The strength and coherent nature of the ripple perturbation make integration along unperturbed trajectories untenable for trapped particles; so we consider trapped particles only in turbulent spectra. In this case, (7) is more difficult to satisfy than (6), by a factor of roughly $m^2 \sim 10^4$. This requires a large turbulence level $\mathcal{B}_{1,0} > 10^{-2}$, a regime which we do not consider here. Because (7) is not satisfied, a particle makes only a fraction of an oscillation in the $\cos(\vec{l} \cdot \vec{\Theta})$ well during the first half of its bounce period, then retraces its motion during the return half. Mathematically, this is manifested by a factor $J_{-1} + J_1 = 0$ ($l_b = 0$) which replaces the factor $J_{l_b - m}$ appearing in $\mathcal{B}_1$. We conclude that trapped electrons should not be stochastic.

Finally, one may consider trapped ions in turbulence. Here, since $l_\varphi \Omega_\varphi / \omega_a \sim \epsilon \ll 1$, the resonance condition requires that $l_b \simeq \omega_a / \Omega_b \approx q\epsilon^{-3/2} \sim 12$. Thus the terms $J_{l_b \pm 1}$ in $\mathcal{B}_1$ greatly reduce $g_{\vec{l}}$, by a factor on the order of $(\epsilon^{3/2})^{l_b}$, and one expects no stochasticity for ions as well. The physical mechanism here is that, for $\omega_a \sim \omega_*$, the ions move too slowly to resonate with the waves.

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Observation of Anomalous Heat Capacity in Liquid ^{3}He near the Superfluid Transition

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The specific heat of liquid ^{3}He from 0.8 to 20 mK at zero pressure has been measured. Above ~ 3 mK the specific heat is linear in temperature and $C/nRT = 2.11\ \mathrm{K}^{-1}$, which is 30% less than the currently accepted value. Below 3 mK, C appears to deviate increasingly from this relationship reaching, at the superfluid transition $T_c = 1.04$ mK, a value 9% in excess of the extrapolated linear specific heat. This Letter discusses the anomalous behavior and its consequences with regard to the interpretation of our data.

There is an urgent need for precise specific-heat data on liquid ^{3}He in the vicinity of the superfluid transition over the whole pressure range. Some of the most fundamental tests of the current theories[1,2] on superfluidity in ^{3}He and of the Fermi-liquid theory can be carried out when accurate specific-heat data become available.

Below the superfluid transition the specific heat of liquid ^{3}He has been measured earlier by several groups.[3-8] The results, however, are not very consistent because of problems associated with thermometry and with background contributions to the heat capacity. The most reliable data seem to be those of Halperin *et al.*[5] along the melting curve. At low pressures no accurate data of the specific heat below 10 mK are available.

We report in this Letter measurements of the specific heat of liquid ^{3}He in the temperature range 0.8–20 mK at zero pressure. In order to be able to determine temperatures precisely we have developed a thermometer based on the magnetic susceptibility of cerous magnesium nitrate, diluted to 3% molar solution in the corresponding lanthanum salt (abbreviated as CLMN, cerium diluted in lanthanum magnesium nitrate).[9,10] In addition, by using a method of analysis based on the variation of the amount of liquid in the cell, we have been able to perform an accurate determination of the background heat capacity.

The experimental cell (Fig. 1), which was made of silver and equipped with two independent thermometers, CLMN and platinum NMR, was connected to the inner copper stage of a double-bundle nuclear refrigerator[11] via a superconducting tin heat switch. The volume for ^{3}He in the chamber is 17.3 cm^3 and the maximum amount of liquid in the fill line is 0.08 cm^3. The area of silver sinter, having an electron microscopically determined average grain size of 0.4 μm, is ~ 10 m^2. The heater was made of 6 m of fine insulated silver wire. The calculated background heat capacity of the cell body in zero magnetic field is negligible, because the indium seals used in the cell are assumed to be in the superconducting state; normal indium would contribute less than 3% of the full cell heat capacity at 1 mK. The ^{4}He content of our sample gas was measured to be about 10 ppm.

The CLMN thermometer was monitored by a SQUID susceptibility bridge. A useful empirical expression for the magnetometer output S_m is

$$S_m = A/(T - \Delta) + S_{ind}, \tag{1}$$

where S_{ind} is the temperature-independent part of the signal, and A and Δ are constants. These parameters were determined by calibrating the CLMN susceptibility against a standard platinum NMR thermometer in a field of 28 mT. We assume that the platinum nuclear spin susceptibility is inversely proportional to T. The constant of proportionality was found by measuring the spin-lattice relaxation time τ_1 of platinum and by using the Korringa relation $\tau_1 T = 29.9$ ms K. No deviation from this law has been observed with the platinum powder used in these measurements.[11] The calibration was done separately for each cooldown. Parameters A and S_{ind} were reproducible within 2.5%, whereas Δ seemed to depend on the magnetic field trapped by the niobium shield. With Earth's field compensated, the value of Δ was found to be -0.13 mK within 3%.

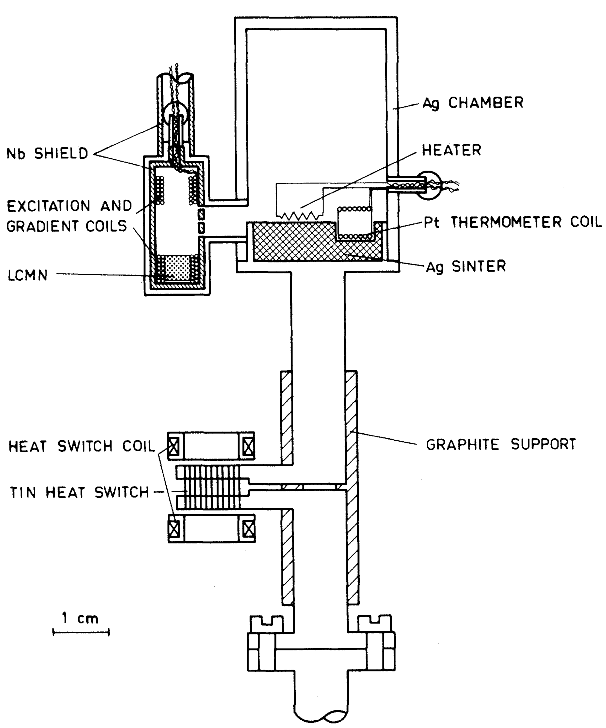

FIG. 1. Experimental cell.

For the superfluid transition temperature we obtained $T_c = 1.04$ mK with 1% reproducibility; the value of S_m at T_c was independent of the externally applied magnetic field at least up to 28 mT. The resolution of our CLMN thermometer is better than one part in 10^4 and the estimated absolute accuracy of the temperature scale is ± 5%.

The heat capacity was determined by applying a heat pulse ΔQ to the sample and by measuring the corresponding increase in temperature ΔT. We then calculated the heat capacity from $C = \Delta Q/\Delta T$, where the value of ΔQ was obtained from the measured heater current, the resistance value of the heater wire, determined separately with use of the four-lead method, and the duration of the pulse. The magnitude of ΔQ was selected so that typically $\Delta T/T$ ranged from 1 to 4%. We did not see any systematic changes in our results when considerably varying the pulse length or the heater current.

In a typical experimental run the heat leak corresponded to a warmup rate of less than 1 μK/min with the heat switch in the nonconducting state. T and ΔT due to a heat pulse were determined by extrapolating the initial and final warmup slopes to the middle of the pulse. Our accuracy in determining ΔT was better than 1%; the scatter of the data is of the same magnitude.

We found that the measured heat capacity at zero pressure has an unexpectedly large background contribution at $T \lesssim 5$ mK when compared with the range $T > 6$ mK, where $C/nR = \gamma T$, as predicted by Fermi-liquid theory. The background increases towards lower temperatures reaching at T_c a value more than 30% of the full cell heat capacity calculated from $C = nR\gamma T_c$.

In order to subtract accurately all calorimetric and nonbulk liquid contributions from heat capacity we made measurements with 15 different amounts of ^{3}He in the cell. The surface level of liquid ^{3}He was above the silver sinter in all runs. By plotting the measured heat capacities at a given temperature against the volume of ^{3}He one obtains a straight line, the slope of which gives the heat capacity per unit volume or the specific heat of ^{3}He; the intersection at zero volume is the nonscaling or the background contribution to the measured heat capacity. The analysis yields for this background the value 1.62 ± 0.05 mJ/K just above T_c and its temperature dependence can be described by an exponential relation $2.6 \exp[-T/(2.3\ \mathrm{mK})]$ mJ/K. A possible source for a background of this order of magnitude is the solidlike ^{3}He layers on the sinter surface. At T_c the background displays a discontinuity, a drop of ~ 20%, which can be ascribed to the suppression of superfluidity within a few coherence lengths, $\xi_0 \sim 150$ Å, at the sinter surface. The magnitude of the drop and the observed steeper temperature dependence below T_c seem to be quantitatively in agreement with this interpretation.

In Fig. 2 we have plotted C/nR and C/nRT versus temperature for two typical runs with the background subtracted, and measured at zero pressure and magnetic field. Below 3 mK we observe an anomalous contribution to the heat capacity, which will be discussed shortly. Above 3 mK, C is proportional to temperature: $C/nR = \gamma T$. The average value of γ for the fifteen runs is $\gamma = 2.11 \pm 0.02\ \mathrm{K}^{-1}$. Two of these runs were extended up to 20 mK and no change in γ was observed within the scatter of our data.

If the linear region corresponds to the $T \to 0$ limiting behavior of a Fermi liquid, then $\gamma = 2.11\ \mathrm{K}^{-1}$ yields for the effective-mass ratio m^*/m the value 2.12, and thus for the Fermi-liquid parameter F_1 the value 3.36. Our γ is 30% smaller than that reported earlier by Mota *et al.*[12] and by Abel *et al.*[13] at low pressures. Their values were determined by extrapolating to zero temperature from a region where C/nRT was found to vary as a function of temperature. However, around 20 mK, where a comparison between the experiments is possible, the values of C/nRT appear to differ by 23%.

In the measurements of Archie *et al.*[14] the stripped normal fluid density of ^{3}He was found to be pressure independent indicating that the observed strong-coupling effects were also independent of pressure in conflict with theoretical predictions. It is interesting to note that with the above F_1 the stripped normal-fluid density curve reduces at low pressures to that predicted by the BCS theory.

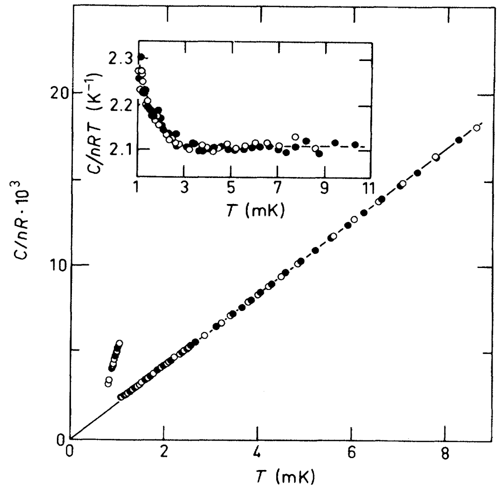

FIG. 2. Molar specific heat of liquid ^{3}He at zero pressure as a function of temperature. The solid line is a least squares fit to the linear region above 4 mK. The inset is a plot of C/nRT vs temperature. The data are from two typical runs.

The inset of Fig. 2 shows that C/nRT starts to increase rapidly at low temperatures, so that at T_c it is 9% higher than at $T > 3$ mK. Our precision in determining the background contribution is ± 0.05 mJ/K; the deviation from the $C/nR = \gamma T$ law, however, corresponds to a roughly 10 times larger effect. A nonlinearity in our temperature scale at $T < 4$ mK seems improbable as an explanation of this anomaly because the CLMN and platinum NMR thermometers were found to be linear against each other within the temperature range of our measurements.

Signs of anomalous heat capacity in ^{3}He experiments have also been reported earlier.[4] Similarly, the measurements of Parpia *et al.*[15] have indicated a deviation from the T^{-2} dependence of the viscosity below about 4 mK.

In our measurements we cannot eliminate a possible anomalous contribution induced by the cell walls above the sinter. The magnitude of the volume-independent background at T_c was found to be twice that of the excess in Fig. 2. Since the surface area of the sinter is three orders of magnitude larger than the area of the cell walls and the volume of liquid inside the sinter is roughly 2 cm^3, the characteristic length for any type of surface induced enhancement has to be a few hundred microns in order to explain the observed excess. However, an estimate[16] for the

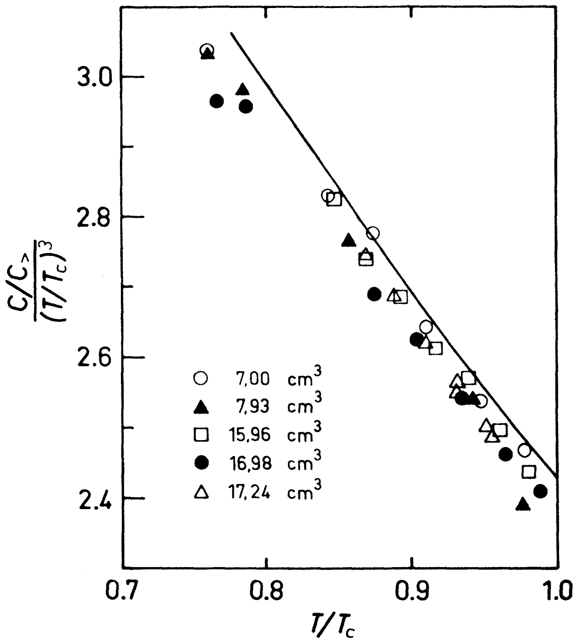

FIG. 3. The specific heat in the superfluid phase, normalized by $C_>$ and divided by $(T/T_c)^3$, is plotted vs T/T_c. The symbols correspond to various sample volumes.

longest plausible characteristic length, the quasiparticle mean free path, is almost an order of magnitude smaller, ~ 50 μm. If the increase in C/nRT still is interpreted as a surface effect, then the values of m^*/m and F_1 given above are the true low-temperature values of these parameters.

For the ratio of specific heats just below and just above T_c we obtain $C_</C_> = 2.39 \pm 0.02$, which is slightly less than 2.43, the BCS prediction for the Balian-Werthamer state. In Fig. 3 we have plotted $(C/C_>)/(T/T_c)^3$ against T/T_c below T_c $= 1.04$ mK. The temperature dependence of C seems to agree with the BCS result, although the quantitative discrepancy of ~ 2% continues in the plot towards lower temperatures. If the excess specific heat is not affected by the transition and is to be subtracted on both sides of T_c, we obtain 2.52 for the specific-heat ratio. For $C_</nR\gamma T_c$ we find 2.61 with use of $\gamma = 2.11$.

In conclusion, our measurements show that there is an anomalous contribution to the heat capacity of ^{3}He in the vicinity of the superfluid transition at zero pressure. Whether the phenomenon reflects a property of bulk liquid or some type of surface effect cannot be fully resolved at present. Therefore, it is not clear, how the excess should be treated when determining the specific-heat ratio and when comparing our results with the current theories.

Our results for C/nRT differ substantially from those obtained in earlier measurements and imply considerable changes in the generally accepted values of the Fermi-liquid parameters.

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