element, so that

$$\frac{\Gamma(D^+ \to \pi^+\pi^0)}{\Gamma(D^+ \to \pi^+ K_S)} = \frac{1}{2}\frac{\lambda^2}{\tan^2\theta_C |1+\lambda\, b/a|^2}, \qquad (12)$$

where b/a is the ratio of reduced matrix elements. If b/a is of order unity, the correction represented by this term will be of order 10%. However, it is not at all unthinkable that $|b/a|$ is considerably larger than unity, in which case the correction could well be several tens of percent.

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Polarization of High-Transverse-Momentum Single Photons as a Test of Quantum Chromodynamics

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It is shown that high-transverse-momentum photons originating from the process quark + gluon → quark + photon exhibit a substantial linear polarization in perturbative quantum chromodynamics. The angular dependence of the polarization is quite sensitive to the non-Abelian structure of the interaction, and can be used to check the equality of the quark-gluon and three-gluon couplings.

While fairly rigorous tests of perturbative quantum chromodynamics (QCD) have been developed,[1] it has not been easy to find observable effects which are sensitive to the specific properties of gluons and the non-Abelian structure of the theory. These features generally come into play only when higher-order corrections are included. For example, we have shown[2] that the transverse polarization of a produced quark, which is determined by one-loop corrections to the lowest-order amplitude, will vanish at large transverse momentum (p_T) because of the vector nature of the quark-gluon coupling. The corrections to this result are of order $(Q^{-2})^{1/2}$ rather than having the logarithmic form usually associated with asymptotic freedom.

In this paper, we show that the linear polarization of photons produced in the reaction quark + gluon → quark + photon[3] is substantial—on the order of a few percent. The polarization depends sensitively on the three-gluon vertex and hence on the non-Abelian character of the interaction. Furthermore, the photon final state has the advantage that no model is needed to describe how a polarized constituent fragments into hadrons in order to test the prediction. If it is technically feasible to measure the photon polarization, as appears to be the case, this will be an important test of QCD.

The generalization of our results to gluons produced at high p_T, which will be reported in a subsequent publication, will eventually allow a test of gluon polarization properties. It is also interesting to note that this calculation, because of its special dependence on helicity-flip amplitudes, is technically far less complicated than most calculations involving observables which depend on one-loop corrections.

Proceeding with an outline of the calculation, we define helicity amplitudes $M_{\lambda_3\lambda_4,\,\lambda_1\lambda_2}$, where the various helicities λ_i are associated with the labeling $q(1)+g(2)\rightarrow q(3)+\gamma(4)$. We assume that the quark mass m is negligible, in which case the quark helicity-flip amplitudes ($\lambda_1\neq\lambda_3$) vanish by γ_5 invariance. This is the reason that all quark polarizations vanish when $m\rightarrow 0$. Parity invariance leaves four independent helicity amplitudes,

$$A=M_{1/2,1;\,1/2,1},\quad B=M_{1/2,-1;\,1/2,1},$$

$$\overline{B}=M_{1/2,1;\,1/2,-1},\quad C=M_{1/2,-1;\,1/2,-1}.$$

Notice that when the interaction is Abelian (e.g., electron-photon scattering), $B=\overline{B}$ by time-reversal invariance. In terms of these amplitudes, the linear polarization of the outgoing photon is

$$\mathcal{P}=\frac{|M_\perp|^2-|M_\parallel|^2}{|M_\perp|^2+|M_\parallel|^2}=\frac{2\,\mathrm{Re}[AB^*+C\overline{B}^*]}{(|A|^2+|B|^2+|\overline{B}|^2+|C|^2)}. \quad (1)$$

Here, $M_\perp$ denotes the amplitude for the outgoing photon to be polarized perpendicular to the scattering (x-z) plane, $M_\parallel$ is the amplitude for linear polarization in the scattering plane, and we sum over all other helicities and color. Under the assumption that $m=0$, the amplitudes B and $\overline{B}$ vanish in the lowest order, while A and C are finite. The calculation of the polarization in leading order therefore requires computing the flip amplitudes B and $\overline{B}$ by considering one-loop corrections to the pole terms. For the nonflip amplitudes A and C, the pole terms suffice.

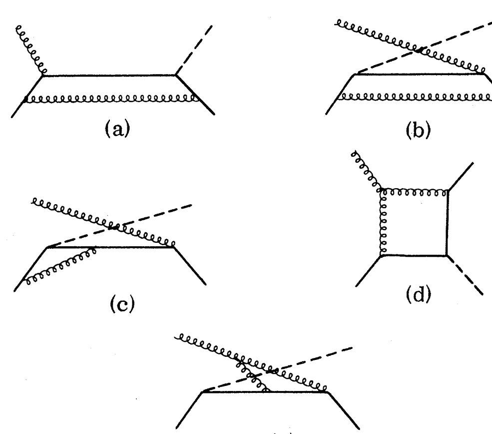

FIG. 1. Contributing diagrams. The wavy line denotes a gluon and the dashed line a photon. One-loop diagrams which do not contribute to helicity flip in our gauge ($\vec{k}\cdot\vec{\epsilon}=0$) are not shown.

TABLE I. Contributions to the amplitude B.

Diagram	Contribution[a]
(a)	$[1/2](-1/5)$
(b)	$[(\hat{t}/\hat{u})\epsilon^{-1}\Gamma^2(1+\epsilon)/\Gamma(2+2\epsilon)+1/2](-1/6)$
(c)	$(\hat{t}/\hat{u})[-\epsilon^{-1}\Gamma^2(1+\epsilon)/\Gamma(2+2\epsilon)+1/2](4/3)$
(d)	$(-\hat{t}/2\hat{u})[\epsilon^{-2}\Gamma^2(1+\epsilon)/\Gamma(1+2\epsilon)](3/2)$

[a] Each contribution to B and $\overline{B}$ has an overall factor $eg(\alpha_S/\pi)(-\hat{u}/\hat{s})^{1/2}(\pi/-\hat{u})^{-\epsilon}\Gamma(1-\epsilon)(\lambda_a/2)_{r'r}$, where the caret (^) denotes invariants for the quark-gluon system. The numbers in parenthses are SU(3) color factors.

Since $m=0$ and the quark-gluon coupling is vector, the spinor matrix element associated with any particular diagram contains an odd number of γ matrices. This product can always be reduced to a sum involving scalar products, and the single matrix element $\bar{u}_{1/2}(p_3)\gamma_\mu u_{1/2}(p_1)$. For this calculation, the matrix reduction together with the subsequent evaluation of parameter integrals was carried out with use of the CERN algebraic manipulation program SCHOONSCHIP.[4] The diagrams which contribute are shown in Fig. 1 and their contributions to B and $\overline{B}$ are given in Tables I and II, respectively. The relative signs of the diagrams are fixed by gauge invariance and the Feynman gauge was used for internal gluons.

Before presenting a formula for the photon polarization, a number of comments about the entries in Tables I and II are in order. First, the various contributions are real since a physical intermediate state, which is necessary to produce an imaginary part, always involves a Born term with helicity flip and all of these are zero.

TABLE II. Contributions to the amplitude $\overline{B}$.

Diagram	Contribution
(a) + (b) + (c)[a]	$[-(\hat{s}/2\hat{u})+1/2](-1/6)$
(d)	$(\hat{t}/\hat{u})[\epsilon^{-1}\Gamma^2(1+\epsilon)/\Gamma(2+2\epsilon)](3/2)$
(e)	$(\hat{t}/2\hat{u})[\epsilon^{-2}\Gamma^2(1+\epsilon)/\Gamma(1+2\epsilon)-1](3/2)$

[a] This contribution was obtained by applying time-reversal invariance to those diagrams which are identical to electron-photon scattering.

Second, the infrared divergences which occur in individual diagrams but cancel in the complete expressions for B and $\overline{B}$ were handled by use of dimension regularization. Specifically, the loop integrals were evaluated in $4+2\epsilon$ dimensions. (This required some care because individual diagrams contain double poles in ϵ.) Finally, the existence of kinematic zeros proportional to $\hat{t}$ in the forward direction is obscured by the $m\to 0$ limit. For example, $\hat{t}/\hat{u}$ is actually $\hat{t}/(\hat{u}-m^2)$ and a constant factor arises from the $m\to 0$ limit of a term like $\hat{t}/(\hat{t}-m^2)$. As a result, our formula for the polarization is inapplicable near the forward and backward directions. Because of the smallness of the mass scale m^2 this is, in practice, no serious restriction.

From Table I, the expression for B in the quark-gluon center-of-mass system is

$$B=\frac{eg\alpha_S}{12\pi}\frac{1}{\cos(\theta/2)}+(3-5\cos\theta)\left(\frac{\lambda_a}{2}\right)_{r'r}, \qquad (2)$$

where the incoming gluon is in the positive z direction, e is the quark charge, and g is the gauge coupling constant. The color factor $(\lambda_a/2)r'r$ is common to all amplitudes and cancels in the expression for the polarization. Table II gives the corresponding expression for $\overline{B}$ in the form

$$\overline{B}=\frac{-eg\alpha_S}{6\pi}\frac{1}{\cos(\theta/2)}(3-2\cos\theta)\left(\frac{\lambda_a}{2}\right)_{r'r}. \qquad (3)$$

Using Eqs. (2) and (3) together with the lowest-order expression[5] for A and C in Eq. (1) gives

$$\mathcal{P}=\frac{\alpha_S}{2\pi}\frac{4}{3}\frac{\cos\theta(3-\cos\theta)}{5+2\cos\theta+\cos^2\theta}=\frac{\alpha_S}{2\pi}P(\theta). \qquad (4)$$

The angular dependence[6] $P(\theta)$ is shown in Fig. 2 where it can be seen to vary between about $\frac{1}{3}$ and $-\frac{4}{3}$. Taking a value of $\alpha_S\approx\frac{1}{3}$ gives photon polarizations between -8 and $+2\%$.

The form of the numerator in Eq. (4) is quite specific to QCD. It involves the quadratic Casimir operators of the triplet and octet representations in the right combination to eliminate the ϵ dependence. Furthermore, the crossover from positive to negative values of the polarization, which occurs at $\cos\theta=0$ [$\cos\theta=(n-3)/(n+1)$ for $SU(n), n\geq 2$] would not occur if the coupling were Abelian. In that case, the polarization would be negative, as indicated by the dashed curve in Fig. 2.

Theoretical estimates[7] of single-hard-photon rates indicate that the process $q+g\to q+\gamma$ is detectable and dominates over other sources of single photons such as the related process $q+\overline{q}\to g+\gamma$. Preliminary data[8] on single-photon rates are reasonably consistent with these estimates. Our conclusion is that a measurement of the polarization of these photons can provide a rigorous test or perturbative QCD as well as an important check on the color hypothesis. The latter aspect is particularly attractive because the polarization involves the three-gluon interaction and the equality of the quark-gluon and three-gluon couplings in an essential way[9] (see Fig. 1).

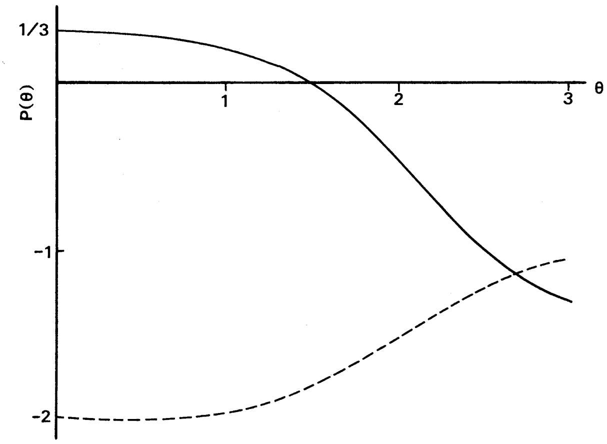

FIG. 2. The angular distribution $P(\theta)$ for SU(3) (solid line) and an Abelian theory (dashed line).

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Note added.—We would like to point out that the time-reversed reaction $\gamma_{\rm pol}+q\to g+q$ can also be used to check Eq. (4) Specifically, for incident photons with linear polarization P the cross section is

$$d\sigma(\gamma_{\rm pol}+q\to g+q)=d\sigma(\gamma+q\to g+q)(1+P\mathcal{P}\cos 2\varphi).$$

Here $\mathcal{P}$ is given by Eq. (4) and φ is the angle between the scattering plane and one of the polarization vectors.

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Direct Photon Production from π^+p Interactions at 10.5 GeV/c

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We have studied inclusive photon production from π^+p interactions at 10.5 GeV/c and found a source of centrally produced direct photons which accounts for $(46\pm9)\%$ of all photons detected with $P_L{}^*$ and P_T between 0 and 20 MeV/c. The experimental data are found to be in good agreement with a calculation of hadronic inner bremsstrahlung.

Experimental information about direct photon production from hadronic interactions is quite limited. Rather detailed measurements have been made of hadronic bremsstrahlung from elastic πp (Ref. 1) and pp (Ref. 2) collisions at low energies (beam momenta below ~1 GeV/c), but there are few data at high energies where inelastic processes are dominant. Three measurements have been reported from pp collisions at high center-of-mass energies (30 to 53 GeV) where direct photons with P_T greater than 2 GeV/c were observed.[3] Other experimental searches have either reported negative results[4] or postulated the decay of new particles as sources of observed anomalous photon signals.[5]

In this Letter we report a measurement of direct photon production from inelastic π^+p interactions with a center-of-mass energy of 4.5 GeV. Our results are relevant to studies of direct electron production since direct photons can produce e^+e^- pairs by means of internal conversion.

The experiment was done in the Stanford Linear Accelerator Center's 82-in. bubble chamber filled with a hydrogen-neon mixture having a radiation length of 125 cm. The chamber was exposed to a 10.5-GeV/c π^+ beam and the resulting interactions selectively scanned for those apparently occurring on free protons. We report here on data from a sample of 33 676 π^+ interactions of which about $\frac{2}{3}$ occur on free protons and $\frac{1}{3}$ on protons loosely bound in neon nuclei; these events consist of 10 275 inelastic two-prongs, 19 990 four-prongs and 3411 six-prongs. The photons produced from these interactions were detected with an efficiency of about 25% by means of e^+e^- pair production in the hydrogen-neon liquid.[6]

The evaluation of background sources of low-energy photons is especially important in this

