Spin-Dependent Forces in Heavy-Quark Systems

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For heavy $q\bar{q}$ bound states in quantum chromodynamics, spin-dependent forces are obtained in a manifestly gauge-invariant formalism. Classical spin-orbit and Thomas precession terms are expressed in general in terms of the static energy, and assuming that the confinement mechanism is electric determines uniquely the spin-dependent forces in terms of the nonrelativsitic potential. The Coulomb plus linear potential model gives P-state splittings correct to better than 25% and a mass for the η_c of 3.030 ± 0.020 GeV.

The fine structure in charmonium has been the focus of intense experimental[1] and theoretical[2] research during the past several years. In the nonrelativistic approximation the Coulomb plus linear potential provides the basis of a phenomenological model which correctly describes the gross features of both the charmonium and the upsilon-family spectra.[3] An understanding of their fine structure, however, would provide additional important insights into the nonrelativistic limit of the strong interactions. Here we use a manifestly gauge-invariant formalism to determine all spin-dependent forces (except those due to fermion-pair creation or annihilation) and then, assuming that the confinement mechanism is electric rather than magnetic, show that the spin-dependent forces in charmonium are determined by the nonrelativistic potential: No new parameters are introduced. The resulting triplet P-state masses agree with the current data within the accuracy of the hypotheses. Moreover, the η_c, the pseudoscalar partner of the J/ψ, is predicted to have a mass of 3.030 ± 0.020 GeV, 65 MeV below the J/ψ.

The static energy (or potential), $\epsilon(R)$, of an infinitely massive fermion-antifermion pair is expressed in terms of a Wilson loop integral by

$$\epsilon(R) = \lim_{T\to\infty} \left[-T^{-1}\ln\left(\mathrm{Tr}\,\langle 0|P\{\exp[ig\oint_C A^\mu(z)\,dz_\mu]\}|0\rangle\right)\right], \tag{1}$$

where $A^\mu(z) \equiv A_a{}^\mu(z)t^a$ is the Yang-Mills potential and t^a the fermion representation matrices which will be suppressed. C is a closed rectangular path of spatial length R and temporal length T. The path ordering, indicated by P, is necessary because the fields A^μ do not commute with each other, and the trace is over group indices. Equation (1) is to be evaluated in Euclidean space. The sections of the path C corresponding to the time integrations arise from the nonrelativistic fermion propagators while the spatial parts ensure gauge invariance. Furthermore, the relativistic spin-dependent corrections to the static energy can be included by retaining corrections to the nonrelativistic limit of the fermion propagation function.

The relativistic corrections to the fermion propagation function are found by iteration of the following integral equation for $S(x,y;A)$, the full fermion propagation function:

$$S(x,y;A) = S_0(x,y;A) + \int d^4z\; S_0(x,z;A)\,\vec{\gamma}\cdot\vec{\mathrm{D}}(z)S(z,y;A) \tag{2a}$$

$\vec{\mathrm{D}} = i\nabla + g\vec{\mathrm{A}}$ is the covariant derivative and $S_0(x,y;A)$ is the nonrelativistic propagator given by

$$iS_0(x,y;A) = \theta(x^0-y^0)\frac{1+\gamma^0}{2}\exp[-im(x^0-y^0)]\,P[\exp(ig\int_{y^0}^{x^0} dz\,A^0(\vec{\mathrm{x}},z))]\,\delta(\vec{\mathrm{x}}-\vec{\mathrm{y}})$$
$$+\theta(y^0-x^0)\frac{1-\gamma^0}{2}\exp[im(x^0-y^0)]\,P[\exp(ig\int_{x^0}^{y^0} dz\,A^0(\vec{\mathrm{x}},z))]\delta(\vec{\mathrm{x}}-\vec{\mathrm{y}}), \tag{2b}$$

where P denotes path ordering. Also, it is useful to project onto the positive- and negative-frequency components of the fermion propagator:

$$S^{++} \equiv \frac{1+\gamma^0}{2} S \frac{1+\gamma^0}{2}, \quad S^{+-} \equiv \frac{1+\gamma^0}{2} S \frac{1-\gamma^0}{2},$$

etc. Introducing these projectors into Eq. (2a), and eliminating the mixed components S^{-+} and S^{+-}, give

$$S^{++}(x,y;A) = S_0^{++}(x,y;A) + \int d^4z\, d^4w\, S_0^{++}(x,z;A)\vec{\gamma}\cdot\vec{\mathrm{D}}(z) S_0^{--}(z,w;A)\vec{\gamma}\cdot\vec{\mathrm{D}}(w) S^{++}(w,y;A) \tag{3}$$

and a similar equation for $S^{--}(x,y;A)$. The first iteration of this equation describes a relativistic correction in which the fermion propagates forward in time from x to z. At z it suffers a $\vec{\gamma}\cdot\vec{\mathrm{D}}$ interaction and then propagates backwards in time to w where the fermion again has a $\vec{\gamma}\cdot\vec{\mathrm{D}}$ interaction and is restored to its original state of motion. This expansion is not perturbative in the coupling constant, but in $1/m$. We perform the time integrals in Eq. (3) by observing that the region between z^0 and w^0, when the fermion travels backwards in time, should be suppressed nonrelativistically. Therefore, expanding the integral in powers of (z^0-w^0), a $1/m$ expansion, and keeping only the leading terms produce

$$\left\{1 + \frac{1}{4m^2}[\vec{\mathrm{D}}^2(x) - g\vec{\sigma}\cdot\vec{\mathrm{B}}(x)]\right\} S^{++}(x,y;A) = S_0^{++}(x,y;A) + \int d^4w\, S_0^{++}(x,w;A)\Big[-\frac{1}{2m}[\vec{\mathrm{D}}^2(w) - g\vec{\sigma}\cdot\vec{\mathrm{B}}(w)]$$
$$+ \frac{g}{4m^2}(\delta_{ij} - i\epsilon_{ijk}\sigma^k)E^i(w)D^j(w)\Big] S^{++}(w,y;A), \tag{4}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + g[A^\mu, A^\nu]$ and the gauge electric and magnetic fields are $E^i = F^{0i}$ and $B^i = \frac{1}{2}\epsilon^{ijk}F^{jk}$. Equation (4) succinctly characterizes all relativistic propagation corrections to order $(1/m)^2$ in a manifestly gauge-invariant form.

To incorporate the spin corrections to the static energy[4] we include the corrections to the propagator S_0 determined by Eq. (4) and express the result in the form of Eq. (1). To simplify the notation we introduce the following expectation value (on the Wilson loop) of any operator $O(x)$ where $x \in C$:

$$\langle O(x)\rangle \equiv \int [dA^\mu]\,\mathrm{Tr}P\{\exp[ig\int_c dz^\mu A_\mu(z)]\,O(x)\}e^{-S(A)}, \tag{5}$$

where $S(A)$ is the classical Yang-Mills action. In this notation

$$\epsilon(R) = \lim_{T\to\infty}\left(-\frac{1}{T}\ln\langle 1\rangle\right). \tag{6}$$

The $1/m$ corrections involve the expectation values $\langle\vec{\mathrm{D}}^2/2m\rangle$ which is spin independent and $(\vec{\sigma}/2m)\cdot\langle\vec{\mathrm{B}}\rangle$ which vanishes because of parity conservation.

The $(1/m)^2$ spin-dependent corrections to $\epsilon(R)$ come from the explicit $(1/m)^2$ term in Eq. (4) as well as iterations of the $1/m$ contributions. Considerable simplification of the expectation values of these operators is found by the use of the following identities:

$$P(x^0, y^0; \vec{\mathrm{x}})P(y^0, z^0; \vec{\mathrm{x}}) = P(x^0, z^0; \vec{\mathrm{x}}), \tag{7a}$$

$$\vec{\mathrm{D}}(x^0,\vec{\mathrm{x}})P(x^0,y^0;\vec{\mathrm{x}}) = P(x^0,y^0;\vec{\mathrm{x}})\vec{\mathrm{D}}(y^0,\vec{\mathrm{x}}) + \int_{y^0}^{x^0} dz^0\, P(x^0,z^0;\vec{\mathrm{x}})E(z^0,\vec{\mathrm{x}})P(z^0,y^0,\vec{\mathrm{x}}), \tag{7b}$$

where $P(x^0, y^0; \vec{\mathrm{x}}) \equiv P\exp[ig\int_{y^0}^{x^0} dz^0 A^0(z^0,\vec{\mathrm{x}})]$. These identities relate the electric field and covariant derivatives to each other. Also, the covariant and regular derivatives are related by

$$P\exp[ig\int_{\vec{\mathrm{y}}}^{\vec{\mathrm{x}}} d\vec{\mathrm{z}}\cdot\vec{\mathrm{A}}(x^0,\vec{\mathrm{z}})]\vec{\mathrm{D}}(x^0,\vec{\mathrm{x}}) \xrightarrow[x^0\to\pm\infty]{} i\vec{\partial}^x P\exp[ig\int_{\vec{\mathrm{y}}}^{\vec{\mathrm{x}}} d\vec{\mathrm{z}}\cdot\vec{\mathrm{A}}(x^0,\vec{\mathrm{z}})]. \tag{7c}$$

The final form for the spin-dependent potential (after several integrations by parts) is $V_{\mathrm{SD}} \equiv V_a + V_b$, where

$$V_a\delta(\vec{\mathrm{x}}_1-\vec{\mathrm{y}}_1)\delta(\vec{\mathrm{x}}_2-\vec{\mathrm{y}}_2) = \lim_{T\to\infty}\left[-\frac{1}{T}\frac{1}{\langle 1\rangle}\epsilon_{ijk}\left(\frac{i\sigma_1^i}{4m_1^2}\partial_k^{x_1}\langle 1\rangle\partial_j^{x_1} + \frac{i\sigma_2^i}{4m_2^2}\partial_k^{x_2}\langle 1\rangle\partial_j^{x_2}\right)\delta(\vec{\mathrm{x}}_1-\vec{\mathrm{y}}_1)\delta(\vec{\mathrm{x}}_2-\vec{\mathrm{y}}_2)\right] \tag{8}$$

and

$$V_b\,\delta(\vec{x}_1-\vec{y}_1)\,\delta(\vec{x}_2-\vec{y}_2)=\lim_{T\to\infty}\left(-\frac{1}{T}\,\frac{1}{\langle 1\rangle}\left\{\left[-\frac{g^2}{2m_1^{\,2}}\int_{x_1{}^0}^{y_1{}^0}dz\,dz'\,\sigma_1{}^i(z'-z)\langle B^i(z,\vec{x}_1)E^j(z',\vec{x}_1)\rangle\,\partial_j{}^{x_1}\right.\right.\right.$$
$$\left.-\frac{g^2}{2m_1m_2}\int_{x_1{}^0}^{y_1{}^0}dz\int_{y_2{}^0}^{x_2{}^0}dz'\,\sigma_1{}^i z'\langle B^i(z,\vec{x}_1)E^j(z',\vec{x}_2)\rangle\,\partial_j{}^{x_2}\right]$$
$$\left.+[1\longleftrightarrow 2]-\frac{g^2}{4m_1m_2}\int_{x_1{}^0}^{y_1{}^0}dz\int_{y_2{}^0}^{x_2{}^0}dz'\,\sigma_1{}^i\sigma_2{}^j\langle B^i(z,\vec{x}_1)B^j(z',\vec{x}_2)\rangle\right\}$$
$$\left.\times\,\delta(\vec{x}_1-\vec{y}_1)\,\delta(\vec{x}_2-\vec{y}_2)\right),\qquad(9)$$

where 1 refers to the fermion and 2 to the antifermion.

The expression for V_a can be explicitly evaluated with use of Eq. (6). The result is

$$V_a=\left(\frac{\vec{\sigma}_1\circ\vec{L}_1}{4m_1^{\,2}}-\frac{\vec{\sigma}_2\circ\vec{L}_2}{4m_2^{\,2}}\right)\frac{1}{R}\,\frac{d\epsilon(R)}{dR}\,,\qquad(10)$$

where the angular momentum $\vec{L}_n=-i\vec{R}\times\nabla_n$. Note that for a bound state with its center of mass at rest $\vec{L}_1=-\vec{L}_2\equiv\vec{L}$ and that V_a, part of the spin-orbit coupling, includes the classical spin-orbit term and the Thomas precession effect. Equation (10) follows solely from the vector nature of the underlying gauge interactions of the fermions.

Equations (9) and (10) together describe the spin-dependent forces in quantum chromodynamics. The terms in V_b, unlike V_a, cannot be reexpressed in terms of ϵ alone. The expression $t\vec{E}\cdot\nabla$ in V_b arises from the kinetic energy $\vec{D}^2$ and, appropriately, is related to the generator of boosts. The second term in V_b corresponds to the $1/m_1m_2$ spin-orbit forces; and spin-spin and tensor forces are contained in the last term. In the Abelian limit, Eqs. (9) and (10) reproduce the standard Breit interaction in lowest-order pertrubation theory. The first term in V_b does not contribute in this limit.

To obtain phenomenological consequences of Eqs. (9) and (10) we assume that the confinement mechanism is electric rather than magnetic.[5] Then the electric field alone is responsible for the long-range part of the potential whereas the magnetic field effects should be short range, calculable in perturbation theory. Explicitly V_b is calculated using lowest-order perturbation theory since it contains the magnetic field and only the term V_a contains the effects of the long-range interaction.[6] The resulting expression for the spin-dependent potential[7] is (with $\vec{S}_i=\frac{1}{2}\vec{\sigma}_i$)

$$V_{\rm SD}(R)=\left(\frac{1}{2m_1^{\,2}}\,\vec{L}\cdot S_1+\frac{1}{2m_2^{\,2}}\,\vec{L}\cdot\vec{S}_2\right)\frac{1}{R}\,\frac{d\epsilon(R)}{dR}$$
$$+\frac{4}{3}\,\frac{\alpha_s}{m_1m_2}\,\vec{L}\cdot(\vec{S}_1+\vec{S}_2)\frac{1}{R^3}+\frac{32\pi}{9m_1m_2}\alpha_s\vec{S}_1\cdot\vec{S}_2\delta(\vec{R})+\frac{4\alpha_s}{3m_1m_2}(3\vec{S}_1\cdot\hat{R}\,\vec{S}_2\cdot\hat{R}-\vec{S}_1\cdot\vec{S}_2)\,\frac{1}{R^3}\,.\qquad(11a)$$

For the static energy we use the form

$$\epsilon(R)=-\tfrac{4}{3}\alpha_s/R+R/a^2,$$

with parameters[3] $\alpha_s=0.39$, $a=2.34\ \text{GeV}^{-1}$, and charmed-quark mass $m_c=1.84$ GeV. Then (with $\vec{S}=\vec{S}_1+\vec{S}_2$)

$$V_{\rm SD}(R)=\frac{1}{2m_c^{\,2}}\,\vec{S}\cdot\vec{L}\left\{\frac{4\alpha_s}{R^3}+\frac{1}{Ra^2}\right\}+\frac{32\pi}{9m_c^{\,2}}\,\vec{S}_1\circ\vec{S}_2\delta(\vec{R})+(3\vec{S}_1\cdot\hat{R}\,\vec{S}_2\cdot\hat{R}-\vec{S}_1\cdot\vec{S}_2)\frac{4\alpha_s}{3m_c^{\,2}R^3}\,.\qquad(11b)$$

In Eq. (11b) the spin-dependent splittings are determined uniquely since the results depend only on the nonrelativistic potential and perturbation theory. The P-state splittings for charmonium predicted by Eq. (11b) are given in Fig. 1 and agree with the experimental values[8] to within 25%. Neglected contributions from higher-order perturbative terms and higher-order relativistic corrections can easily account for effects of this order of magnitude. Note that neither pure $\vec{L}\cdot\vec{S}$ nor pure Coulombic terms can account for the magnitude of the experimental splittings. The ratio of splittings $r\equiv[m(P_1)-m(P_0)]/[m(P_2)-m(P_0)]\sim\frac{5}{8}$ experimentally, where $r_{\vec{L}\cdot\vec{S}}=\frac{1}{3}$ and $r_{\rm Coul}=\frac{5}{9}$. Our model gives $r\sim\frac{1}{2}$.

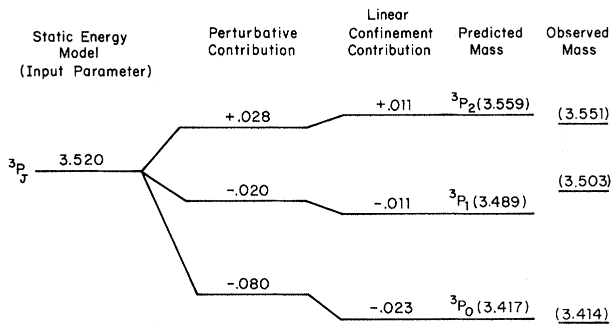

FIG. 1. Fine structure of 1^3P_J states in charmonium. The splittings from perturbative electric and magnetic interactions are in the second column, and the additional splittings due to the nonperturbative spin-orbit contribution are in the third column. All energies are in GeV.

The spin-dependent potential of Eqs. (11a) and (11b) has many other applications to the phenomenology of the J/Ψ and Υ family of resonances.[9] Of particular interest is the mass of the $^1S_0$ state, η_c, the spin-0 partner of the J/Ψ. From Eq. (11b) and the wave function computed with use of the parameters of the Cornell model,[10]

$$m(\eta_c) = 3.030 \pm 0.020 \text{ GeV}.$$

The error is the approximate magnitude of the neglected theoretical contributions. Confirmation of the existence of the η_c close to the J/ψ would provide strong supporting evidence for the assumption of short-range magnetic and long-range electric gauge interactions between the fermions postulated here.

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Note added.—Recent evidence [E. Bloom, in Proceedings of International Symposium on Lepton and Photon Interactions at High Energies, Batavia, Illinois, 26–31 August, 1979 (to be published)] indicates the existence of a narrow state, seen in the inclusive photon spectrum of ψ', with a mass of 2.976 ± 0.020 GeV and an exclusive photon branching ratio from ψ' of $\sim 0.5\%$. If this state is the η_c, higher-order perturbative corrections to the J/ψ-η_c mass splitting are substantially different than assumed in Ref. 10. The corrections are discussed in detail in Ref. 9.

[1]For a general experimental review see, e.g., G. J. Feldman and M. L. Perl, Phys. Rep. 33C, 285 (1977).

[2]See, e.g., T. Appelquist, R. M. Barnett, and K. D. Lane, Annu. Rev. Nucl. Part. Sci 28, 387 (1978).

[3]E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T.-M. Yan, Cornell University Report No. CLNS-425, 1979 (to be published).

[4]There are other approaches for computing spin-dependent corrections. The formulations of C. G. Callan, R. Dashen, D. J. Gross, F. Wilczek, and A. Zee [Phys. Rev. D 18, 4684 (1978)], and N. Parsons and P. Senjanović [Phys. Lett. 79B, 273 (1978)] are the most similar to the method presented here.

[5]The dominance of the electric field contributions to the potential is suggested even in perturbation theory. See S. Davis and F. Feinberg, Phys. Lett. 78B, 90 (1978).

[6]This result differs from the treatment of spin-dependent forces within the Massachusetts Institute of Technology bag model although the magnetic forces are also short range (perturbative). There the only long-range term is the Thomas term. K. Johnson, private communication.

[7]Comparing this form for the spin-dependent potential in QCD to the phenomenological parametrization of previous models [see Ref. 2, Eqs. (3.36) and (3.37 a)], we would conclude that this potential arises from a mixture of a scalar and vector confining potentials (scalar fraction $\eta = \frac{1}{2}$). Actually the form follows directly from the vector nature of the underlying gauge interaction.

[8]The P-state masses quoted here are the results of the Mark-I detector group at SPEAR: W. Tanenbaum *et al.*, Phys. Rev. D 17, 1731 (1978).

[9]A more detailed analysis of the phenomenological implications of the form of the spin-dependent potential and a treatment of instanton effects is discussed in E. Eichten and F. Feinberg, to be published.

[10]There is a large correction to the Van Royen–Weisskopf relation between the wave function at the origin and the leptonic width in the next order of perturbation theory (see Ref. 3 and the sources cited therein); although the perturbative corrections in the J/ψ-η_c splitting are in general different, the same large correction appears in the relation between the wave function at the origin and J/ψ-η_c splitting. This large correction has been included in the quoted mass.