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Evidence for the Tetrahedral Nature of ^{16}O

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Evidence is given to show that ^{16}O behaves like a tetrahedral rotor with a level sequence $0^+, 3^-, 4^+, 6^+, 7^-, 8^+, \ldots$. The charge form factors for excited states can be predicted from the ground-state form factor and excellent agreement with experiment is obtained for the 3^- and 4^+ states at 6.13 and 10.35 MeV, respectively. The elastic-scattering data are fitted using deformed rather than spherical α clusters.

For many years the collective $E3$ transition strength for the 3^- state at 6.13-MeV excitation energy in ^{16}O has interested nuclear theorists. This state is often said[1] to be predominantly a particle-hole shell-model state with the configuration $d_{5/2}p_{1/2}^{-1}$. Except for Dennison's early work[2] the 3^- state has always been regarded as basically a vibrational excitation. We present here new evidence based on electron scattering that this 3^- state and the 4^+ state at 10.35 MeV are rotational excitations of a tetrahedrally deformed nucleus. As shown below, a pure rotational excitation in lowest order leads to a factorization which allows inelastic form factors to be completely predicted from the ground-state form factor. The major change in the present model from that of Dennison is the use of deformed α clusters.

For a nucleus with a cluster distribution yielding an intrinsic deformation with tetrahedral symmetry, one obtains[2,3] a rotational band with the spin and parity sequence

$$J^\pi = 0^+, 3^-, 4^+, 6^+, 7^-, 8^+, \ldots .$$

The relative energy of these states in lowest order is given by

$$E_0^{J\pi} = (\hbar^2/2I)J(J+1) ,$$

with

$$I \approx I_0 = \tfrac{8}{3} M_\alpha R^2$$

being the moment of inertia of the semirigid system calculated at the equilibrium radial positions R of identical clusters of mass M_α.

Higher-order rotation-vibration corrections have been given by Hecht[4] and involve only two corrections for the ground-state band, i.e.,

$$\Delta E^{J\pi} = -D_s J^2(J+1)^2 - D_t\langle O_{pppp}(\text{tensor})\rangle$$

in which D_s is positive and determines the degree of stretching and the fourth-rank tensor term is an interesting term tabulated by Hecht[4] which gives deviations from the $J^2(J+1)^2$ form. This tensor term is important to explain the fact[2] that the 4^+ level at 10.35 MeV excitation in ^{16}O cannot be understood in a simpler theory with $D_t = 0$. Using the above higher-order theory, one can fit the levels suggested from zeroth-order theory to the sequence 0^+ (g.s.), 3^- (6.13), 4^+ (10.35), and 6^+ (16.29) which requires $B = \hbar^2/2I = 0.5627$ MeV, $D_s = 3.202\times10^{-3}$ MeV, and $D_t = -0.448\times10^{-3}$ MeV. Such a set of parameters yields the higher members[5] with $J^\pi = 7^-$ and 8^+ at 21.19 and 29.18 MeV, respectively. For $J^\pi \gtrsim 8^+$ the theory is rapidly showing signs of not being appropriate as the "correction" terms are too large. It suffices for the present discussion to note that the above simple theory involves only small $\Delta E^{J\pi}$ corrections for the lower-spin members with $J^\pi = 3^-$ and 4^+. The value for B above is consistent with an equilibrium distance of $R = 1.86$ fm, which is close to the value used in the electron-scattering analysis below.

The equilibrium radius vectors $\vec{R}_i$ for the $i = 1, 2, 3, 4$ clusters are arranged tetrahedrally[2] and define a set of body-fixed axes x', y', z'. To calculate electron scattering we transform the position vector $\vec{r}_{ai}$ of a given nucleon (a) belonging to a given cluster (i) to more appropriate coordinates:

$$\vec{r}_{ai} = \vec{r}_{ai} - \vec{R}_i + \vec{R}_i = \Delta\vec{r}_{ai} + \vec{R}_i\,.$$

For a semirigid "molecular" system the ground-state band has a zeroth-order wave function of a product type:

$$\psi_{RV}{}^J = \psi_R{}^J \psi_V{}^J$$

in which the rotational state is a specific linear combination of D-matrix elements for each value of J. The transition-matrix element appropriate to electron scattering exciting a state with spin J in the ground-state band then has a factorized form

$$M(0\to J) = [M_R(0\to J)]M_V$$

in which

$$M_V = \langle\psi_V{}^0|\exp(i\vec{q}\cdot\Delta\vec{r}_{ai})|\psi_V{}^0\rangle$$

is the internal vibrational matrix element (common to the entire band in lowest order) for a given momentum transfer $\vec{q}$.

The rotational term is easily evaluated by integrating $\exp(i\vec{q}\cdot\vec{R}_i)$ over the Euler angles α, β, and γ which define the body-fixed system relative to space-fixed axes x, y, and z. We find

$$\langle\psi_R{}^J|\exp(i\vec{q}\cdot\vec{R}_i)|\psi_R{}^0\rangle = g_J j_J(qR) Y_{JM}{}^*(\hat{q})\,,$$

where $j_J(x)$ is a spherical Bessel function, $Y_{JM}{}^*$ is a spherical harmonic, and g_J is a constant which is determined entirely by the tetrahedral geometry, i.e.,

$$g_J \sim \textstyle\sum_K a_K{}^J Y_{JK}(\Omega_i{}')$$

with $a_K{}^J$ being the coefficients in the D-matrix expansion for the state $\psi_R{}^J$ and $\Omega_i{}'$ represents the polar angles of $\vec{R}_i$ relative to the body axes x', y', z'.

The properly normalized charge form factor factorizes in a similar way to M above and yields the desired relationship between the inelastic and elastic form factors:

$$F_{0J}(q^2) = C_J j_J(qR) F_V(q^2) = \frac{C_J}{C_0}\frac{j_J(qR)}{j_0(qR)} F_{00}(q^2)$$

in which C_J is proportional to g_J from the norm[6] of $F_{0J}(q^2)$ when $F_V(0) = 1$. We find for the present case that $C_0{}^2 = 1$, $C_3{}^2 = 3.89$, and $C_4{}^2 = 2.29$. Since C_J is known and $F_V(q^2)$ is the same vibrational form factor for all J in the band we only need to determine R and $F_V(q^2)$ from the $0^+\to0^+$ data in order to fully predict $F_{0J}(q^2)$ for $J = 3$ and 4.

The results for $|F_{03}|^2$ and $|F_{04}|^2$, with $R = 1.96$ fm and $F_V(q^2)$ taken from the fit to $|F_{00}|^2$, are shown in Fig. 1. The close agreement for both the 3^- discussed below and 4^+ states is quite remarkable in view of the fact that no additional parameters are invoked. As indicated by Bergstrom *et al.*[8] some difficulties are experienced in fitting the 4^+ form factor with more conventional theories.[9] Measurements of the 4^+ form factor at higher q would be interesting since the alternative theories available[8,9] predict a peak at $q \sim 1.3$ fm^{-1}. The $BE(4)$ for the 10.35-MeV level is calculated here to be about 2600 $e^2\cdot\text{fm}^8$ which corresponds to about 3 s.p.u. (single-particle units, as defined by Bernstein[10] for ^{16}O).

The form factor for the 3^- state (also completely *predicted* by the model from the 0^+ form factor) is a remarkably good fit to the data. Although

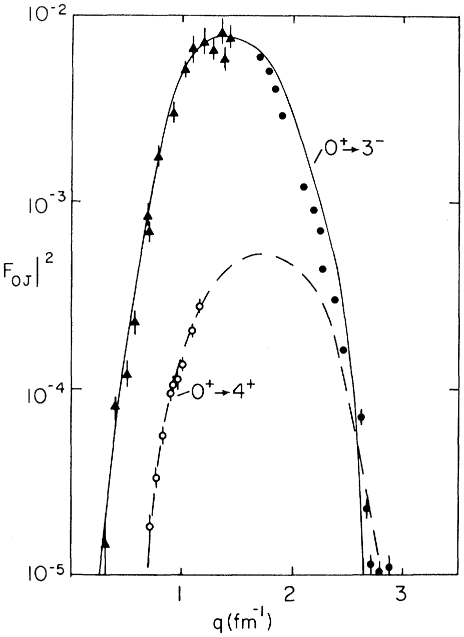

FIG. 1. Theoretical predictions for the inelastic charge form factors for the $0^+ \to 3^-$ transition (solid line) and the $0^+ \to 4^+$ transition (broken line). Experimental points for the 3^- state are from Ref. 7 and for the 4^+ state from Ref. 8. Note that the high-q data and some of the points around the maximum of the 3^- data involve a weak contribution from the unresolved 0^+ state at 6.05 MeV.

this is only a lowest-order calculation we expect from our estimates above of $\Delta E^{J\pi}$ that corrections to F_{03} (and F_{04}) will also be at the 10% level. The $BE(3)$ value is easily calculated from the $|F_{03}|^2$ curve and is found to be 1200 $e^2 \cdot \mathrm{fm}^6$ which is in close agreement with experimental values[10] (1150–1500 $e^2 \cdot \mathrm{fm}^6$). The result for $|F_{03}|^2$ in Fig. 1 is the major result of this work and represents strong evidence for the rotational character of the 3^- (6.13) level in ^{16}O.

I note at this point that I have not shown any evidence for the deformed nature of the α clusters in ^{16}O. To do this requires a model-dependent calculation for $F_V(q^2)$ and a comparison with $F_{00}(q^2)$ from electron scattering. To calculate $F_V(q^2)$, I make the following assumptions: (1) The internal vibrations of a cluster are independent of the relative vibrations of clusters; (2) the zero-point motions, which are all we need here, are harmonic oscillations about the equilibrium position. In this case we have a further factorization,

$$F_V(q^2) = F_{\mathrm{cluster}}(q^2) \exp(-\alpha q^2),$$

with α being related to the average oscillator frequency and $F_{\mathrm{cluster}}(q^2)$ being a form factor for the cluster itself. If the α cluster is spherical, we expect

$$F_{\mathrm{cluster}}(q^2) \approx F_{00}{}^{\alpha}(q^2)$$

as measured via electron scattering from ^{4}He. I was unable, as others have been,[11] to obtain a very good fit to the ^{16}O form factor $F_{00}(q^2)$ with this assumption. Changes in $F_{00}{}^{\alpha}(q^2)$ corresponding to the use of clusters larger or smaller than the free α particle yield no fits at all because of the additional zeroes appearing in $F_{00}(q^2)$. The calculation for spherical clusters (using the above approximation of a free α particle) is shown in Fig. 1 and requires a very small value of $\alpha = 0.06$ fm^2 which is not consistent with the physical values (see below) of $\hbar\omega$ for the cluster model.[2,3] The fit for spherical α particles is not as good as that for deformed clusters for large q values but it could be argued that the oscillator approximation is the cause of this discrepancy. Our concern in the case of the spherical α-particle model is the need for very small values of the oscillator length which appear to be totally inconsistent with the calculated vibrational spectra[3] and the α-particle binding energy of ^{16}O.

If we deform the α cluster in a similar manner (tetrahedrally) to ^{16}O then

$$F_{00}{}^{\alpha}(q^2) = j_0(qR_\alpha) F_V{}^{\alpha}(q^2)$$

and we expect the embedded cluster to satisfy

$$F_{\mathrm{cluster}}(q^2) \approx F_V{}^{\alpha}(q^2),$$

provided that the relative orientations of the deformed clusters in ^{16}O undergo zero-point harmonic vibrations. These latter degrees of freedom are then absorbed into the $\exp(-\alpha q^2)$ term. Using $R_\alpha = 0.98$ fm $= R/2$ as suggested from close packing yields an excellent fit[12] to the ^{4}He charge form factor. Using a value of $\alpha = 0.23$ fm^2 then gives a very good description of $F_{00}(q^2)$ as shown in Fig. 2. The value of the fitting parameter α corresponds to an average oscillator length a (defined by $a^2 = 4\alpha = \langle \hbar / M_\alpha \omega \rangle$ of 0.96 fm for the relative cluster motions. Earlier work[3] on the specific values of $\hbar\omega$ suggest an E-type vibration with $\hbar\omega \sim 4$ MeV and an F-type vibration with $\hbar\omega \sim 6$ MeV. The remaining thirteen dimensions of relative motion between clusters we estimate to have $\hbar\omega \gtrsim 14$ MeV, which yields an average oscillator length satisfying $0.8 < a \lesssim 1.07$ fm in rough agreement with the value needed to describe the electron-scattering data.

In conclusion we emphasize that the relation-

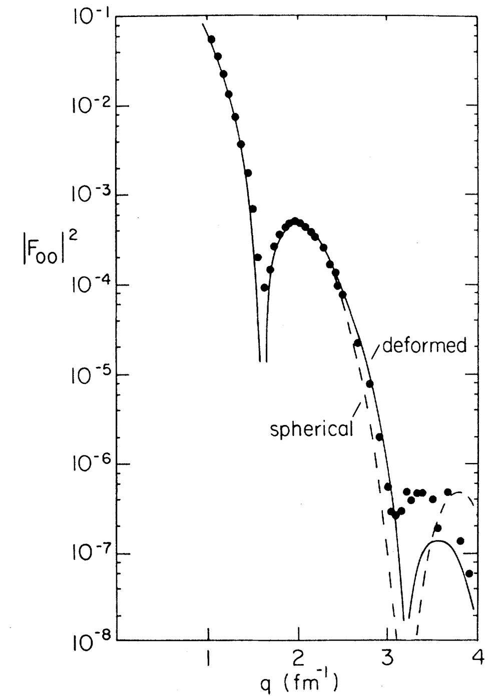

FIG. 2. Theoretical calculations for the elastic charge form factor squared for deformed clusters (full line) with $\alpha = 0.23$ fm^2 and spherical clusters (broken line) with $\alpha = 0.06$ fm^2. The value of $R = 1.96$ fm is taken from the close-packing arguments of Ref. 12. The experimental points are from Ref. 13.

ships proposed here between the 3^-, 4^+ form factors and the elastic form factor for ^{16}O depend only upon the assumption of a tetrahedral rotor and not on the details of the clusters themselves. The arguments for a deformed α cluster given here are not totally convincing, but when combined with a study of other light nuclei[3,12] suggest growing evidence for a tetrahedral α particle as well as for a tetrahedral ^{16}O. A more detailed analysis of the ^{16}O data and other $4N$ nuclei will be presented at a later time.

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