tion is then given by

$$\sigma(\gamma p \to \omega p) = \frac{p_\pi^2}{k_\gamma}\frac{e^2}{4\gamma_\omega^2}\frac{1}{2}[\sigma^{\rm el}(\pi^+ p) + \sigma^{\rm el}(\pi^- p)] \times \exp(-b|t_{\rm min}|), \quad (2)$$

where p_π and k_γ are the momenta of the π and the photon, respectively, in the πp and γp center-of-mass systems (evaluated at the same s). Here γ_ω^2 is the γ-ω coupling constant, and the factor $\exp(-b|t_{\rm min}|)$ allows for the minimum $|t|$ in ω photoproduction. Equation (2) is plotted in Fig. 4 with use of smoothed πp elastic-scattering data.[9] The curve is normalized to the ω cross section of this experiment. The value of the coupling constant resulting from the normalization is

$$\gamma_\omega^2/4\pi = 5.4 \pm 0.4,$$

and compares with 4.6 ± 0.5 from colliding-beam measurements[10] and 7.5 ± 1.3 from photoproduction on complex nuclei.[10] The deviation of the lower-energy ω cross section from the VMD–quark-model prediction is attributed to unnatural-parity (pion) exchange in the t channel, which decreases with increasing photon energy approximately as $1/E^2$. The cross section which is due to natural-parity exchange only has been measured by Ballam *et al.*[4] and is also shown in Fig. 4. There is good agreement with the VMD–quark-model prediction.

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[1]R. M. Egloff *et al.*, Phys. Rev. Lett. 43, 657 (1979).
[2]D. O. Caldwell *et al.*, Phys. Rev. Lett. 40, 1222 (1978); J. P. Cumalat, thesis, University of California, Santa Barbara, 1977 (unpublished).
[3]D. O. Caldwell *et al.*, Phys. Rev. Lett. 42, 553 (1979).
[4]J. Ballam *et al.*, Phys. Rev. D 7, 3150 (1973).
[5]$\sigma(\gamma p \to \rho p) = 9.3\ \mu$b from Ref. 1, and $\rho \to \pi^0\gamma$ branching ratio = 0.024%.
[6]J. Whitmore, Phys. Rep. 10C, 273 (1974).
[7]This fraction is consistent with diffraction-dissociation cross sections measured in pp and πp scattering: See Ref. 6 and also M. G. Albrow *et al.*, Nucl. Phys. B108, 1 (1976); G. Wolf, Nucl. Phys. B26, 317 (1971).
[8]Aachen-Berlin-Bonn-Hamburg-Heidelberg-Munchen Collaboration, Phys. Rev. 175, 1669 (1968); Y. Eisenberg *et al.*, Phys. Rev. D 5, 15 (1972); J. Ballam *et al.*, Phys. Rev. D 7, 3150 (1973).
[9]D. S. Ayres *et al.*, Phys. Rev. D 15, 3105 (1977); K. J. Foley *et al.*, Phys. Rev. Lett. 11, 425 (1963); I. Ambats *et al.*, Phys. Rev. D 9, 1179 (1974).
[10]D. W. G. S. Leith, in *Electromagnetic Interactions of Hadrons*, edited by A. Donnachie and G. Shaw (Plenum, New York, 1978).

Calculability in $SU(2)_L \otimes SU(2)_R \otimes U(1)$: The Mass Matrix and *CP*-Invariance Violation

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It is shown that calculability of the mixing angles in a class of n-generation, left-right–symmetric $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models restricts the fermion mass matrix to be of the form $\sum_{m=0}^{n} \oplus I_m A_m$ with the constraint $\sum_{m=0}^{n} m I_m = n$, where A_m is a nonsingular $m \times m$ Hermitian or complex symmetric block containing $2m-1$ elements and I_m is the number of such blocks. Such a mass matrix *cannot* supply *CP*-invariance-violating phases.

There have been, in the last few years, many attempts to derive expressions for the Cabibbo mixing angles, θ_i, as a function of quark masses only. These results were obtained by requiring the Lagrangian to be invariant under additional symmetries—Abelian[1] or non-Abelian[2] discrete models only slightly distort the initial spectrum tries—above and beyond the specific gauge group employed. The motivation for these works has been "calculability." Flavor mixing from the standpoint of natural flavor conservation is discussed elsewhere.[4] In some of these models[5] a mechanism combining the *CP*-invariance-violating phases and mixing angles was generated by diagonalizing the fermion mass matrices.

The most natural attempts to date are built within the framework of a left-right–symmetric gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)$. We would like to offer, in this Letter, some general conclusions that result from requiring the following: [1] calculability of the mixing angles, $\theta_i = \theta_i(m_j)$, where the m_j's are the masses of the quark fields; [2] nonsingularity of the fermion mass matrices, M ($\det M \neq 0$), so that all the quarks are massive. We will also assume throughout that the coupling constants are distinct and that we have no charge-conjugate Higgs scalars, $\tilde{\varphi} = \tau_2 \varphi^* \tau_2$, coupling to the fermions.[6]

The principal results are as follows: (A) The nontrivial mass matrices will be, up to the interchange of quarks, block diagonal, each *block* A_i being of the form

$$A_i = \begin{bmatrix} a_{i1} & a_{i2} & 0 & 0 & \cdot & \cdot & \cdot \\ a_{i2}' & 0 & a_{i3} & 0 & \cdot & \cdot & \cdot \\ 0 & a_{i3}' & 0 & a_{i4} & & & \\ 0 & 0 & a_{i4}' & 0 & \cdot & & \\ \cdot & \cdot & & \cdot & \cdot & \cdot & \\ \cdot & \cdot & & & \cdot & \cdot & \cdot \\ \cdot & \cdot & & & & \cdot & \cdot \end{bmatrix}$$

with $|a_{ij}| = |a_{ij}'|$. (B) It is impossible to generate CP-invariance-violating phases from the Yukawa couplings. The above form of M enables us to extract all of the arbitrary phases in the diagonalizing unitary matrices. (I) The mass matrix.—We begin the derivation of these results by considering an n-generation $SU(2)_L \otimes SU(2)_R \otimes U(1)$ gauge model of the weak and electromagnetic interactions. The quark fields will have the assignments

$$\vec{\psi}_{iL[R]} = \begin{pmatrix} \psi^u \\ \psi^d \end{pmatrix}_{iL[R]} \tag{1}$$

transforming as $(\frac{1}{2}, 0, 1)$ $[(0, \frac{1}{2}, 1)]$. We employ an arbitrary number, m, of Higgs scalar φ^k transforming as $(\frac{1}{2}, \frac{1}{2}, 0)$, where the vacuum expectation values are diagonal:

$$\langle \varphi^k \rangle = \begin{pmatrix} \alpha_k^{\,1} & 0 \\ 0 & \alpha_k^{\,2} \end{pmatrix}. \tag{2}$$

The Yukawa couplings of quarks with Higgs scalars will be

$$\mathcal{L}_Y = \sum_{i,j,k} g_{ij}^{\,k} \bar{\psi}_{Li} \varphi^k \psi_{Rj} + \text{H.c.} \tag{3}$$

After spontaneous symmetry breaking, we obtain the quark mass matrices $M^{u(d)}$ for the up (down) sector:

$$(M^{u(d)})_{ij} = \textstyle\sum_k g_{ij}^{\,k} \alpha_k^{\,1(2)}. \tag{4}$$

These mass matrices can be diagonalized with a bilinear transformation

$$U_L M U_R^{\dagger} = M_D, \tag{5}$$

where $(M_D)_{ij} = \delta_{ij} m_j$. Equivalently,

$$U_L \mathfrak{M} U_L^{\dagger} = \mathfrak{M}_D, \quad U_R \mathfrak{M}' U_R^{\dagger} = \mathfrak{M}_D, \tag{6}$$

where

$$(\mathfrak{M}_D)_{ij} = \delta_{ij} |m_j|^2, \quad \mathfrak{M} \equiv MM^{\dagger}, \quad \mathfrak{M}' = M^{\dagger}M.$$

The generalized Cabibbo mixing angles, θ_i, will be defined through the unitary matrices $U_L = U_L^{\,u} U_L^{\,d\,\dagger}$. Since the coupling constants are distinct, $\mathfrak{M}^{u(d)}$ carries the same number of parameters as $M^{u(d)}$. Given these arbitrary $n \times n$ mass matrices, $\mathfrak{M}^{u(d)}$, determined by $n_p^{\,u(d)}$ parameters, we proceed to show that a necessary condition for calculability is $n_p^{\,u(d)} \leq n$.

Invariance of the nth-order characteristic equations of $\mathfrak{M}^u$ and $\mathfrak{M}^d$ under unitary transformations will yield $2n$ invariant quantities corresponding to the coefficients of the characteristic polynomials. Thus, in diagonalization we obtain $2n$ equations relating the $n_p^{\,u}$ complex parameters of $\mathfrak{M}^u$ and the $n_p^{\,d}$ complex parameters of $\mathfrak{M}^d$ to the $2n$ diagonal $|m_j|^2$ terms. Since we construct $U_L^{\,u(d)}$ from the eigenvectors of $\mathfrak{M}^{u(d)}$, calculability $[\theta_i = \theta_i(m_j)$ or $U_L = U_L(m_j)]$ will imply that $\mathfrak{M}^{u(d)}$ be functions of $|m_i^2|$ only. This then necessitates $n_p^{\,u} + n_p^{\,d} \leq 2n$. If we use our condition of no $\tilde{\phi}$ coupling, then the up and down sectors will be disconnected, $n_p^{\,u} \leq n$ and $n_p^{\,d} \leq n$, and we can speak of calculability in the up and down sectors separately. Calculability also limits the numbers of mixing angles in $U_L^{\,u(d)}$ to be $\leq n$. Because our coupling constants are distinct, the only mixing angles that can be generated by the $n_p \leq n$ parameter mass matrices $M^{u(d)}$, assuming condition [2], will be trivial.

If we want nontrivial θ_i's then we must require that the Lagrangian be invariant under some additional symmetries which relate the coupling constants $g_{ij}^{\,k}$. The most natural additional symmetry will be an extension of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ gauge theories—left-right symmetry (LR): $\psi_{iL} \to \psi_{iR}$, $\phi^k \to \phi^{k\dagger}$. Then it follows that

$$g_{ij}^{\,k} = g_{ji}^{\,k\,*}. \tag{7}$$

If the $g_{ij}{}^k$'s and α_k's are both complex, then we will again induce only trivial θ_i's. Otherwise we have the relation

$$|M_{ij}|=|M_{ji}|. \tag{8}$$

Specifically, if the $g_{ij}{}^k$'s are real (complex) [real] and the α_k's are real (real) [complex], then M will be real symmetric (Hermitian) [complex symmetric]. For these cases, conditions [1] and [2] prohibit $n_p<n$ and thus we must have[7] $n_p=n$.

It is then easy to show that, up to an interchange of quarks that leaves the characteristic equation invariant, such a mass matrix, defined by distinct coupling constants, will be block diagonal with between n and $2n-1$ nonzero elements. We reiterate that $n_p=n$ is a *necessary* condition. The *sufficient* condition will be that every $m\times m$ block must be characterized by m parameters. The fermion mass matrix M must then be of the form stated in result (A).

We can classify the mass matrices according to the number and dimension of the blocks using direct-sum notation. For example, any $n\times n$ mass matrix M can be written in the form[8]

$$M=\sum_{m=0}^{n}\oplus I_m A_m, \tag{9}$$

where A_m is an $m\times m$ block and I_m is the number of $m\times m$ blocks, with the constraint

$$\sum_{m=0}^{n} mI_m=n. \tag{10}$$

The above result is a statement of the allowed forms of the mass matrices if we require conditions [1] and [2]. It does not state *how* that form might be generated or what the specific form will be. In order to do this we can employ an additional discrete symmetry, K:

$$\psi_{iL}\to X_i{}^\dagger\psi_{iL},\quad \psi_{jR}\to Y_j\psi_{jR},\quad \varphi^k\to Z_k\varphi^k, \tag{11}$$

where X_i, Y_j, and Z_k are diagonal unitary operators. Under K the mass matrix transforms as

$$(M)_{ij}\to(M)_{ij}'=\textstyle\sum_k g_{ij}{}^k Z_k X_i\langle\varphi^k\rangle X_j. \tag{12}$$

The invariance of $\mathcal{L}$ under K implies that

$$X_i\langle\varphi^k\rangle X_j=Z_k{}^\dagger\langle\varphi^k\rangle \tag{13}$$

for all nonzero couplings. LR requires that $X_j=Y_j$. We can always find a K symmetry of the above form to generate an allowed mass matrix.

(II) *CP-invariance violation.*—We now apply the results of the previous section to the question of CP-invariance violation. For this discussion we need only work with an arbitrary $m\times m$ block, A, since the disconnected quark families do not interact.

If the $g_{ij}{}^k$'s and α_k's are real, then it is obvious that there will be no phases. For the cases when the $g_{ij}{}^k$'s are real (complex) and the α_k's are complex (real) we argue as follows. We diagonalize the complex symmetric (Hermitian) block A with a bilinear transformation, Eq. (5). We can find[9] U_L and U_R by solving for the m eigenvectors, χ_i, of $\mathcal{A}=AA^\dagger$ and $\mathcal{A}'=A^\dagger A$. For both cases it is straightforward to show that χ_i will have the form

$$\vec{\chi}_i=\sum_{j=1}^{m} R_i{}^j\exp(ir_j)\hat{e}_j, \tag{14}$$

where $R_i{}^j$ depends only on the real eigenvalue λ_i and the magnitudes of the parameters in $\mathcal{A}$, r_j is a real constant parameter *independent* of λ_i, and $\hat{e}_j$ is the jth unit vector.

The unitary matrix, $U_L(U_R)$, that diagonalizes $\mathcal{A}$ ($\mathcal{A}'$) will be constructed from these χ_i as columns. Since r_j is not a function of the eigenvalues, each *row* will carry a common phase which can be absorbed into the quark fields. The CP-invariance-violating phases will thus disappear from the generalized Cabibbo rotations.[10]

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[1]S. Weinberg, in *A Festschrift for I. I. Rabi*, edited by L. Motz (N. Y. Academy of Sciences, New York, 1977); A. De Rújula, H. Georgi, and S. L. Glashow, Ann. Phys. (N.Y.) 109, 258 (1977); F. Wilczek and A. Zee, Phys. Lett. 70B, 418 (1977); K. Kang and A. C. Rothman, Brown University Report No. HET-371, 1978 (unpublished); Ling-Fong Li, Carnegie-Mellon University Report No. COO-3066-121, 1979 (to be published); A. Ebrahim, Phys. Lett. 72B, 457 (1978), and 73B, 181 (1978), and 76B, 605 (1978); T. Kitazoe and K. Tanaka, Phys. Rev. D 18, 3476 (1978); M.-A. de Crombrugghe, Phys. Lett. 80B, 365 (1979).

[2]S. Pakavasa and H. Sugawara, Phys. Lett. 73B, 61 (1978); H. Harari, in *Proceedings of the Nineteenth International Conference on High Energy Physics, Tokyo, Japan, 1978*, edited by S. Homma, M. Kawaguchi, and H. Miyazawa (International Academic Printing

Co., Ltd., Tokyo, Japan, 1979); G. Segrè, H. A. Weldon, and J. Weyers, University of Pennsylvania Report No. UPR-0105T, 1979 (to be published); V. S. Mathur and T. Rizzo, University of Rochester Report No. UR-667, 1979 (to be published); D. Wyler, Phys. Rev. D 19, 330 (1979).

[3]F. Wilczek and A. Zee, Phys. Rev. Lett. 42, 421 (1979); A. Davidson, M. Koca, and K. Wali, Phys. Rev. Lett. 43, 92 (1979), and to be published.

[4]R. Barbieri, R. Gatto, and F. Strocchi, Phys. Lett. 74B, 344 (1978); R. Gatto, G. Morchio, and F. Strocchi, Phys. Lett. 80B, 265 (1979); G. Segrè and A. Welson, University of Pennsylvania Report No. UPR-0125T, 1979 (to be published); R. Gatto, in Jerusalem Einstein Centennial Symposium, Jerusalem, Israel, 14–23 March, 1979 (unpublished).

[5]R. N. Mohapatra and D. P. Sidhu, Phys. Rev. D 17, 1876 (1978); R. N. Mohapatra, in *Proceedings of the Nineteenth International Conference on High Energy Physics, Tokyo, Japan, 1978,* edited by S. Homma, M. Kawaguchi, and H. Miyazawa (International Academic Printing Co., Ltd., Tokyo, Japan, 1979); R. N. Mohapatra and G. Senjanović, Phys. Lett. 73B, 176 (1978); T. Hagiwara, T. Kitazoe, G. B. Mainland, and K. Tanaka, Phys. Lett. 76B, 602 (1978).

[6]See, for example, De Rújula, Georgi, and Glashow, Ref. 1, and Mohapatra and Senjavović, Ref. 5.

[7]If $n_p < n$ it can be trivially shown that either $\det M = 0$ or the mass spectrum is degenerate.

[8]For example, in the case of three generations we have the mass matrices A_3 and $A_1 + A_2$ only, as correctly observed by de Crombrugghe in Ref. 1. If nature demands that the b quark couple significantly to the c quark, then A_3 is preferred. The general mass matrices containing no disconnected quark families, $M = A_n$, have been considered by T. Hagiwara, "Quark Mass Formulae and the Number of Quark Flavors" (to be published). See also H. Fritzsch, CERN Report No. TH-2672, 1979 (to be published).

[9]In general if we diagonalize a Hermitian (complex symmetric) matrix using a bilinear transformation $U_L A U_R^{-1} = A_D$, then $U_R = K U_L$ ($U_R = K U_L^*$), where K is an arbitrary diagonal unitary matrix that can be absorbed by the quark fields. Then $\theta_L^i = \theta_R^i$ for both cases and the phases will be related by $\delta_L^i = \delta_R^i$ ($\delta_L^i = -\delta_R^i$).

[10]In general one cannot absorb all the phases of the unitary matrices for three generations or more. See M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

Multimuon Final States in Muon-Nucleon Scattering at 270 GeV

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Data from a new muon-nucleon scattering experiment includes 513 events, of which 449 are dimuons and 64 are trimuons. Conventional hadronic and electromagnetic processes account for less than 15% of these events. Model calculations suggest that the majority of the dimuons result from associated charm production with a total cross section of about 3 nb.

Events with two (or more) final-state muons have been observed in experiments using muons,[1] neutrinos,[2] or hadrons[3] as incident particles. Possible interpretations of these events in the lepton-scattering experiments include such processes as the production (and semileptonic decay) of charmed mesons,[4] heavy-lepton production and decay,[5] and more conventional hadronic and electromagnetic mechanisms.[1] This Letter describes the characteristics of a large sample of dimuon events from a new muon-scattering experiment. The improved acceptance and increased statistics of these data have reduced previous uncertainty about the origin of these events.

The experimental apparatus (Fig. 1) consisted of four major parts: the beam, target/calorimeter, final-state muon spectrometer, and trigger/veto counters. The Fermilab muon (μ^+) beam energy was 270 GeV (with a mean spread of 5 GeV) and the intensity within the beam telescope averaged 6×10^5 muons/spill with an almost equal number of halo muons outside the beam. This halo was prevented from triggering the apparatus by a large bank of scintillation counters at the

