E. Seiler, "On the Construction of Quantized Gauge Fields I. General Results" (to be published); in four dimensions the result is known for the case of constant field strength. Analogous inequalities for the relativistic situation have been shown by J. Schwinger, Phys. Rev. 93, 615 (1954).

[13]By stability theorems for the essential spectrum, the equality sign in (7) extends to a larger class of B's than the one considered here. In particular, it extends to smooth B's with falloff at infinity.

[14]For example, $B(x)=x^2+y^2$ (more generally, any $B \to \infty$ at infinity); see Ref. 6 for details. Note that Ref. 9 and the positivity of $H(\vec{a})$ imply that $H(\vec{a})$ has no discrete ground state.

[15]Indeed for spin-0 particles

$$\inf\{\mathrm{Spec}(p-a)^2\} \geq \inf_{\vec{x}\in R^3} |\vec{B}(\vec{x})|.$$

[16]The theorem of Lieb appears in an appendix to Ref. 11.

[17]For example, if $V \in L^{3/2}(R^3)+L^\infty(R^3)$, $a_\mu \in L_{\mathrm{loc}}{}^2(R^3)$; this involves no loss of generality. Let $V_\epsilon = V+\epsilon\vec{x}^2$. $H(0)+V_\epsilon$ has a discrete ground state that converges to $\inf\{\mathrm{Spec}[H(0)+V]\}$. Define $H(a)+V_\epsilon$ by the Friedrichs extension on C_0^∞. The inequality (7) is then proven by passing to the limit $\epsilon=0$.

[18]We use the natural identification of $f\psi_{\{d\}}$ with the spinor-valued function $(0,\psi_{\{d\}})$.

Implications of General Covariance and Maximum Four-Dimensional Yang-Mills Gauge Symmetry

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General covariance and maximum four-dimensional Yang-Mills gauge symmetry lead to these results: (1) Gravity is characterized by a dimensionless constant $F \sim 10^{-19}$; (2) the Newtonian force is always attractive; (3) space-time has a torsion; (4) gravitational spin-force between two protons is about 10^{19} times stronger than the corresponding Newtonian force. A possible experimental test is discussed.

The idea of gauge symmetry has been developed to obtain simple and elegant spin-1 fields by Yang and Mills.[1] When this idea was extended to spin-2 fields such as gravity, the dynamics of interactions becomes extremely complicated.[2-4] This may be due to the conceptual bondage of the conventional approach which postulates the Riemannian metric tensor as basic field variables. The Yang-Mills-type gauge symmetry for gravity has been studied by many physicists.[3-6] The results are stimulating but not completely satisfactory. Also, previous formulations of gravity involves a dimensional coupling constant, which leads to serious divergences in higher orders.

In this Letter, we explore a different approach in which Yang-Mills gauge fields, associated with maximum four-dimensional symmetry (i.e., the de Sitter group), are regarded as basic dynamical fields and the metric tensor is postulated to be a function of gauge fields. The physical motivation is to combine the two basic principles, i.e., the Yang-Mills gauge symmetry and the Einstein general covariance, in such a way that the formulation of gravity, including fermions, involves a small *dimensionless* coupling constant and agrees with experiments. Furthermore, the dynamics of the gravitational interaction and the maximum four-dimensional gauge symmetry are interlocked in the same way as that in electromagnetism. Thereby, serious divergences could be reduced and other problems[4] can be resolved as well.

We stress that the de Sitter group is used only as the gauge group so that the dynamics of interaction between fields are uniquely specified by such a gauge symmetry. One should not interpret the de Sitter-group operators to be the translational and rotational operators of physical space-time. In other words, the physical space may not be the same as the de Sitter space.[7]

We first observe that in analogy with the electric force, the gravitational force can be written as $-F_1F_2/r^2$, where $F_1=G^{1/2}m_1$ and $F_2=G^{1/2}m_2$ are dimensionless (for $c=\hbar=1$). This suggests that the de Sitter group is natural for the gauge group of gravity because it involves a length L. The matrix representation[6,8] Z_A of the de Sitter-group generator is given by

$$Z_A=(Z_i, Z_a)=(\gamma_i/2L,\ i\gamma_j\gamma_k/2),$$

with

$$[Z_B, Z_C] = if_{BC}{}^A Z_A, \quad A, B, C = i, a, \tag{1}$$

where $a \equiv jk$ denotes a pair of antisymmetric indices and γ_k is the constant Dirac matrix. The gauge-covariant derivative has the form[1]

$$D_\mu \psi = (\partial_\mu + iFh_\mu{}^A Z_A)\psi$$

$$(F, \text{ real and dimensionless}) \tag{2}$$

in a Yang-Mills gauge theory, and $h_\mu{}^A = (h_\mu{}^i, h_\mu{}^{ik})$ are the gauge fields.

The action must be invariant under the combined local SO(4, 1) gauge and general coordinate transformations:

$$\begin{aligned} &\delta x^\mu = \epsilon_\nu{}^\mu(x) x^\nu + \epsilon^\mu(x), \\ &\delta\psi(x) = i\epsilon^A(x) Z_A \psi(x), \\ &\delta h_\mu{}^A(x) = -F^{-1}\partial_\mu \epsilon^A(x) + f_{BC}{}^A h_\mu{}^B \epsilon^C(x). \end{aligned} \tag{3}$$

As noted by West,[5] $F_{\mu\nu}{}^A F_{\alpha\beta}{}^B g_{AB} E^{\mu\nu\alpha\beta}$ is the only SO(4, 1) invariant. The action is

$$S = \int d^4x \left(F_g L_\psi - \tfrac{1}{4} F_{\mu\nu}{}^A F_{\alpha\beta}{}^B E^{\mu\nu\alpha\beta} g_{AB} + L'\right), \quad L' = -\tfrac{1}{2} E_g \partial_\mu h_\nu{}^A \partial^\mu h^{\nu B} g_{AB}, \tag{4}$$

where $E_g = (-\det g_{\mu\nu})^{1/2}$, $g_{AB} = f_{AC}{}^D f_{DB}{}^C/6$ and

$$L_\psi = \tfrac{1}{2} i\bar\psi \gamma^\mu (\partial_\mu + iFh_\mu{}^A Z_A)\psi - \tfrac{1}{2} i\bar\psi(\overleftarrow{\partial}_\mu - iFh_\mu{}^A Z_A)\gamma^\mu \psi - m\bar\psi\psi + (\psi \to \psi_e, \; m \to m_e), \tag{5}$$

$$F_{\mu\nu}{}^A = \partial_\nu h_\mu{}^A \partial_\mu h_\nu{}^A - F f_{BC}{}^A h_\nu{}^B h_\mu{}^C, \quad \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}. \tag{6}$$

The nucleon and the electron wave functions are ψ and ψ_e, respectively. Note that the action (4) is physically meaningful if and only if there is a relation between the metric tensor $g_{\mu\alpha}$ and the basic fields $h_\mu{}^A$. We define

$$\begin{aligned} &g_{\mu\alpha}(x) = e_\mu{}^i e_\alpha{}^k \eta_{ik}, \quad e_\mu{}^i = (\exp h)_\mu{}^i = \delta_\mu{}^i + h_\mu{}^i + \tfrac{1}{2} h_\mu{}^k h_k{}^i + \ldots, \\ &S_\sigma{}^{\mu\nu} = e_i{}^\mu e_k{}^\nu h_\sigma{}^{ik} = \text{torsion}, \end{aligned} \tag{7}$$

where the gauge fields $h_\mu{}^A(x)$ vanish at infinity, just like ordinary fields. The tetrads $e_\mu{}^i$ and $e_k{}^\nu$ satisfies $e_\mu{}^i e_i{}^\nu = \delta_\mu{}^\nu$ and $e_\mu{}^i e_k{}^\mu = \delta_k{}^i$. And the matrix γ^μ in (5) can be expressed as $e_i{}^\mu \gamma^i$, where γ^i is the usual constant Dirac matrix, $\gamma_i\gamma_k + \gamma_k\gamma_i = 2\eta_{ik}$, $\eta_{ik} = (1, -1, -1, -1)$.

We may remark that the covariant field tensor $F_{\mu\nu}{}^A$ satisfies the (gauge) Bianchi identity

$$\begin{aligned} &D_\mu F_{\nu\lambda}{}^A + D_\nu F_{\lambda\mu}{}^A + D_\lambda F_{\mu\nu}{}^A = 0, \\ &D_\mu F_{\nu\lambda}{}^A \equiv \partial_\mu F_{\nu\lambda}{}^A - F f_{BC}{}^A h_\mu{}^B F_{\nu\lambda}{}^C. \end{aligned} \tag{8}$$

Also, L' in (4) corresponds to the usual gauge-fixing term, which is necessary for all components of $h_\mu{}^A$ to have well-defined wave equations.[9]

The fermion equation can be derived from (4):

$$\left[-\tfrac{1}{2} i(\partial_\mu + iFh_\mu{}^A Z_A)\gamma^\mu - \tfrac{1}{2} i\gamma^\mu(\partial_\mu + iFh_\mu{}^A Z_A) + m\right]\psi = 0. \tag{9}$$

To see the implication of (9) for the classical motion of a particle, let us consider Dirac's equation in electrodynamics. We can multiply the Dirac equation by some factor and neglect small spin effects, so that we have $[(P_\mu + eA_\mu)^2 - m^2]\psi = 0$, in which the operator is just the classical relativistic Hamiltonian[10] (which is equivalent to the well-known Lorentz equation, i.e., the Hamiltonian equation of motion of a charge). By the same procedure as in electrodynamics and neglecting small terms, we obtain the approximate equation of motion of a particle in an external field:

$$(g^{\mu\nu} P_\mu P_\nu - m^2)\psi = 0, \tag{10}$$

in which the operator is interpreted as the relativistic Hamiltonian for classical motion. The approximate relation $g^{\mu\nu}P_\mu P_\nu = m^2$ can also be derived from $\delta S_g = -m\delta\int (g_{\mu\nu} dx^\mu dx^\nu)^{1/2} = 0$, but this extra assumption is not needed here.

In the static case, the action (4) leads to the equations

$$\nabla^2 h_i{}^\nu \approx L^2[\bar\psi\gamma_i P^\nu \psi - (\tfrac{1}{2} FL)\delta_i{}^\nu \bar\psi\psi + (\psi \to \psi_e)], \quad \nabla^2 h_{jk}{}^\nu \approx -\tfrac{1}{4} F\bar\psi\gamma_5\gamma_m \psi \epsilon_{jk}{}^{im}\delta_i{}^\nu + (\psi \to \psi_e) \equiv F\Sigma_{jk}{}^\nu, \tag{11}$$

in the weak-field limit, where only linear terms in gauge fields and "large" source term are retained.

To be consistent with experiments, we should have

$$F=\tfrac{1}{2}L(m+m_e)\approx\tfrac{1}{2}Lm\,, \tag{12}$$

as we shall see later. In order to determine $g_{\mu\nu}$, we apply the approximation of classical physics and of point particle: $P_\mu\approx(m,0,0,0)$, $\bar{\psi}\gamma_0\psi\approx\bar{\psi}\psi\approx\delta^3(\vec{r})$. We find that the nonvanishing components are

$$h_0{}^0\approx-L^2m/8\pi r,\quad h_1{}^1=h_2{}^2=h_3{}^3\approx-h_0{}^0; \tag{13}$$

$$h_{jk}{}^\nu=-\int\frac{F\Sigma_{jk}{}^\nu(r')d^3r'}{4\pi|\vec{r}\,'-\vec{r}\,|}\approx-\int\left(\frac{F\epsilon_{jkm}\bar{U}^P(r')\sigma_m U^P(r')}{16\pi|\vec{r}\,'-\vec{r}\,|}+(U-U_e)\right)d^3r',\quad \text{for } \nu=0,$$

when U^P is the positive-energy Pauli spinor. These relations are sufficient for our purpose because only g_{00} is needed to have the second-order terms in the potential φ. It follows from (7), (12), and (13) that

$$g_{00}\approx1+2\varphi+2\varphi^2,$$
$$g_{kk}\approx-1+2\varphi-2\varphi^2,\quad k=1,2,3; \tag{14}$$
$$\varphi=-L^2m/8\pi r,$$

This agrees with previous experiments because the relativistic Hamiltonian for classical motion is given by (10), provided that $g_{\mu\nu}$ are given by (14) with

$$L^2=8\pi G. \tag{15}$$

Since the Newtonian gravitational constant G is 2.6×10^{-66} cm^2, the relations (12) and (15) give

$$L=8.1\times10^{-33}\text{ cm}, \tag{16}$$

$$F=2\times10^{-19}, \tag{17}$$

where we have used $m=m_p\approx0.94$ GeV in (17).

It is worthwhile to compare the action (4) with those in previous works in which the Lagrangians are also quadratic in the gauge-field strengths.[3-6] The main differences are that the equations for gravity derived from (4) do not involve the third-order differentiation of $g_{\mu\nu}$ and that we have the correct limit for weak fields because of the definition (7) for $g_{\mu\nu}$. Interesting new features in this theory are that gravitational interaction is characterized by the dimensionless constant F and that spin density $\Sigma_{jk}{}^\mu$ is the source of "rotational" gravitational field $h_{jk}{}^\mu$. Because of this we may introduce a physically meaningful torsion $S_\sigma{}^{\mu\nu}$ for space-time by interpreting $e_i{}^\mu e_k{}^\nu h_\sigma{}^{ik}$ as $S_\sigma{}^{\mu\nu}$. Yet the result (13) shows that torsion will have little effect on macroscopic gravitational phenomena because the spin density usually averages to zero on a large scale. However, the spin force between two point protons coming from $h_{jk}{}^\nu$ is about 10^{19} times stronger than that which comes from $h_i{}^\nu$ as one can see from (13):

$$\left|\frac{\text{gravitational spin force}}{\text{Newtonian force}}\right|=\left|\frac{d(\langle\sigma^{jk}\rangle h_{jk}{}^0)/dr}{d(m\varphi)/dr}\right|\sim f^{-1}\sim10^{19} \tag{18}$$

for two protons. Suppose we use strong magnetic field ($H=10^4$ G) to orient the proton spin in a body containing N protons. At temperature $T\sim300$°K, the ratio of the number of protons in two energy states is $N'/N=\exp(-2\mu_pH/kT)$. We can estimate the absolute value of the ratio of the gravitational spin force and the Newtonian force for two such bodies:

$$\frac{f\Delta N^2/r^2}{L^2(Nm)^2/r^2}\approx\frac{\Delta N^2}{N^2f}\approx10^7, \tag{19}$$

where $\Delta N/N=(N-N')/N\approx10^{-6}$. Thus if the magnetic force can be properly shielded and treated, the predicted gravitational spin-force can be detected in a Cavendish-type experiment involving spin densities.

To conclude, the Yang-Mills SO(4, 1) gauge symmetry for gravity implies that the gravitational interaction between fields is characterized by a dimensionless coupling constant and that the gravitational strength is $F^2\approx4\times10^{-38}$. Since $L^2>0$ for the SO(4, 1) group, the usual gravitational force is always attractive, as shown in (14). This is to be contrasted with the SO(3, 2) de Sitter group in which $L^2<0$. Furthermore, the theory agrees with experiments including three classical tests and the time delay of radar echoes passing the sun; it also predicts the existence of the gravitational spin force between fermions. This new force may be either attractive or repulsive, depending on directions of spins. The existence of such a new long-range force may be significant because it could affect the evolution of the universe and stars. Also, the gauge field $h_\mu{}^{jk}$, produced by spin densities of matter, may be reasonably interpreted to be related to the torsion of space-time. Although the spin force may not be detected in the usual gravitational phenomena or in the (Stanford University–National Aeronautics and Space Administration) gyro experiment, it may be tested by a Cavendish-type experiment. It is a challenging problem to carry out this difficult experiment to test the interesting prediction (18).

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Evidence for Breather Excitations in the Sine-Gordon Chain

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Using a molecular-dynamics technique for the classical sine-Gordon chain, we have found that nonlinear breather modes give rise to two excitation branches. A low-frequency resonance is associated with the propagating envelope and a high-frequency peak with the internal oscillations.

Recently, it has been demonstrated that the excitation spectrum of the sine-Gordon chain, associated with density fluctuations, is dominated by a low-frequency resonance, due to kink and antikink solitons and a high-frequency phonon peak. Moreover, the time-dependent mean-square displacement revealed a long-time tail implying the nonexistence of the self-diffusion and diffusion coefficient.[1]

Recognizing that the dynamics of the continuous limit of the sine-Gordon chain is described by the ubiquitous sine-Gordon equation,[2-5] it is known that in addition to the kinks (solitons) and antikinks, there is another important nonlinear mode, the kink-antikink bound state. This mode has been referred to as a "breather"[2] and a "bion."[4] Although breatherlike excitations can be anticipated in a variety of nonlinear equations,[6] their lifetimes are limited. The complete integrability of the sine-Gordon system enhances the lifetime and makes this example especially interesting. Furthermore, breathers have an internal oscillatory degree of freedom which increases their physical potential.[5,6] Unlike the kink, the breather need not require an activation energy, because its rest energy can range from 0 to $2E_0$, where E_0 is the rest energy of the kinks or antikinks.

It is the purpose of this Letter to investigate whether or not the kink and breather modes do give rise to new excitation branches in the discrete and thermalized system. In view of the lack of any reliable analytic estimates for dynamic statistical properties, we used a molecular-dynamics technique simulating a canonical ensemble. Details of this have been given by Schneider and Stoll.[7]

