PHYSICAL REVIEW LETTERS

VOLUME 43 22 OCTOBER 1979 NUMBER 17

Photocoupling of the Roper Resonances in a Relativistic Quark Model with a Scalar Confining Potential

Kohichi Ohta
Institute of Physics, University of Tokyo, Komaba, Meguro-ku, Tokyo 153, Japan
(Received 22 March 1979)

The Roper resonance photoexcitation amplitude is calculated by use of a quark model with a Lorentz scalar confining potential perturbed by a Coulomb-type vector potential. All the necessary complications due to SU(6) admixture, relativistic effects, and non–single-quark-transition interactions are considered. The result is found to be in good agreement with experiment.

It has long been recognized that although the success of the nonrelativistic harmonic-oscillator quark model is remarkable in predicting the nucleon resonance photoexcitation amplitudes, there is a serious discrepancy in the sign of the photocoupling of $P_{11}(1470)$, the Roper resonance.[1] The four-dimensional oscillator model of Feynman, Kislinger, and Ravndal[2] also yields the opposite sign to experiment. The algebraic approach[3] of the Melosh transformation[4] successfully predicts the ratio $A_{1/2}{}^p/A_{1/2}{}^n = -\frac{3}{2}$, but it fails to get $A_{1/2}{}^p$ or $A_{1/2}{}^n$ independently since it requires $A_{1/2}{}^p$ as an input to determine a single reduced matrix element for the $[\underline{56}, 0^+]_2$ multiplet. In order to calculate the reduced matrix element itself, we must use some explicit quark model.

On the other hand, the study of hadron spectroscopy motivated by quantum chromodynamics has developed the earlier naive model into a modern variant which emphasizes valence-quark dynamics. This gluon-perturbed quark-binding model[5] has become a powerful tool for ordinary meson[6] and baryon,[7] as well as charmonium, spectroscopy. It is therefore desirable to reconsider the problem of the Roper resonance in the light of recent researches. The purpose of this Letter is to show that the difficulty about the photocoupling of the Roper resonance can be cleared up within the framework of a relativistic quark model with an effective scalar confinement perturbed by one-gluon exchange.

For a Lorentz scalar binding potential, together with a Coulomb-type vector potential, three quarks with mass m are described by the effective Hamiltonian

$$H = H_{\rm NR} + H_{\rm RC}, \tag{1}$$

where $H_{\rm NR}$ is the nonrelativistic Hamiltonian

$$H_{\rm NR} = \sum_{i=1}^{3} \vec{p}_i^{\,2}/2m + \sum_{i<j} V_{ij}, \tag{2}$$

with $\vec{p}_i$ ($i = 1, 2, 3$) for the quark momentum operator. The static potential V_{ij} depends only on the separation between a pair of quarks, $r_{ij} = |\vec{r}_i - \vec{r}_j|$, and consists of the long-range potential v_{ij} and the Coulomb potential $-\frac{2}{3}\alpha_s/r_{ij}$. The nonrelativistic quark model is concerned only with $O(m^{-1})$ terms but the Lorentz character of the potential manifests itself in $O(m^{-3})$ terms which are denoted by $H_{\rm RC}$ in Eq. (1). It is the sum of spin-orbit, spin-spin, tensor, and spin-independent terms. The spin-spin and tensor interactions, H_{SS} and H_T, come only from the one-gluon–exchange potential.

It is well known that if we express (1) in terms of the internal position and momentum operators $\vec{\rho}_i = \vec{r}_i - \vec{R}$ and $\vec{\pi}_i = \vec{p}_i - \frac{1}{3}\vec{P}$, the total momentum $\vec{P}$ conjugate to the c.m. position $\vec{R}$ cannot be separated from the internal motion. A way out of this difficulty is to make a unitary transformation[8] $H \to H' = e^{i\varphi} H e^{-i\varphi}$, with

$$\varphi = -\frac{1}{12m}\sum_{i=1}^{3}\left[\vec{p}_i\cdot\vec{P}, \frac{1}{m}\vec{\pi}_i\cdot\vec{p}_i + \sum_{j\neq i} V_{ij}\right] + \frac{1}{12m^2}\sum_{i=1}^{3}\vec{\sigma}_i\times\vec{\pi}_i\cdot\vec{P}. \qquad (3)$$

The transformed H' is then cast in the form $[\vec{P}^2 + (3m+h)^2]^{1/2}$ and has eigenstates of the simple form $e^{i\vec{P}\cdot\vec{R}}|n\rangle$. The above procedure seems in effect unnecessary for the calculation of rest-frame wave functions $|n\rangle$, but it should be noted that this simplification results from the unitary transformation of H which modifies all other operators as well.

Decomposing the rest-energy operator h into the nonrelativistic internal Hamiltonian h_{NR} and the relativistic correction h_{RC}, I first consider h_{NR}. Following Isgur and Karl,[9] I write the potential term of h_{NR} in the form

$$V_{ij} = \tfrac{1}{6} m\omega^2 r_{ij}^2 + U(r_{ij}). \qquad (4)$$

They show that for any unknown deviation U from the harmonic-oscillator potential, the first-order perturbation theory suffices for a satisfactory description of the low-lying baryon states including the Roper resonance. From this, as they argue, the oscillator wave functions remain an adequate approximation to the true wave functions.

Considering their success, we can assign the Roper resonance to the $[\underline{56}, 0^+]_2$ multiplet, a radial excitation of the ground state $[\underline{56}, 0^+]_0$ in the harmonic-oscillator spectrum.

In the presence of the relativistic correction h_{RC}, the Roper resonance is mixed. The spin-independent interactions in h_{RC} only cause interband mixings which, as with Isgur and Karl,[9] are neglected in the present Letter. There is a controversy[9,10] about the spin-orbit interaction, but since it does not affect the composition of the Roper resonance, H_{SS} and H_T alone are involved for mixing. The result of Isgur and Karl[9] is

$$|P_{11}(1470)\rangle = 0.99|\underline{56}, 0^+\rangle + 0.17|\underline{70}, 0^+\rangle \qquad (5)$$

with negligibly small admixture of other multiplets.

Now that the wave function of the Roper resonance is given, I turn to the calculation of the photocouplings. The electromagnetic interaction of composite systems bound by a scalar or vector potential was essentially given by Faustov.[11] One may also refer to Ref. 12. The results of their work can be derived from a slightly different viewpoint which follows. The interaction of quarks with a vector potential $\vec{A}$ consists of two terms

$$H_{\rm em} = H_{\rm em}^{\rm I} + H_{\rm em}^{\rm II}. \qquad (6)$$

If quarks are free, photocouplings of baryons are described by a single-quark transition operator $H_{\rm em}^{\rm I}$. In the nonrelativistic quark model, we use the $O(m^{-1})$ term[13]

$$H_{\rm em}^{(1)} = -\sum_{i=1}^{3}\left[\frac{e_i}{2m}\left\{\vec{p}_i, \vec{A}(\vec{r}_i)\right\} + \mu_i\vec{\sigma}_i\cdot\vec{H}(\vec{r}_i)\right] \qquad (7)$$

in the m^{-1} expansion of the quark-photon interaction. Here e_i is the quark charge, $\mu_i = (1+\kappa)e_i/2m$, κ is the anomalous magnetic moment of the quark, and $\vec{H}$ is the magnetic field. As we are considering relativistic corrections to $O(m^{-3})$, I add to (7) the $O(m^{-2})$ (Ref. 14) and $O(m^{-3})$ Foldy-Wouthuysen (FW)[15] interactions

$$H_{\rm em}^{(2)} = \frac{1}{8m^2}(1+2\kappa)\sum_{i=1}^{3} e_i\{\vec{\sigma}_i\times\vec{p}_i, \vec{E}(\vec{r}_i)\}, \qquad (8)$$

$$H_{\rm em}^{(3)} = \frac{1}{8m^3}\sum_{i=1}^{3} e_i\{\vec{p}_i^{\,2}, \vec{\sigma}_i\cdot\vec{H}(\vec{r}_i)\} + \frac{\kappa}{8m^3}\sum_{i=1}^{3} e_i\{\vec{\sigma}_i\cdot\vec{p}_i, \vec{p}_i\cdot\vec{H}(\vec{r}_i)\}. \qquad (9)$$

In the above, $\vec{E}$ is the electric field and I have omitted terms which have no effect on the Roper excitation.

For the bound system, the electromagnetic interactions must appear in such a way as to leave the equations of motion gauge invariant. The non–single-quark transition operator $H_{\rm em}^{\rm II}$ arises from the momentum dependence of the quark-quark interactions through the minimal substitution of $\vec{p}_i$. From the spin-orbit force in $H_{\rm RC}$ we obtain

$$H_{\rm em}^{V} = -\frac{\alpha_s}{6m^2}\sum_{i\neq j} e_i\vec{r}_{ij}\times\vec{A}(\vec{r}_i)\cdot(\vec{\sigma}_i + 2\vec{\sigma}_j)\frac{1}{r_{ij}^3} + \frac{1}{4m^2}\sum_{i\neq j} e_i\vec{r}_{ij}\times\vec{A}(\vec{r}_i)\cdot\vec{\sigma}_i\frac{v_{ij}'}{r_{ij}}. \qquad (10)$$

Of all others in $H_{\rm RC}$, only the term $-(1/8m^2)$

$\times\sum_{i\neq j}\{\vec{p}_i^2, v_{ij}\}$ generates a spin-dependent electromagnetic interaction. Interpreting $\vec{p}_i^2$ as $(\vec{\sigma}_i\cdot\vec{p}_i)^2$, we are to replace $\vec{\sigma}_i\cdot\vec{p}_i$ by $\vec{\sigma}_i\cdot[\vec{p}_i - e_i\vec{A}(\vec{r}_i)]$ to get

$$H_{\rm em}{}^G = \frac{1}{2m^2}\sum_{i\neq j} e_i\vec{\sigma}_i\cdot\vec{H}(r_i)v_{ij}. \qquad (11)$$

Evidently, the total Hamiltonian $H + H_{\rm em}$ becomes gauge invariant. The interactions $H_{\rm em}{}^V$ and $H_{\rm em}{}^G$ are the common ingredients of Refs. 11 and 12, though the derivation is different. Finally, in order to compute current matrix elements between eigenstates of H' rather than H, we must use the interaction $e^{i\varphi}H_{\rm em}e^{-i\varphi}$, i.e., we must add to $H_{\rm em}$ an interaction of $O(m^{-3})$

$$H_{\rm em}{}^{\Delta\rm FW} = i[\varphi, H_{\rm em}{}^{(1)}]. \qquad (12)$$

As U is treated as a perturbation, we may put $U=0$ in the $O(m^{-3})$ interactions (10)–(12) as a crude approximation, though U may be substantial. We retain the short-range part of the static potential, however, to see its effect on photoelectric matrix elements.

Before calculating the Roper excitation amplitude, consider the magnetic moment of the proton,

$$\mu_p = \mu_0(g + g_l + g_s), \qquad (13)$$

where μ_0 is the quark Dirac moment, $g = 1+\kappa$, and

$$g_l = \frac{2\epsilon_0}{3m} + \frac{1}{4m^2}(2-\kappa)\langle p_\perp{}^2\rangle - \frac{1}{12m^2}\langle p_\perp{}^2\rangle, \qquad (14)$$

$$g_s = (\alpha_s/9)(\Omega/3\pi m^2)^{1/2}. \qquad (15)$$

I have used the notation, $p_\perp{}^2 = p_{1x}{}^2 + p_{1y}{}^2$, $\epsilon_0 = 3\omega$, and $\Omega = 6m\omega$. The last term in (14) arises from $H_{\rm em}{}^{\Delta\rm FW}$. Some authors[16] calculated relativistic corrections to μ_p but they did not consider the two-quark interactions (10) and (11). Seeing that $\langle p_\perp{}^2\rangle = \Omega/9$ for the harmonic oscillator, we can use (13) to fix the value of κ, once m, ω, and α_s are specified. We choose $m = 313$ MeV, $\omega = 300$ MeV, and $\alpha_s = 0.5$ to get $\kappa = 1.83$. This choice of small ω follows from the view of Isgur and Karl[9] that deviations from the harmonic oscillator decrease the energy of the Roper resonance more or less in parallel with that of the nucleon.

It is now straightforward to calculate the photocoupling of the Roper resonance. When the proton with helicity $-\frac{1}{2}$ absorbs a photon of helicity $+1$, the matrix elements of (7)–(12), apart from the common factor

$$F = \begin{bmatrix}\sqrt{2}\\ -1\end{bmatrix} \times \mu_0\left(\frac{k}{6}\right)^{1/2} e^{-\zeta} \text{ for } \begin{bmatrix}\underline{56}, 0^+\\ \underline{70}, 0^+\end{bmatrix} \qquad (16)$$

are, in the resonance rest frame,

$$M^{(1)} = -g\zeta, \qquad (17)$$

$$M^{(2)} = (k/12m)(1+2\kappa)(2-\zeta), \qquad (18)$$

$$M^{(3)} = \tfrac{1}{6}\lambda(3+\zeta+5\zeta^2) + \tfrac{1}{6}\kappa\lambda(1+\zeta), \qquad (19)$$

$$M^V = g_s I^V - \tfrac{1}{6}\lambda(1-\zeta), \qquad (20)$$

$$M^G = -\tfrac{1}{2}\lambda([{}^2_1] - 4\zeta + \zeta^2), \qquad (21)$$

$$M^{\Delta\rm FW} = gg_s I^{\Delta\rm FW} + \tfrac{1}{18}\lambda(1+\zeta) - \tfrac{1}{12}g\lambda\zeta([{}^4_5] - 19\zeta + 13\zeta^2), \qquad (22)$$

where k is the wave number of the photon, $\zeta = k^2/\Omega$, and $\lambda = \Omega/6m^2$. The radial integrals I^V and $I^{\Delta\rm FW}$ related to the short-range potential are written in terms of the confluent hypergeometric functions. The helicity amplitude $A_{1/2}{}^p$ is obtained from (17)–(22) by multiplying each of them by the sign of the πN amplitude.[17] The resulting amplitude together with $A_{1/2}{}^n$ is given in Table I in comparison with experimental figures taken from Barbour, Crawford, and Parsons.[18] In the last row we give the value computed by Metcalf and Walker[19] from the nonrelativistic model. Our value for $A_{1/2}{}^{p,n}$ is found to have the correct sign and magnitude in good agreement with experiment.

To check that the present model does not ruin

TABLE I. Photocouplings in units of 10^{-3} GeV$^{-1/2}$. Each term in Eqs. (19), (20), and (22) for $H_{\rm em}{}^{(3)}$, $H_{\rm em}{}^V$, and $H_{\rm em}{}^{\Delta\rm FW}$ is given separately in two or three rows.

	$P_{11}(1470)$		$P_{33}(1232)$	
	$A_{1/2}{}^p$	$A_{1/2}{}^n$	$A_{1/2}{}^p$	$A_{3/2}{}^p$
$H_{\rm em}{}^{(1)}$	109	−78	−292	−505
$H_{\rm em}{}^{(2)}$	−103	73	−33	−57
$H_{\rm em}{}^{(3)}$	−72	51	59	102
	−45	32	30	52
$H_{\rm em}{}^V$	4	0	0	0
	12	−9	16	28
$H_{\rm em}{}^G$	50	−32	93	161
$H_{\rm em}{}^{\Delta\rm FW}$	−1	0	0	0
	−8	9	8	14
	−9	6	−4	−7
Total	−63	52	−123	−212
Expt	−75±15	59±16	−142±7	−271±10
NR	29	−19	−101	−175

TABLE II. Photocouplings for the $[\underline{56}, 0^+]_2$ multiplet in units of 10^{-3} $\text{GeV}^{-1/2}$ (a) predicted by the present model compared with (b) the predictions with the two-quark transitions deleted and (c) the predictions of Backcock and Rosner (Ref. 3).

	$^2 8_{1/2}[\underline{56}, 0^+]_2$ $A_{1/2}^p$	$A_{1/2}^n$	$^4 10_{3/2}[\underline{56}, 0^+]_2$ $A_{1/2}^p$	$A_{3/2}^p$
a	-77	57	-59	-103
b	-76 (input)	53	-48	-84
c	-76 (input)	50	-47	-81

the success of the naive model for other resonances, I also calculate $A_{1/2}^p$ and $A_{3/2}^p$ for $P_{33}(1232)$ and display them in Table I. An improvement on the naive model is again obtained. Finally in Table II, I compare the results for the $[\underline{56}, 0^+]_2$ multiplet with the predictions of Babcock and Rosner.[3] If one deletes H_{em}^V and H_{em}^G and renormalize all the matrix elements to fit $A_{1/2}^p$ to experiment, one can reproduce the Melosh algebraic structure, with weak effects of nonadditive terms in $H_{em}^{\Delta FW}$ which cause SU(6) breaking.

The model I have developed differs from the Melosh method in that one can calculate all the photocouplings explicitly. Since the Melosh approach is based on the free-quark model, the FW interactions I have used are very much related to the Melosh current operators.[20] The recent success of baryon spectroscopy based on the gluon-perturbed confining potential model makes it necessary for us to go beyond the free-quark model. The intention of this Letter has been to incorporate physically real complications by the addition of H_{em}^V and H_{em}^G which makes a step out of the single-quark transition picture. These interactions are brought about by a transformation which eliminates gluons from field theory to go to the potential model. Since it is not possible at present to derive this transformation from the fundamental theory, one cannot help using phenomenology to determine the form of the potential. For charmonium a small fraction of vector confinement is necessary, but for ordinary quarks the study of the mass spectrum[6,7,9,10] seems in favor of pure scalar confinement. The present analysis of photocouplings complements such study.

[1]For a review, see R. G. Moorhouse, in *Electromagnetic Interactions of Hadrons*, edited by A. Donnachie and G. Shaw (Plenum, New York, 1978), Vol. 1. He remarks that out of sixteen well-determined signs, fifteen agree with the quark-model predictions, the exception being the proton coupling of the Roper resonance.

[2]R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971).

[3]J. Babcock and J. L. Rosner, Ann. Phys. (N.Y.) 96, 191 (1976); R. Cashmore, A. J. G. Hey, and P. Litchfield, Nucl. Phys. B89, 237 (1975).

[4]H. J. Melosh, Phys. Rev. D 9, 1095 (1974).

[5]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[6]H. J. Schnitzer, Phys. Rev. D 18, 3482 (1978).

[7]D. Gromes and I. O. Stamatescu, Nucl. Phys. B112, 213 (1976); W. Celmaster, Phys. Rev. D 15, 1391 (1977); D. Gromes, Nucl. Phys. B130, 18 (1977); N. Isgur and G. Karl, Phys. Lett. 72B, 109 (1977), and Phys. Rev. D 18, 4187 (1978).

[8]F. E. Close and H. Osborn, Phys. Rev. D 2, 2127 (1970); R. A. Krajcik and L. L. Foldy, Phys. Rev. D 10, 1777 (1974).

[9]N. Isgur and G. Karl, to be published.

[10]D. Gromes and I. O. Stamatescu, to be published.

[11]R. Faustov, Nuovo Cimento 69A, 37 (1970).

[12]F. E. Close and H. Osborn, Phys. Lett. 34B, 400 (1971); G. Feinberg and J. Sucher, Phys. Rev. Lett. 35, 1740 (1975); J. Sucher, Rep. Prog. Phys. 41, 1781 (1978).

[13]For any two vectors $\vec{a}$ and $\vec{b}$, we use the notation $\{\vec{a}, \vec{b}\} = \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a}$.

[14]K. C. Bowler, Phys. Rev. D 1, 929 (1970); L. A. Copley, G. Karl, and E. Obryk, Nucl. Phys. B19, 477 (1970).

[15]L. L. Foldy and S. A. Wouthuysen, Phys. Rev. 78, 29 (1950).

[16]P. N. Bogolioubov, Ann. Inst. Henri Poincaré 8, 163 (1968); A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Rev. D 9, 2636 (1974); B. H. Kellett, Ann. Phys. (N.Y.) 87, 60 (1974).

[17]In parallel with the photoelectric amplitude, the πN amplitude is calculated. Its sign is found to coincide with the prediction of Ref. 2 but its magnitude is much increased to get the rate $\Gamma = 107$ MeV in good agreement with experiment ≈ 120 MeV.

[18]I. M. Barbour, R. L. Crawford, and N. H. Parsons, Nucl. Phys. B141, 253 (1978).

[19]W. J. Metcalf and R. L. Walker, Nucl. Phys. B76, 253 (1974).

[20]G. Morpurgo, in *Quarks and Hadronic Structure*, edited by G. Morpurgo (Plenum, New York, 1977); F. E. Close, *ibid*.