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## Disorder and Superconductivity in A-15 Compounds

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The universal depression of the superconducting transition temperature  $T_c$  in disordered 4-15 compounds is examined. Existing energy-band calculations are used to calculate the density of electron states, which is possibly enhanced by disorder in some cases such as  $Nb_3Ge$ . The dramatic drop in  $T_c$  in  $Nb_3Ge$  at a critical value of the resistivity is attributed to overdamping of acoustic plasmons which decreases the electron pairing interaction despite small changes in the density of states

Disorder induced by crystal growth conditions,  $\alpha$ -particle irradiation, and neutron damage yields a remarkable depression of the superconducting transition temperature  $T_c$  for the high-temperature superconducting  $A-15$  compounds Nb<sub>3</sub>Ge, V,Si, and similar materials. Testardi, Poate, and Levinstein<sup>1</sup> have pointed out the universal nature of the depression of  $T<sub>c</sub>$  by correlating it with the residual resistance ratio for various types of damage. <sup>A</sup> thorough investigation of neutron irradiation by Sweedler, Cox, and Moehlecke' demonstrated that the relative decrease of  $T_c$  in many  $A-15$  compounds followed a similar pattern as a function of the neutron flux. The measured transition temperatures of disordered  $V_s$ Si samples are plotted versus the residual resistance  $R(T = 25 \text{°K})$  in Fig. 1. The drop in  $T_c$  from 18 to 4'K is characteristic of the general trend for  $Nb<sub>3</sub>Ge$ ,  $Nb<sub>3</sub>Al$ , and similar materials. Also the



FIG. 1. Superconducting transition temperature of  $V<sub>3</sub>Si$  subjected to various types of damage as a function of residual resistivity. The universal drop in  $T_c$  is common to other  $A-15$  compounds, and is in sharp contrast to the behavior of Nb shown by the dotted curve.

persistence of low-temperature super conductivity  $(T_e \approx 4 \text{ K})$  at rather large resistance values is evident from Fig. 1.

By contrast, disorder appears to have very little influence on Nb samples which remain superconducting to  $T_c \approx 9$ °K over a wide range of residual resistance as shown in Fig.  $1<sup>1</sup>$  Further clues to the problem are provided by the disordered-enhanced superconducting temperature (from 1 to 5'K) in Mo,Ge which also exhibits the  $A-15$  structure.<sup>3</sup>

On the basis of the phonon-exchange mechanism of the superconducting electron pairing it is difficult to reconcile changes in the phonon spectrum with the dramatic variations in  $T_c$  induced by disorder. However, changes in the electron density of states may have a profound influence on the transition temperature according to the BCS weak-coupling formula

$$
T_c \simeq 0.7\theta_{\rm D} \exp[-1/(\lambda_{\rm ph} - \mu^*)], \qquad (1)
$$

where  $\theta_{\rm D}$  is the phonon Debye temperature,  $\mu^*$  $\simeq 0.1$  represents the Coulomb repulsion between electron pairs, and  $\lambda_{\rm ph}$  is the attractive pairing parameter originating from exchange of phonons. Since  $\lambda_{\rm ph} \simeq N(0)V^2$ , where  $N(0)$  is the electron density of states at the Fermi surface and  $V^2$  is an averaged electron-phonon matrix element, it is evident that changes in  $N(0)$  will significantly alter the superconducting properties.

Measurements of the specific heat  $C<sub>v</sub>$  and magnetic susceptibility of  $V<sub>3</sub>Si$  indeed suggest a reduction in the density of states with increased neutron damage, but a quantitative analysis gives discrepancies with the measured transition temperatures of this and other  $A-15$  compounds.<sup>5</sup>

The purpose of the present work is twofold. First we compute the density of states including disorder using band-structure results of augmented-plane-wave (APW) and pseudopotential methods. These calculations will show the general trends in  $T_c$  to be expected with disorder, and, in particular, will exhibit possible  $T_c$  enhancement with disorder. We then calculate the influence of the electron relaxation time  $\tau$  on an alternative pairing mechanism based on exchange of acoustic plasmons; the usual criterion for overdamped excitations will be shown to correspond to the universal drop in  $T<sub>c</sub>$  for disordered Nb<sub>3</sub>Ge and some other A-15 superconductors.

A primary and general consequence of disorder is to limit the electron relaxation time  $\tau$  and thereby increase the resistivity  $R$  by the Einstein relation

$$
R = (4\pi/\omega_{\rm pl}^2)\tau^{-1},\tag{2}
$$

where the plasma frequency  $\omega_{\rm pl}^2 = 4\pi n e^2/m^*$  may be extracted from optical data. Hence the electron damping  $\Gamma = \tau^{-1}$  will cause a broadening of structure in the density of states  $N(E)$  according to

$$
N(E, \Gamma) = \int S(E, E', \Gamma) N^{(0)}(E') dE', \qquad (3)
$$

where S is an appropriate broadening function. We consider a Lorentzian form

$$
S(E, E', \Gamma) = \pi^{-1} \Gamma / [(E - E')^{2} + \Gamma^{2}], \qquad (4)
$$

and impose the essential constraint of the conservation of total states. The latter point is crucial since a finite range of energies is considered, which is not necessarily much greater than  $\Gamma$  in the extremely disordered cases. Thus we compute  $N(E, \Gamma)$  from Eqs. (3) and (4) numerically by allowing a shift of the Fermi energy  $E_F'$  to preserve the total density.

We present here the analysis of Nb<sub>o</sub>Ge using the self -consistent APW band-structure calculathe sen-consistent AP w band-structure calculations of Klein *et al.*<sup>6</sup> and independently using the pseudopotential bands of Ho, Cohen, and Pickett. ' A final element of our calculation is the value of the plasma frequency  $\omega_{\rm pl} \approx 2.3$  eV obtained from optical data. ' Our results for the density of states including damping are shown in Fig. 2: They demonstrate the expected smearing of structure in  $N(E)$  for  $\Gamma \sim 0.05$  eV, but show relatively small changes in  $N(E_{\rm F})$  with disorder! The significance of these changes in the density of states is greatly amplified in the superconducting transition temperature  $T_c$  shown in Fig. 3. The  $T_c$ was evaluated from the McMillan equation which incorporates strong-coupling corrections to Eq. (1), and the values of the electron-phonon coupling parameter  $\lambda = V^2 N(E_F)$  were scaled to yield  $T_c = 23$ <sup>o</sup>K for the ordered Nb<sub>3</sub>Ge: This requires  $V^2$  = 0.13 eV for the pseudopotential results and  $V^2$  = 0.25 eV for the APW bands. Since the Fermi level lies near a peak in  $N(E)$  in the calculations of Ref. 7, the lower portion of the shaded region in Fig. 3 represents a *lower* bound on  $T_c$ , whereas the results of Ref. 6 indicate an  $\mu p e r$  bound on the  $T_c$  variation reflecting a Fermi energy near a valley in  $N(E)$ .

Clearly the results of the independent bandstructure calculations shown in Fig. 3 yield a much smaller variation in  $T_c$  than observed experimentally for  $Nb<sub>3</sub>Ge$ . The discrepancy be-



FIG. 2. Electron density of states for  $Nb<sub>3</sub>Ge$  from (a) pseudopotential calculations (Ref. 7) and (b) from APW band structures (Ref. 6). The solid curves are in the limit of infinite electron lifetime  $\Gamma = \tau^{-1} = 0$ . Structure is significantly broadened for  $\Gamma = 0.05$  eV. although  $N(E_F)$  is only slightly modified in both calculations even in the extreme disordered case of  $\Gamma = 0.2$ eV.

tween  $T_c$  variations and the disordered  $N(E)$  is considerably less for  $V_3Si$  as shown by the independent work of Testardi and Mattheiss using a similar analysis of APW band calculations.<sup>9</sup> However, the cases of  $Nb<sub>3</sub>Al$  and other A-15's resemble the Nb<sub>2</sub>Ge situation.<sup>5</sup>

To explain the anomalous depression of superconductivity in disordered  $Nb<sub>3</sub>Ge$ , we consider the additional pairing mechanism achieved by exchange of acoustic plasmons.<sup>10</sup> The acoustic branch represents  $d$ -electron oscillations screened by lighter-mass electrons. A priori, it is clear that disorder will strongly affect the electron damping and thus eliminate the plasmon mechanism at some critical resistivity, even though the phonons may persist as well-defined excitations.

Denoting the electron pairing parameters induced by phonons and plasmons, respectively, as



FIG. 3. Superconducting transition temperature for  $Nb<sub>3</sub>Ge$  as a function of resistivity is indicated by experimental points from Ref. 1. The shaded region indicated the expected decrease in  $T_c$  from variations in the density of states obtained from pseudopotential calculations (Ho, Cohen, and Pickett), and the predicted increase from the APW band structure (Klein et al.). The solid curve results from the damping of the acoustic plasmons which becomes critical at the value  $\rho_0 \approx 120 \mu\Omega$  cm.

 $\lambda_{ph}$  and  $\lambda_{pl}$ , the transition temperature may be expressed as<sup>10</sup>

$$
T_c \approx 0.7\theta_{\rm ph} \left(\frac{\theta_{\rm pl}}{\theta_{\rm ph}}\right) \frac{\lambda_{\rm pl}}{\lambda_T} \exp\left(-\frac{1+\lambda_T}{\lambda_T-\mu^*}\right),\tag{5}
$$

where  $\lambda_T = \lambda_{ph} + \lambda_{pl}$ ,  $\theta_{ph} \approx 300$  °K is the phonon<br>Debye energy, and  $\theta_{pl}$  represents the corresponding maximal acoustic plasmon energy. The Coulomb repulsion term is  $\mu^* \cong 0.1$ . Noting that the transition temperature persists at  $T_c \approx 5$  °K at large disorder for both  $V_3Si$  and  $Nb_3Ge$ , we take  $\lambda_{ph}$  = 0.45 to achieve this saturation in the regime of overdamped plasmons.

The plasmon damping  $\Gamma$  is related to the resistivity R by Eq.  $(2)$ . The relevant dimensionless parameter is  $x = \Gamma / \theta_{\text{pl}}$ , so that the acoustic plasmons cease to exist for  $x > 1$ . Analysis<sup>10</sup> of the band-structure results for  $Nb_3Ge$  yields  $\theta_{p1}$  $\approx 0.15$  eV, so that we expect the plasmon mechanism to become inoperative at  $R_c \approx 4 \pi \theta_{\text{pl}}/\omega_{\text{pl}}^2$  $\approx$  200  $\mu\Omega$  cm. This value is in good agreement with experiment, as shown in Fig. 3.

To make a realistic comparison over a wider range of resistivity, we consider a linear acoustic plasmon dispersion,  $\Omega = sq$ , where s is a "sound" velocity, and a matrix element  $g_2$  for the coupling of electrons to the acoustic plasmon mode. Treating the damping  $\Gamma$  as independent of

momentum and energy, we find the pairing parameter

$$
\lambda_{\rm pl} = \frac{N(0)g_2^2 \theta_{\rm pl}}{\Gamma(s k_{\rm F})^2} \times \left[ \frac{x}{4} \ln \left( 1 + \frac{4}{x^2} \right) + \tan^{-1} \left( \frac{2}{x} \right) - \frac{x}{2} \right],
$$
 (6)

where  $k_F$  is the Fermi momentum. Microscopic calculations of  $g_2$  are faced with difficulties and uncertainties similar to the evaluation of the electron-phonon coupling, and thus we adjust  $\lambda_{p1}$  (x =0) to fit  $T<sub>e</sub>$  at the lowest resistivity value. The resulting behavior of the calculated  $T_c$  from Eqs. (5) and (6) is shown by the solid curve in Fig. 3, in comparison to the data on Nb, Ge. At the highest point,  $T_c = 23 \text{ K}$ , we find  $\lambda_{\text{pl}} \approx 0.22$ , in comparison to the phonon contribution  $\lambda_{ph} = 0.45$ . We have included the additional Landau damping of the acoustic plasmons by decay into the screening electron continuum: This yields a corresponding width  $X_L = \Gamma_{\text{Landau}} / \theta_{\text{pl}} = 0.4$ .

In conclusion, the damping of acoustic plasmons explains the anomalous behavior of  $T_c$  in Nb, Ge even though the electron density of states remains unchanged with disorder. In view of detailed uncertainties in the band-structure calculations which limit our knowledge of the acoustic plasmon parameters, the agreement with existing experiments is quite reasonable.

Optical data on the disordered A-15 materials would provide independent tests of the electron damping as a function of resistivity. Preliminary data<sup>8</sup> on one disordered sample of  $V<sub>3</sub>Si$  gives a positive real part of the dielectric function which may be associated with overdamped plasmon modes. Our analysis of this data suggests a high value of resistivity  $(R \sim 400-700 \ \mu\Omega \text{ cm})$ . Nevertheless the sample is superconducting with  $T_c$  $\simeq 4$  K. It is interesting to note that the highly disordered  $V<sub>3</sub>Si$  samples exhibit resistivities<sup>1</sup> an order of magnitude greater than the room temperature value of  $R(300 \text{°K}) \approx 80 \mu \Omega$  cm for pure  $V_s$ Si. Thus the peculiar temperature-dependent resistivity is far from a saturation value, in agreement with the estimated long mean free paths of electrons in  $V_sSi.^8$ 

Finally, we comment on the prospects of achieving higher superconducting temperatures by shifting the Fermi energy to a favorable maximum in

 $N(E_F)$ . Alloying may reduce the electron lifetime and thereby reduce the acoustic plasmon mechanism. However, it may be of interest to change the electronic structure by ion implantation of hydrogen in compounds such as Nb, A1, with the goal of increasing  $N(E_{\rm F})$ , while minimizing disorder. Furthermore our results may be relevant to Nb, Si, whose band structure favors the acoustic plasmon formation, so that a high  $T_c$  would require rather well-ordered crystals with minimal defects and other electron scattering centers.

As an independent check on the analysis we have calculated the influence of compositional order on  $T<sub>c</sub>$  for a realistic three-dimensional band model  $T_c$  for a realistic three-dimensional band model<br>by extending the method of Labbé and Van Reuth.<sup>11</sup> These results support the present conclusions and will be published elsewhere.<sup>5</sup>

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