to add the pseudoscalar and vector contributions. Given the importance of those effects, they must be included and better magnetic couplings determined before detailed comparison with experiment can be made.

We have presented calculation of charge-symmetry breaking in the N-N system arising from intrinsic SU(2)-symmetry breaking in the strong interaction arising from quark mass differences. There are large effects on the scattering length, of order 1 fm, even after considerable apparently accidental cancellations in both the pseudoscalar- and vector-meson contributions. These cancellations imply that the effects of heavier mesons cannot be ignored *a priori* simply because of their shorter range. Further, isospin seems likely to be a better symmetry for the nucleon-nucleon system than even the small quark-mass ratios would suggest. Finally, we have explicitly exhibited these cancellations, as well as the importance of treating the ρ width correctly.

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Effects of Heavy Particles through Factorization and Renormalization Group

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For renormalizable theories without spontaneous symmetry breaking, a formalism is developed to systematically extract and evaluate the effects of heavy particles in low-energy physics through factorized local operators and the renormalization group. As an application, quantum electrodynamics with electrons and muons is considered and the effects of muons on the electron anomalous magnetic moment are assessed.

It is an interesting view which is shared by many of us that the dimensionless coupling constants in various interactions are almost universal in strength. The breakup into various hierarchies is really due to the mass scales involved. In other words, the apparent strengths are intimately tied up with the relative lightness and heaviness of species of particles and the experi-