Velocity Autocorrelation Function in a Strongly Coupled, Magnetized, Pure Electron Plasma

R. L. Varley and J. E. Tigner[a]
Department of Physics and Astronomy, Hunter College of the City University of New York, New York, New York 10021
(Received 28 December 1978)

The Alder-Wainwright effect for the velocity autocorrelation function is small and difficult to observe in a system of neutral particles but is more prominent in a strongly coupled plasma. The pure electron plasma recently produced by Malmberg and deGrassie provides an opportunity to observe the Alder-Wainwright effect experimentally. We speculate on the behavior of the velocity autocorrelation function for this system.

Alder and Wainwright[1] (AW) first observed an anomalous nonexponential "tail" in velocity autocorrelation functions calculated from computer molecular dynamic studies of classical hard core systems. They found empirically that the velocity autocorrelation function (VAF) behaves asymptotically as $t^{-d/2}$ after several collision times where d is the dimension of the system. One of several explanations[2] of the long-time "tails" was provided by Ernst, Hauge, and van Leeuwen[3] utilizing the concept of local equilibrium and linearized hydrodynamics. Landau and Placzek[3] are often given credit for introducing these assumptions for the calculation of various correlation functions but certainly Ernst, Hauge, and van Leeuwen presented a novel application of the ideas that the result is unusual.

The AW effect has proven difficult (but not impossible) to observe in the laboratory since the velocity autocorrelation function for a fluid system of neutral particles falls exponentially initially. After a few collision times the "long-time tail" becomes evident, but by then the VAF is but a fraction of a percent of its initial value. Kim and Matta utilizing latex spheres moving in a shock tube and Kim and Modla utilizing direct position measurement of a Brownian test particle illuminated with a laser beam have obtained some success in experimentally discerning what is a very small effect.[4] The AW effect is of great importance in the theoretical understanding of time correlation functions even though experimentally the effect is small for most circulstances in systems consisting of neutral particles.[2]

The one-component plasma system has recently attracted some interest. The computer molecular-dynamics studies of Hansen, McDonald, and Pollock[5] revealed a velocity autocorrelation for a strongly coupled plasma which oscillates at the plasma frequency and decays as $t^{-d/2}$ in the manner of AW. Subsequently Gould and Mazenko[6] provided a kinetic-theory explanation for the behavior of the VAF and Varley[6] provided a Landau-Placzek argument *à la* Ernst, Hauge, and van Leeuwen.[3] These studies are of importance in their own right but in addition they provide a new system which manifests the AW mode-mode coupling phenomena. Furthermore, in a *strongly coupled* plasma the AW effect for the velocity autocorrelation function becomes the dominant effect since *there is no initial exponential decay.*[5] The coupling of the plasma is usually given by the dimensionless parameter $\Gamma \equiv (Ze^2)/ak_BT$, where $a \equiv (3/4\pi\rho)^{1/3}$ is the ion-sphere radius and strong coupling is the regime where $1 < \Gamma < 155$.

The production of a nonneutral plasma by Malmberg and deGrassie[7] provides an opportunity for the experimental observation of the AW effect in a plasma and a recent investigation of Malmberg and O'Neil[7] indicates that the strong-coupled regime should be accessible. Here we describe the results of a calculation of the VAF via a Landau-Placzek argument (*à la* Ernst, Hauge, and van Leeuwen[3]) for a nonneutral plasma in a constant external magnetic field which is appropriate to the experiment of Malmberg and deGrassie.

The velocity autocorrelation function $C_D(t)$ is defined by

$$C_D(t) = \lim_{\{N,V\to\infty\}} \frac{N}{V} \langle V_{1x}(t=0)V_{1x}(t)\rangle ,$$

but it is possible to express the VAF in terms of the single-particle nonequilibrium distribution function.[3] Ernst, Hauge, and van Leeuwen argue that this single-particle nonequilibrium distribution function will take on the local equilibrium form a sufficiently long time after a spontaneous fluctuation occurs, and Pomeau and Résibois[3] showed that this assumption follows from general kinetic-theory arguments under "normal" circumstances for neutral-particle systems. Baus and Wallenborn[6] noted that this assumption is problematic for a plasma except when the binary collision processes of the strongly coupled plasma dominate the mean-field, Vlasov processes, and thereby enforce local equilibrium. The local equilibrium distribution[8] is a functional of the local density $n(\vec{r},t)$, velocity $\vec{v}(\vec{r},t)$, and temperature $T(\vec{r},t)$, and it seems reasonable to assume that the time evolution of these hydrodynamic quantities is determined by the usual linearized magnetohydrodynamic (MHD) equations in an external field.[3] [The calculation of $C_D(t)$ requires us to consider the local equilibrium distribution with $n(\vec{r},t)$ replaced by $P(\vec{r},t)$ the test-particle probability density; see Ernst, Hauge, and van Leeuwen[3] in this regard.] Specifically, we utilize the continuity equation

$$[\partial n(\vec{r},t)/\partial t] + \nabla\cdot\{n(\vec{r},t)\vec{v}(\vec{r},t)\} = 0$$

and the Navier-Stokes equation in the form

$$mn(\vec{r},t)[\partial/\partial t + \vec{v}(\vec{r},t)\cdot\nabla]\vec{v}(\vec{r},t) = -\nabla p(\vec{r},t) + \nabla\cdot\Pi(\vec{r},t) + qn(\vec{r},t)\vec{E}(\vec{r},t) + (q/c)n(\vec{r},t)\vec{v}(\vec{r},t)\times\{\vec{B}_0 + \vec{B}_s(\vec{r})\},$$

where $p(\vec{r},t)$ is the pressure, $\vec{B}_0$ is a magnetic field produced by external sources, and $\vec{B}_s(\vec{r})$ is the self-magnetic field produced by currents inside the plasma. The electric field $\vec{E}(\vec{r},t)$ is determined from the density through the Gauss law $\nabla\cdot\vec{E}(\vec{r},t) = 4\pi qn(\vec{r},t)$ where in the case of the pure electron plasma $q=-e$ where e is the charge on an electron. We will be considering the case of a strong, uniform external field $\vec{B}_0$ and we will neglect the diamagnetic field $\vec{B}_s(\vec{r})$ since it is small in comparison with $\vec{B}_0$ for the strongly coupled (low temperature) plasma. The general form of the viscous stress tensor $\Pi(\vec{r},t)$ for a nonisotropic system with a preferred direction (the direction of $\vec{B}_0$) is given by Braginskii[9] in a form applicable to both the strong- and weak-coupled plasma provided one does not utilize the specific values for the viscosities he provides for the weak-coupled domain. Finally, since we are considering a strongly coupled plasma (with low temperature) we neglect the heat equation and all temperature fluctuations.

The above hydrodynamic description is applicable to the inertial, laboratory frame of the experiment. It proves convenient to write the hydrodynamic equations in a noninertial, rotating frame with use of the Coriolis theorem[10] since the electrons of the pure electron plasma must have a rotational motion to provide stability to the system.[11] We then proceed to write the density as $n(\vec{r},t) = n_0(\vec{r},t) + \delta n(\vec{r},t)$, the velocity as $\vec{v}(\vec{r},t) = \vec{v}_0(\vec{r},t) + \delta\vec{v}(\vec{r},t)$, the electric field as $\vec{E}(\vec{r},t) = \vec{E}_0(\vec{r},t) + \delta\vec{E}(\vec{r},t)$, and the pressure as $p(\vec{r},t) = p_0(\vec{r},t) + \delta p(\vec{r},t)$, where we have separated the steady, nonfluctuating zeroth-order parts of the hydrodynamic quantities from their fluctuating parts. Substitution of the above into the Navier-Stokes equation in the rotating frame leads (in the zeroth order) to a force-balance equation. We then choose the angular velocity ω of the rotating frame so that the $\vec{v}_0(\vec{r},t)$ of the electron fluid vanishes in the rotating frame. There is thus no viscous term in this order and the pressure term is negligible.[12] The force-balance equation yields two possible angular velocities $\omega_e^{\pm}$ for which an equilibrium exists.[10] We assume an experiment where the density n_0 is constant out to some radius and consider the case of a strong external field $\vec{B}_0$ so that ω_e^+ is approximately the gyrofrequency $\Omega_e = eB_0/m_ec$ and ω_e^- becomes approximately $-cE_0/rB_0$, a much slower "drift" angular velocity mode. The thermal gyroradius associated with ω_e^+ is much smaller than the interparticle spacing in a strongly coupled plasma so that the dominant collective motion which maintains stability in a pure electron plasma must be the slower "drift" mode ω_e^-.

We then proceed to the solution of the first-order MHD equations for the fluctuating quantities. Terms of second order in fluctuations are neglected and since the rotating frame was chosen so that $\vec{v}_0 = 0$ there is no contribution from the convective terms. The Coriolis term is negligible in comparison with the strong external field $\vec{B}_0$. Thus the Navier-Stokes equation for the fluctuating hydrodynamical quantities appears in the rotating frame as[13]

$$\partial\delta\vec{v}(\vec{k},t)/\partial t = -i(1/n_0^2 m_e K_T)\vec{k}\delta n(\vec{k},t) - \nu k^2\delta\vec{v}(\vec{k},t) - (e/m_e)\delta\vec{E}(\vec{k},t) - (e/m_ec)\delta\vec{v}(\vec{k},t)\times\vec{B}_0.$$

In obtaining the above equation, we have performed a spatial Fourier transform as a preliminary in solution. Also, we have utilized a simplified expression for the viscosity, retaining only the shear viscosity ν which dominates. The electric field satisfies the Gauss law $i[\vec{k}\cdot\delta\vec{E}(\vec{k},t)] = -4\pi e\,\delta n(\vec{k},t)$ and the continuity equation takes the form

$$\partial\delta n(\vec{k},t)/\partial t = -in_0[\vec{k}\cdot\delta\vec{v}(\vec{k},t)].$$

Finally, the test-particle density $P(\vec{k},t)$ satisfies a simple diffusion equation of the form $\partial P(\vec{k},t)/\partial t = -Dk^2P(\vec{k},t)$ in the frame rotating with respect to the laboratory.

Utilization of the local-equilibrium form of the Maxwellian distribution[3,8] in the rotating frame together with the solutions to the first-order linearized MHD equations described above yields $C_D(t)=t^{-1}\{K_1+K_2\cos(\overline{\omega}t+\varphi)\}$ as the asymptotic form for the VAF in a strongly coupled, two-dimensional plasma immersed in a strong, uniform magnetic field which is perpendicular to the plane of motion. We give the above result for $C_D(t)$ in the frame rotating with respect to the laboratory with a constant angular velocity ω_e^- as described previously. We give $C_D(t)$ in the rotating frame since the physical interpretation of the processes contributing to the behavior of $C_D(t)$ are more easily understood there than in the laboratory frame where the actual experiments are performed. The above result for $C_D(t)$ reduces to our previous result[3] for the unmagnetized case in the appropriate limit, and that result for $C_D(t)$ is in substantial agreement with the data of Ref. 5 beyond a short time (a small fraction of a plasma period) after the fluctuation occurs. The magnetic field modifies the oscillation frequency $\overline{\omega}$ of $C_D(t)$ to $\overline{\omega}=(\Omega_e^2+\omega_p^2)^{1/2}$, where $\omega_p=(4\pi e^2 n_e/m_e)^{1/2}$ is the plasma frequency and $\Omega_e = eB/m_e c$ is the gyrofrequency. K_1 and K_2 are constants given in terms of the transport coefficients by

$$K_1 = \{8\pi\beta m\nu(1+D/\Delta_2)\}^{-1},$$

$$K_2 = 2\Delta_1/\{8\pi\beta m\nu[(\Delta_1+D)^2+\Delta_3^2]\}^{1/2},$$

where $\Delta_1 = (\nu/\overline{\omega}^2)(\Omega^2+\omega_p^2/2)$, $\Delta_2 = (\nu\omega_p^2\overline{\omega}^2)$, and $\Delta_3 = (1/2\overline{\omega}\rho K_T)$. The phase angle φ is given by $\varphi = \tan^{-1}[\Delta_3/(\Delta_1+D)]$. The three-dimensional problem is considerably more complicated but we expect the oscillatory behavior of the velocity autocorrelation function to persist in three dimensions for motion transverse to the external magnetic field and the decay to be modified to $t^{-3/2}$.

The physical picture corresponding to our velocity autocorrelation function is revealing. The VAF requires us to focus on the decay of the x component of the velocity of the Brownian particle as it interacts with the electrons of the medium. The movement of a Brownian particle is produced by relatively large-scale fluctuations in density and velocity of the medium. Once in motion, the Brownian particle in turn produces hydrodynamic fluctuations in the medium which evolve according to the equations of linearized MHD. It is the effect of these induced hydrodynamic modes acting back on the Brownian particle which is responsible for the nonclassical (nonexponential) decay of $C_D(t)$ that we have calculated. There are several separately discernible effects in this reaction of the medium on the Brownian particle. As in the case of a medium of neutral particles,[1,3] a part of the transverse velocity (vortex) mode provides an explanation for the power-law t^{-1} aspect of the decay. The velocity of the Brownian particle is also affected periodically by its interaction with the plasma oscillation which was excited with the initial motion of the Brownian particle.[5,6] Superimposed on the medium plasma oscillation is the gyromotion of the electrons due to the presence of the external magnetic field. The net effect of the plasma oscillation and gyromotion of the medium particles (electrons) is to cause the x component of the velocity of the Brownian particle to oscillate at the composite frequency $\overline{\omega}$. The final form we obtained for the VAF is a consequence of coupling between the self-diffusion mode of the Brownian particle and the velocity mode of the medium just described. We emphasize that the composite oscillation frequency of the Brownian particle $\overline{\omega}$ is a consequence of the effect of the motion of the medium electrons on the Brownian particle (note that Ω_e and ω_p depend upon the properties of the medium electrons and do not depend upon the properties of the Brownian particle).

A self-consistent calculation of the diffusion coefficient utilizing the above result for the VAF in the Green-Kubo formula yields a Bohm-like dependence on the magnetic field. While the theoretical foundations of such a self-consistent calculation are somewhat tenuous, Krommes and Oberman and others[10] working with the weakly coupled, strongly magnetized plasma have found good agreement between their results for the diffusion coefficient and the computer molecular dynamics results of Okuda and co-workers.[9] Such agreement presents indirect evidence to support the assumption of local equilibrium as utilized here. Presumably, the magnetic field provides the necessary localization for making valid both the assumptions of linearized MHD and local equilibrium in the case of a weakly coupled plasma. The strong-coupled nature of the plasma discussed here should enhance the localization effect of the magnetic field, providing even more assurance of the validity of these two Landau-Placzek assumptions.

In conclusion, the Landau-Placzek method provides a result for the VAF more quickly than a kinetic-theory calculation and in addition it provides physical insight into the relevant nonequilibrium processes involved in the VAF. We hope this calculation will encourage an experimental measurement of the VAF and motivate subsequent kinetic-theory and molecular-dynamics calculations.

This research was supported in part by a grant from the Faculty Research Award Program of the City University of New York.

[(a)] Now at Science Applications, Inc., McLean, Va. 22101.

[1]B. J. Alder and T. E. Wainwright, Phys. Rev. Lett. 18, 988 (1967), and Phys. Rev. A 1, 18 (1970).

[2]Y. Pomeau and P. Résibois, Phys. Lett. 19C, 64 (1975).

[3]M. H. Ernst, E. H. Hauge, and J. M. J. van Leeuwen, Phys. Rev. A 4, 2055 (1971); L. Landau and G. Placzek, Z. Phys. Sowjetunion 5, 172 (1934); R. D. Mountain, Rev. Mod. Phys. 38, 205 (1966); P. Résibois and Y. Pomeau, Physica (Utrecht) 72, 493 (1974).

[4]Y. W. Kim and J. E. Matta, Phys. Rev. Lett. 31, 208 (1973); J. E. Matta, Ph.D. thesis, Lehigh University, 1974 (unpublished); J. C. Modla, Ph.D. thesis, Lehigh University, 1977 (unpublished); J. P. Boon and A. Bouiller, Phys. Lett. 55A, 391 (1976).

[5]J. P. Hansen, I. R. McDonald, and E. L. Pollock, Phys. Rev. A 11, 1025 (1975).

[6]H. Gould and G. F. Mazenko, Phys. Rev. Lett. 35, 1455 (1975), and Phys. Rev. A 15, 1274 (1976); R. L. Varley, Phys. Lett. 62A, 340 (1977); M. Baus and J. Wallenborn, Phys. Lett. 55A, 90 (1975); M. Baus, Phys. Lett. 24, 261 (1973), and Physica (Utrecht) 79A, 377 (1975), and 66, 421 (1973), and Phys. Rev. A 15, 790 (1977); S. Sjodin and S. K. Mitra, in *Strongly Coupled Plasmas*, edited by G. Kalman and P. Carini (Plenum, New York, 1978).

[7]J. H. Malmberg and J. S. deGrassie, Phys. Rev. Lett. 35, 577 (1975); J. S. deGrassie and J. H. Malmberg, Phys. Rev. Lett. 39, 1077 (1977); J. H. Malmberg and T. M. O'Neil, Phys. Rev. Lett. 39, 1333 (1977).

[8]S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases* (Cambridge Univ. Press, Cambridge, 1970), 3rd ed.

[9]S. I. Braginskii, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1965), Vol. 1, especially p. 218. Note that here we are considering a two-dimensional system with the external field $\vec{B}_0$ perpendicular to the plane of motion and so we have specialized Braginskii's three-dimensional results to this case.

[10]The Coriolis theorem is discussed in numerous mechanics books; see, for example, K. R. Symon, *Mechanics* (Addison-Wesley, Reading Mass., 1960), 2nd ed., especially p. 276. For applications in hydrodynamics one can consult G. K. Batchelor, *An Introduction to Fluid Mechanics* (Cambridge Univ. Press, Cambridge, 1967), especially p. 139 ff.

[11]R. C. Davidson, *Theory of Nonneutral Plasmas* (Benjamin, Reading, Mass., 1974); A. W. Trivelpiece, Comments Plasma Phys. Contr. Fusion 1, 57 (1972); R. C. Davidson, J. Plasma Phys. 6, 229 (1971); R. C. Davidson and N. A. Krall, Phys. Fluids 13, 1543 (1970).

[12]The zeroth-order force-balance equation is obtained by utilizing the Coriolis theorem (for a frame having constant ω) to write the Navier-Stokes equation in the rotating frame of the electrons. Keeping only zeroth-order terms in the hydrodynamical variables one obtains $\vec{\omega}\times(\vec{\omega}\times\vec{r}) = -eE_0(\vec{r}) - (e/c)(\vec{\omega}\times\vec{r})\times\vec{B}_0$, where we have chosen $\vec{\omega}$ such that the zeroth-order fluid velocity $\vec{v}_0$ vanishes in the rotating frame. The viscous terms vanish since (to zeroth order) the fluid is in uniform rotation and the pressure term vanishes since considering $p=p(n,T)$ we write $\nabla p = (\partial p/\partial n)_T\nabla n + (\partial p/\partial T)_n\nabla T$. We assume that ∇T is negligible (strong coupling) and ∇n_0 vanishes since we assume an experiment where the density is constant. The force-balance equation above is treated by Davidson (Ref. 11) on p. 5 ff. and he solves for the two roots $\omega_e^{\pm}$. Davidson's low-density limit is our strong-field $\vec{B}_0$ limit.

[13]The pressure term of the first-order Navier-Stokes equation is replaced by a density term by considering $p=p(n,T)$ as in Ref. 12 above. The isothermal compressibility is $K_T = -n_0^{-1}[\partial\,\delta n/\partial\,\delta p]_T$ and thus

$$-(n_0 m_e)^{-1}\nabla\delta p = (n_0^2 m_e K_T)^{-1}\nabla\delta n\,.$$

[14]H. Okuda, J. M. Dawson, and R. N. Carlile, Phys. Rev. Lett. 27, 491 (1971); H. Okuda and J. M. Dawson, Phys. Fluids 16, 408 (1973); H. Okuda, C. Chu, and J. M. Dawson, Phys. Fluids 18, 243 (1975).

[15]J. B. Taylor and B. McNamara, Phys. Fluids 14, 1492 (1971); P. B. Corkum, Phys. Rev. Lett. 31, 809 (1973); D. Montgomery, Physica (Utrecht) 82C, 111 (1976); J. A. Krommes and C. Oberman, J. Plasma Phys. 16, 229 (1976); J. Krommes, Ph.D. thesis, Princeton University, 1975 (unpublished); J. Weinstock, Phys. Fluids 11, 354 (1968).