Evidence from Very Large Transverse Momenta of a Change with Temperature of Velocity of Sound in Hadronic Matter

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(Received 8 June 1978)

The most recent CERN intersecting storage-rings data on very large transverse momenta show that the velocity of sound in hadronic matter increases from $\simeq 1\sqrt{7}$ to $\simeq 1\sqrt{3.5}$, implying a phase transition from a strongly interacting (hadron) phase to a weakly interacting (parton?) phase.

Strong-interaction physics can hardly be understood without an adequate explanation for the specific features of the transverse-momentum distribution[1] $f(p_T)$ of secondaries produced in high-energy hadronic collisions, viz. (i) its exponential shape below $p_T \simeq 1$ GeV/c with a slope of ~ 6 $(\text{GeV}/c)^{-1}$ which is independent of the c.m. system energy $\sqrt{s}$ of the reaction; (ii) a striking deviation from exponentiality beyond 1 GeV/c; local logarithmic slopes show a significant increase with $\sqrt{s}$; (iii) resumption of exponential behavior beyond $p_T \simeq 5$ GeV/c with an (almost energy-independent) slope of 1.3 $(\text{GeV}/c)^{-1}$.[2]

Feature (i) could be explained so far only by thermodynamical[3-5] and hydrodynamical[6] models. For the more recently observed (ii) and (iii), various explanations have been suggested, which fall into two main classes, viz. (a) statistical[7,8] and hydrodynamical[9] models, (b) constituent models.[10] To date no single model describes all three properties.

It is the purpose of this paper to show that (A) a hydrodynamical model,[11] which also explains a variety of other effects,[12] can account for experimental facts, (i)–(iii), over the *whole* range of p_T; (B) there is evidence for an increase of the velocity of sound with temperature, which suggests the approach to a new noninteracting phase as implied by asymptotic freedom.

The hydrodynamical model (h.m.) of Landau contains as an essential ingredient Pomeranchuk's observation[3] that in hadron-hadron collisions the system is initially at such a high pressure that the mean free path of the created particles is much smaller than the dimensions of the system; thus no emission of particles can take place before the system has expanded and hence cooled down to a "decay" temperature $T_c \sim m_\pi$. This explains why the bulk of the particles have limited transverse momenta ($\langle p_T\rangle \simeq 0.3$ GeV/c). It is clear, however, that emission at $T > T_c$ cannot be absolutely forbidden and this must lead to leakage of particles from the excited system before expansion has ended and equilibrium has been reached. Such a process of preequilibrium emission is known to take place in nuclear physics and has been proposed by Weiner[13] in connection with large-p_T phenomena. In the hydrodynamical context this idea was stated in Ref. 9 and used explicitly by Gorenstein *et al.*[9] in an attempt to explain the behavior of f at large p_T. However, because of the approximations used, the formula for f derived by Gorenstein *et al.* applies only to large p_T and therefore it was not clear at all whether the h.m. can indeed predict $f(p_T)$ over the whole accessible range of p_T. Moreover, at that time data on very large p_T were not available and it is in this region, where new and important effects are now observed.

We use the one-dimensional solution[14] of the Khalatnikov equation for the relativistic hydrodynamical potential in order to derive the functional dependence of the temperature T on time t, viz., $T(t)$ given implicitly at $y=0$ by

$$t(T)_{y=0} = (d/2uw)\{\int_0^\tau \exp(-w\tau)I_0[(w-1)t']dt' + \exp(-w\tau)I_0[(w-1)]\}, \tag{1}$$

where d is the proton diameter, I_0 is the modified Bessel function, u is the velocity of sound assumed to be constant, $w=(1+u^2)/2u^2$, $\tau=\ln(T/T_0)$; T_0 is the initial temperature given (in units of m_π) by

$$T_0 = (\epsilon_0/\epsilon_\pi\lambda)^{1/2w}. \tag{2}$$

In Eq. (2), $\epsilon_0 = E^2 w m_\pi^3/\pi M_p$ is the energy density; $\epsilon_\pi = m_\pi/V_\pi$; E is the total available energy in the system in which target and projectile have equal and opposite velocities, V the normalization volume, m and M_p are the pion and proton rest masses, respectively; λ is an integration constant of the equation of state $u^2 = dp/d\epsilon$ as evaluated by Cooper *et al.*[15] Values for $T_c(u)$ have also been taken from this reference and approximated by a smooth function; T_c is the decay temperature, as a function of the sound velocity.

As in Ref. 9, we treat the emission process as a succession of local equilibrium states. The invariant cross section $f(p_T)$ reads[16]

$$f(p_T)_{y=0} \sim p_T^{-1} \int_0^{t_c} dt \int_0^R b\, db \Phi(p_T, T(t)), \qquad (3)$$

where

$$\Phi = p_T^2/\{\exp[(p_T^2 + m^2)^{1/2}/T(t)] - 1\}$$

and t_c is the "moment of decay" defined by $T(t_c) = T_c$; R is the target radius, and m the mass of the secondary. The integration over the impact parameter b in (3) is evaluated by observing that the only dependence on b is contained in T_0 via the available energy $E = K(b)\sqrt{s}$, where K is the inelasticity of the collision. (It is known[11] that not the entire c.m. energy is converted into secondaries.) Since K is known from experiment to be approximately uniformly distributed between 0 and 1, integration over b is equivalent to integration over K, which we approximate by fixing the integrand at the mean value of $K^{1/w}$.

We have applied the results of the model discussed above to the analysis of p-p collisions at the CERN intersecting storage rings (ISR).[2,17,18] Besides normalization the only free parameter is u. The results are represented in Figs. 1 and 2 and can be summarized as follows:

(1) From $p_T \simeq 0.1$ to $\simeq 5$ GeV/c the data are well described by our model with a value of u in the narrow range $(6.4)^{-1/2} - (6.8)^{-1/2}$; this range is compatible with values obtained for u from the h.m. when analyzing rapidity distributions in p-p and p-nucleus[12] collisions and is also in agreement with theoretical predictions.[19]

(2) The fits are rather sensitive to small (~5%) variations in u^2.

(3) In the hydrodynamical approach the "large-p_T" region (1–5 GeV/c) appears as a smooth and natural continuation of the "low-p_T" region, and only in the very large-p_T regime (>5 GeV/c) does something "new" appear.

(4) For $p_T \gg T_0$ (at $\sqrt{s} = 53$ GeV, this happens for $p_T > 5$ GeV/c). Equation (3) becomes essentially $f(p_T) \sim \exp(-p_T/T_0)$.

In the "very large-p_T" region (5–15 GeV/c) the (most recent) data deviate strongly from this asymptotic form as long as u remains unchanged ($\sim 1/\sqrt{6.8}$). (See Fig. 2.) However, they are remarkably well fitted by an exponential with a higher initial temperature T_0 ($\simeq 5m_\pi$ instead of $2m_\pi$) corresponding via Eq. (2) to $u \simeq 1/\sqrt{3.5}$. This can be interpreted as follows: *The sound velocity u is a* (step?) *function of temperature.* While the sharp increase of u with T means that in the transition region solution (1) is not applicable, the fact that beyond $p_T \simeq 5$ GeV/c Eq. (3) again reproduces the data albeit with another u implies

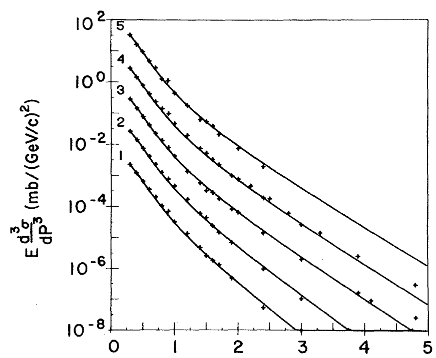

FIG. 1. Transverse momentum spectra at 90° (c.m. system) from the h.m. for $u^2 = 1/6.55$ for various values of $\sqrt{s}$ (GeV): curve 1, 23 ($\times 10^{-4}$); curve 2, 31 ($\times 10^{-3}$); curve 3, 45 ($\times 10^{-2}$); curve 4, 53 ($\times 10^{-1}$); curve 5, 63. Experimental data ($\pi^\pm$) from Ref. 18. Statistical errors are smaller than the size of data points.

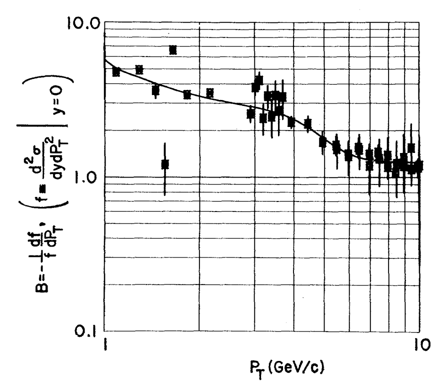

FIG. 2. Logarithmic slope of the transverse momentum spectra at 63 GeV. Data from Refs. 18 ($\pi^\pm$), and 2 and 19 (π^0). The curve is computed from the h.m. with $u^2 = 1/6.55$ with 0.1% of the particles "leaked" from a phase with $u^2 = 1/3.5$.

that solution (1) and hence hydrodynamics is valid again beyond $p_T \simeq 5$ GeV/c. However this region represents a different physical situation, probably a new phase, characterized by another value of u. We stress that the h.m. with only one free parameter, viz. u (which is already fixed to within a few percent of our fitted value by independent experimental facts, and whose numerical value can be understood theoretically[19]) gives a consistent description of the p_T spectra over the whole energy range and in the entire p_T range, for twelve orders of magnitude in cross section. Therefore we propose that the change of u with T is a genuine effect.

The increase of the velocity of sound with temperature has not been observed so far, because most of the data to which the h.m. had been applied are sensitive to the last (expansion and breakup) stage of the fireball evolution, characterized by a temperature range $m_\pi < T < 2m_\pi$, where the T dependence of u is not pronounced.[19] This is the case for the rapidity dependence of spectra of secondaries, and the p_T dependence below 5 GeV/c. An exception is the energy dependence of the total multiplicity for p-p collisions in the ISR range, which has been observed[20] to be consistent with $u^2 \simeq \frac{1}{4}$ rather than the "canonical" value of $\frac{1}{6}-\frac{1}{7}$. This exception can now be understood, since the total multiplicity is determined within the h.m. by the value of the entropy corresponding to the initial temperature T_0, the same initial T_0 responsible for the p_T events above 5 GeV/c (cf. conclusion 4), and therefore both effects are sensitive to the change of u. The sharp increase of u with T is in itself not a theoretically unexpected feature of strong-interaction physics. Zhirov and Shuryak[19] calculated u in a statistical-kinetical model for various hadronic spectra, and found that u^2 increases sharply with T from zero to $\simeq \frac{1}{6}$ in the region $0 < T < m_\pi$, and then remains almost constant. This would explain why a *single* "canonical" value of $u^2 \simeq \frac{1}{6}-\frac{1}{7}$ was sufficient to explain the data which are only sensitive to the temperature region $(m_\pi, 2m_\pi)$. On the other hand, the jump of u^2 from $\frac{1}{7}$ to 1/3.5 reported here represents a transition from a strongly interacting hadronic phase to a weakly interacting [quantum chromodynamic (QCD)] phase. It is known that for an ideal relativistic gas $u^2 = \frac{1}{3}$, while for a weakly interacting system such as an ensemble of quarks and gluons, one expects $\frac{1}{7} \ll u^2 < \frac{1}{3}$. At $T = 0$ and high densities this inequality can easily be derived from the results of Baym and Chin[21] (the actual value of u depends on the pressure). At $T \neq 0$ the calculations are more involved, but the technique is understood[22] and progress along these lines is expected soon. (The temperature should affect the value of u, but not the inequality.)

The existence of an upper bound in u^2 $(=\frac{1}{3})$, which follows from QCD, permits one to predict, within the h.m. as formulated in the present paper, a *nonvanishing* lower bound for $B = -(d/dp_T)(\ln E\, d^3\sigma/dp^3)_{y=0}$, in the limit $p_T \to \infty$, which is energy dependent. On the other hand, the value of $u^2 \simeq 1/3.5$ found at present energies (50–60 GeV) defines an upper bound for B (assuming u to be a monotonically increasing function of T). These bounds are reflected in a certain sense in the parton approach by the bounds on the p_T behavior of the parton-parton cross section (p_T^{-4} vs p_T^{-8}). A power behavior for p_T as predicted by parton models leads to a *vanishing* lower bound for B for $p_T \to \infty$. Thus the two approaches predict different asymptotic p_T behavior of B. We have compared the predictions of the h.m. with those of Field's[23] parton approach (Fig. 3). The two approaches appear to be compatible at present energies and might continue to be so at much higher energies ($\sqrt{s} = 1000$ GeV) as long as p_T is not too large. On the other hand, at very large p_T the predictions of the two models diverge. The onset of this domain depends on E and the details of $u(T)$.

It is relevant to consider the relation between the parton model, the h.m., and QCD. Both the parton model and the h.m. are phenomenological

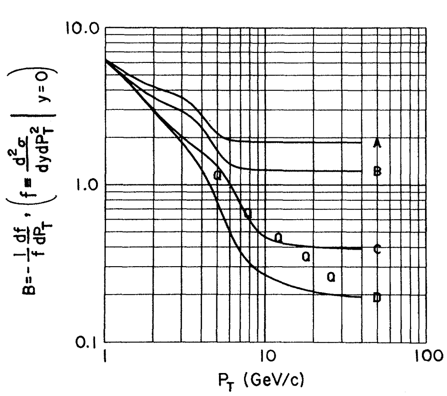

FIG. 3. Logarithmic slope of the transverse-momentum spectra: A and B fitted to experimental data at $\sqrt{s} = 23$ and 63 GeV, respectively; C, prediction for $\sqrt{s} = 1000$ GeV assuming the same parameters as at 63 GeV; D, same as C but assuming $u^2 = \frac{1}{3}$ in the weakly interacting phase. Points labeled Q are from Field's parton-model predictions (Ref. 23).

approaches which use as microscopic input some QCD ingredients. Although there can be overlap between the approaches the transition regime between the weakly and the strongly interacting phases lends itself naturally to the h.m., since it provides an adequate description of the low-p_T phenomena (strongly interacting regime), a domain which is not accessible to the parton models. Therefore these two approaches should be considered complementary, rather than competing, as has been stressed by Hwa[24] and Chiu and Wang.[24]

Phase transitions in QCD and hadronic matter[25] are a subject of considerable interest since it appears to be connected with one of the least understood aspects of QCD, confinement. Furthermore the existence and properties of quark matter have important implications for the early history of the universe and the structure of stars.[21,26]

We wish to thank F. Cooper for useful discussions. We are indebted to H. Schnitzer for a careful reading of the manuscript and for suggestions which made our message clearer. One of us (E.M.F.) is indebted to the University of Marburg and especially to R. Brandt for the kind hospitality extended to him in the initial stage of this work.

This research was supported in part by the Nuclear Physics Division of the U. S. Department of Energy, by the Gesellschaft für Schwerionenforschung, Darmstadt, Federal Republic of Germany, and by the Deutsche Forschungsgemeinschaft.

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[1] Here we consider invariant differential cross sections $f(p_T) = E\,d^3\sigma/dp^3 = (1/\pi)d^2\sigma/dy\,dp_T^2$ for pion secondaries measured at $y=0$ arising from p-p collisions. Predictions for $y \neq 0$ can be worked out along similar lines.

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