Vacuum Instability and New Constraints on Fermion Masses

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I show that, in the Weinberg-Salam model with one Higgs doublet, in order for the physical vacuum to be an absolute minimum (in the one-loop approximation), certain requirements on the fermion masses have to be met. Specifically, the quantity $\{\sum_i m_{f_i}^4\}^{1/4}$, where the summation extends over fermions, is bounded from above by approximately 133.5 GeV ($\sin^2\theta_W = 0.25$) or 137.7 GeV ($\sin^2\theta_W = 0.2$).

Model-independent analyses[1] of neutral-current data as well as the recent Stanford Linear Accelerator Center polarized-electron scattering experiment[2] have revealed the remarkable fact that the only viable (and by far the simplest) SU(2) $\otimes$U(1) gauge theory of the electromagnetic and weak interactions is the Weinberg-Salam model[3] (often referred to as the standard model). However, the data only tell us, so far, about the symmetry nature of the neutral current and its relative strength to the charged current. These striking facts, although in very good agreement with the standard model, are *not* sufficient[4] to prove its main ingredient, namely the spontaneously broken symmetry nature of gauge theories. Until one actually finds the Higgs boson(s) with couplings which are characteristic of spontaneously broken gauge theories, one must look for indirect effects or requirements that follow from the intrinsic nature of spontaneously broken gauge theories. It is the purpose of this note to point out that one does indeed obtain nontrivial constraints on the fermion masses by just looking at the vacuum instability of the standard model with one Higgs doublet.[5]

From the pioneering works of Coleman and Weinberg[6] and of Weinberg,[6] we know that one-loop radiative corrections to the classical (tree) Higgs potential can drastically change the vacuum structure of the theory. If spontaneous symmetry breaking is to occur, certain relationships between various coupling constants of the theory have to be satisfied. These requirements turn into restrictions on the Higgs and fermion masses. More specifically, we obtain an upper bound (and under a special circumstance, even a lower bound) on the quantity $\{\sum_i m_{f_i}^4\}^{1/4}$, where "$\sum_i$" stands for the sum over fermions.

I restrict myself to the case of a single scalar doublet in the standard model. The zero- and one-loop contributions of the effective Higgs potential $V(\varphi_c)$ are given by[6]

$$V(\varphi_c) = -\tfrac{1}{2}\mu_R^2\varphi_c^2 + (\lambda/4!)\varphi_c^4 + \kappa\varphi_c^4[\ln(\varphi_c^2/\langle\varphi\rangle^2) - \tfrac{25}{6}], \quad (1)$$

where I have used the renormalization conditions

$$[d^2V(\varphi_c)/d\varphi_c^2]_{\varphi_c=0} = -\mu_R^2 \quad (\mu_R^2 > 0), \quad (2)$$

$$[d^4V(\varphi_c)/d\varphi_c^4]_{\varphi_c=\langle\varphi\rangle} = \lambda, \quad (3)$$

with $\kappa = (64\pi^2)^{-1}\{3(2g_{WH}^4 + g_{ZH}^4) + \lambda^2/4 - 4\sum_i g_{f_iH}^4\}$ and $\varphi_c^2 = \varphi_c^\dagger\varphi_c$. The constants g_{WH}, g_{ZH}, and g_{f_iH} stand for the couplings of the Higgs boson to the $W^\pm$, Z bosons and the fermions, respectively. They are given, in the standard model, by $g_{WH}^2 = g^2/4$, $g_{ZH}^2 = g^2/(4\sec^2\theta_W)$, where $g^2/8m_W^2 = G_F/\sqrt{2}$. We can rewrite Eq. (1) as follows:

$$V(\varphi_c) = -\tfrac{1}{2}\mu_R^2\varphi_c^2 + \frac{\lambda_{int}}{4!}\varphi_c^4 + \kappa\varphi_c^4\ln\left(\frac{\varphi_c^2}{\langle\varphi\rangle^2}\right), \quad (4)$$

where $\lambda_{int} = \lambda - 100\kappa$. For the one-loop approximation to be reliable, one needs $\lambda, g^2, g_{f_iH}^2 \ll 1$ and $(\lambda, g^2, g_{f_iH}^2)\ln(\varphi_c^2/\langle\varphi\rangle^2) \ll 1$. Since we are looking for an upper bound on the fermion masses, we *will not* neglect the contributions to $V(\varphi_c)$ from fermion loops in Eq. (4).

The local minimum of $V(\varphi_c)$ and the physical mass of the Higgs boson are defined by

$$[dV(\varphi_c)/d\varphi_c]_{\varphi_c=\langle\varphi\rangle} = 0, \quad (5)$$

$$[d^2V(\varphi_c)/d\varphi_c^2]_{\varphi_c=\langle\varphi\rangle} = m_H^2. \quad (6)$$

Using Eqs. (4), (5), and (6), one then obtains

$$\langle\varphi\rangle^2[2\kappa + \tfrac{1}{6}\lambda_{int}] = \mu_R^2, \quad (7)$$

$$m_H^2 = 8\langle\varphi\rangle^2[\tfrac{3}{2}\kappa + \lambda_{int}/4!]. \quad (8)$$

The effective potential evaluated at $\varphi_c = \langle\varphi\rangle$ is given by

$$V(\langle\varphi\rangle) = -\langle\varphi\rangle^4(\kappa + \lambda_{int}/4!). \quad (9)$$

We then require the local minimum to be *absolute*, i.e.,

$$V(0) - V(\langle\varphi\rangle) > 0. \qquad (10)$$

With $V(0) = 0$ and using Eq. (9), the condition (10) turns into $\kappa + \lambda_{\rm int}/4! > 0$, which in turn implies

$$m_{\rm H}^2 > 4\langle\varphi\rangle^2\kappa. \qquad (11)$$

Notice that the right-hand side of the inequality (11) becomes the Weinberg-Linde[7] lower bound on the Higgs mass if we neglect the contributions from the scalar and fermion loops.

So far I have obtained only a constraint on the Higgs-boson mass as represented by the inequality (11). To get a condition on fermion masses, we need to take a closer look at the effective potential $V(\varphi_c)$ as given by Eq. (4). We may ask the following question: Is the local minimum given by Eq. (7) truly an *absolute* minimum? Under what conditions does that vacuum become unstable?

To answer the above questions, let us examine $V(\varphi_c)$ carefully. From Eq. (4), one can see that for $\kappa < 0$, the effective potential $V(\varphi_c)$ is unbounded from below for asymptotic values of φ_c. What $\kappa < 0$ means is $\sum_i g_{f_i\rm H}^4 > \frac{1}{4}\{3(2g_{W\rm H}^4 + g_{Z\rm H}^4) + \frac{1}{4}\lambda^2\}$. Repeating an argument due to Krive and Linde[8] who examined a simplified version of the σ model, one can say that for $g^2, \lambda \ll g_{f_i\rm H}^2$ ($\kappa < 0$), there is a certain value of φ_c, say $\tilde\varphi > \langle\varphi\rangle$, for which $g_{f_i\rm H}^2 \ln(\tilde\varphi^2/\langle\varphi\rangle^2) \simeq \lambda/g_{f_i\rm H}^2$ or $g^2/g_{f_i\rm H}^2 \ll 1$, $\lambda \times \ln(\tilde\varphi^2/\langle\varphi\rangle^2) \simeq \lambda^2/g_{f_i\rm H}^4 \ll 1$, $g^2\ln(\tilde\varphi^2/\langle\varphi\rangle^2) \simeq g^4/g_{f_i\rm H}^4 \ll 1$, and $V(\varphi) < V(\langle\varphi\rangle)$. In such a case the one-loop approximation is still reliable. One can see that the local minimum at $\langle\varphi\rangle$ is unstable and is not the true vacuum of the theory. The true stable vacuum then occurs only at an asymptotically large value of φ_c, in which case one cannot rely on perturbation theory anymore. In Coleman's terms,[9] the local minimum at $\langle\varphi\rangle$ is then a false vacuum.

Let us *assume* that the minimum at $\langle\varphi\rangle$ is actually the *true* vacuum which we live in. We then have the condition $\kappa \geq 0$, which means that

$$\sum_i g_{f_i\rm H}^4 \leq \tfrac{1}{4}\{\tfrac{3}{16}g^4(2 + \sec^4\theta_{\rm W}) + \tfrac{1}{4}\lambda^2\}. \qquad (12)$$

With $m_{f_i}^2 = g_{f_i\rm H}^2\langle\varphi\rangle^2$, $m_W^2 = \frac{1}{4}g^2\langle\varphi\rangle^2$, $m_Z^2 = \frac{1}{4}g^2 \times \langle\varphi\rangle^2\sec^2\theta_{\rm W}$, $\langle\varphi\rangle^2 = (\sqrt{2}\,G_{\rm F})^{-1}$, we can rewrite the constraint (12) as

$$\sum_i m_{f_i}^4 \leq \tfrac{3}{4}m_W^4(2 + \sec^4\theta_{\rm W}) + \lambda^2/32G_{\rm F}^2, \qquad (13)$$

where m_{f_i} is the mass of the ith fermion. Equation (13) is the basic constraint imposed on the fermion masses in the standard model with a single Higgs doublet.

For the validity of the one-loop approximation, one needs, of course, $\lambda \ll 1$. I shall indulge myself in letting $\lambda \leq 1$ as required for the validity of perturbation theory. Armed with this requirement, I now distinguish two cases.

Case (a), $\kappa > 0$.—The bound (13) now becomes

$$\sum_i m_{f_i}^4 < G_{\rm F}^{-2}\{\tfrac{3}{8}\pi^2\alpha^2\csc^4\theta_{\rm W}(2 + \sec^4\theta_{\rm W}) + \tfrac{1}{32}\}, \qquad (14)$$

where I have used $m_W^2 = (\pi\alpha/\sqrt{2}G_{\rm F})\sin^{-2}\theta_{\rm W}$, $\alpha \equiv e^2/4\pi$. I obtain

$$\{\textstyle\sum_i m_{f_i}^4\}^{1/4} \begin{cases} < 133.5 \text{ GeV } (\sin^2\theta_{\rm W} = 0.25) \\ < 137.7 \text{ GeV } (\sin^2\theta_{\rm W} = 0.2) \end{cases}. \qquad (15)$$

Taking the masses of known quarks to be $m_u \sim 4$ MeV, $m_d \sim 7$ MeV, $m_c \sim 1.2$ GeV, $m_s \sim 150$ MeV, (?), $m_b \sim 4.6$ GeV, and also taking into account the masses of e, μ, and τ, one can see that the upper bound (15) is rather insensitive to those "light" fermion masses. Therefore in (14), we can make the following replacement $\{\sum_i m_{f_i}^4\}^{1/4} \to \{\sum_i m_{f_i}^4\}^{1/4}_{\rm heavy}$, where the term "heavy" means that $\sum_i$ is to extend over fermions other than known ones. I will neglect the effects of strong interactions on the quark masses.

If there are heavy fermions or a large number of lighter ones which obey the bounds (15), one can see that the lower bound (11) on the Higgs mass can be significantly smaller than the Weinberg-Linde value which is just $(3\sqrt{2}G_{\rm F}/16\pi^2) \times [m_W^4(2 + \sec^4\theta_{\rm W})]$ ($m_{\rm H} > 5$ GeV for $\sin^2\theta_{\rm W} \simeq 0.25$). Furthermore, as pointed out by some authors,[10] the vacuum at $\langle\varphi\rangle \neq 0$ is a metastable one if $m_{\rm H}^2 < 8\langle\varphi\rangle^2\kappa$, for then the other minimum is at $\varphi_c = 0$. As Linde[11] has shown, if the early Universe was in a metastable vacuum, then for spontaneous symmetry breaking to occur the Higgs mass has to obey $m_{\rm H} > 260$ MeV, where the contributions from fermions to the effective potential have been neglected. If, however, heavy fermions do exist, we would expect the bound to be much lower than 260 MeV.[12]

Case (b), $\kappa = 0$.—This case is interesting in its own right. The contributions from gauge boson, Higgs boson, and fermion loops miraculously cancel each other. In this case, we have an equality in (13) and, for $0 \leq \lambda \leq 1$, the fermion masses obey

$$96.8 \text{ GeV} \leq \{\textstyle\sum_i m_{f_i}^4\}^{1/4}_{\rm heavy} \leq 133.5 \text{ GeV } (\sin^2\theta_{\rm W} = 0.25), \qquad (16)$$

$$106.6 \text{ GeV} \leq \{\textstyle\sum_i m_{f_i}^4\}^{1/4}_{\rm heavy} \leq 137.7 \text{ GeV } (\sin^2\theta_{\rm W} = 0.2), \qquad (17)$$

where again the contributions from known fermions to both upper and lower bounds are negligible.

The effective potential then takes the following familiar form

$$V(\varphi_c) = -\tfrac{1}{2}\mu_R^2\varphi_c^2 + (\lambda/4!)\varphi_c^4. \quad (18)$$

The *only* minimum occurs at $\langle\varphi\rangle^2 = 6\mu_R^2/\lambda$, and is *absolutely* stable. The Higgs mass is then given by

$$m_H^2 = \tfrac{1}{3}\lambda\langle\varphi\rangle^2. \quad (19)$$

One can see from (19) that there no longer exists any restriction on the Higgs mass and it can be arbitrarily small if λ and μ_R^2 are themselves sufficiently small. If the Higgs boson is discovered to have an *extremely* small mass, one is tempted to *conjecture* that the case $\kappa = 0$ is what happens in nature and one should be ready for another generation of heavy fermions with masses in the range of 40–100 GeV.

My discussions can be generalized to the case where we have more than one Higgs doublet. I suspect that the results presented here will not be much affected by the inclusion of many Higgs doublets. I also wish to point out that using partial-wave unitarity at high energies, Chanowitz, Furman, and Hinchliffe[13] have established upper bounds on quark and lepton masses which are significantly higher than the ones presented here. Their bounds are $(500/\sqrt{N})$ GeV and $(1.0/\sqrt{N})$ TeV for quark and leptons separately, with N being the number of flavor doublets.

The discovery of *new* flavors of quarks and leptons with high masses would be extremely important for our understanding of the nature of spontaneously broken gauge theories. One can look for these fermions either by the proposed LEP (Large Electron-Positron) machine,[14] or indirectly through radiative corrections to low-energy processes such as the ones discussed by Veltman[13] or Chanowitz *et al.*[13]

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Note added.—A weaker constraint is imposed on fermion masses if, instead of $\lambda \leq 1$, we require $\lambda^2/256\pi^2 \leq 1$ or $\lambda \leq 16\pi$. This requirement coincides with that of Lee, Quigg, and Thacker[13] who, using partial-wave unitarity, found $m_H^2 \leq 8\pi\sqrt{2}/3G_F$ [or $m_H^2 \lesssim (1.0\ \text{TeV})^2$] with m_H^2 given by Eq. (19). In such a case, the *absolute* upper

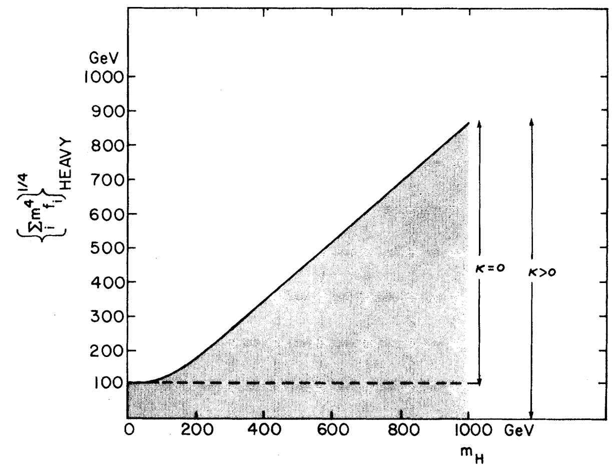

FIG. 1. The allowed region (indicated by the shaded area) of $\{\sum_i m_{f_i}^4\}^{1/4}_{\text{heavy}}$ as a function of m_H (1.0 GeV is the absolute upper bound on m_H) for $\kappa \geq 0$ and for $\sin^2\theta_W = 0.2$.

bound on $\{\sum_i m_{f_i}^4\}^{1/4}$ is now given by

$$\{\textstyle\sum_i m_{f_i}^4\}^{1/4} < 873\ \text{GeV}, \quad (20)$$

instead of the upper bonds found in (15), (16), and (17). Ignoring radiative corrections to the Higgs-boson mass, one can rewrite the constraint (13), using Eq. (19), as

$$\{\textstyle\sum_i m_{f_i}^4\}^{1/4} < \{\tfrac{3}{4}m_W^4(2+\sec^4\theta_W) + \tfrac{9}{16}m_H^4\}^{1/4}. \quad (21)$$

The constraint (21) is then plotted in Fig. 1 as a function of the Higgs-boson mass m_H.

I wish to thank Professor J. D. Bjorken for valuable comments and for suggesting the idea of a mass plot as depicted in Fig. 1.

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Evidence for the Tetrahedral Nature of ^{16}O

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Evidence is given to show that ^{16}O behaves like a tetrahedral rotor with a level sequence $0^+, 3^-, 4^+, 6^+, 7^-, 8^+, \ldots$. The charge form factors for excited states can be predicted from the ground-state form factor and excellent agreement with experiment is obtained for the 3^- and 4^+ states at 6.13 and 10.35 MeV, respectively. The elastic-scattering data are fitted using deformed rather than spherical α clusters.

For many years the collective $E3$ transition strength for the 3^- state at 6.13-MeV excitation energy in ^{16}O has interested nuclear theorists. This state is often said[1] to be predominantly a particle-hole shell-model state with the configuration $d_{5/2}p_{1/2}^{-1}$. Except for Dennison's early work[2] the 3^- state has always been regarded as basically a vibrational excitation. We present here new evidence based on electron scattering that this 3^- state and the 4^+ state at 10.35 MeV are rotational excitations of a tetrahedrally deformed nucleus. As shown below, a pure rotational excitation in lowest order leads to a factorization which allows inelastic form factors to be completely predicted from the ground-state form factor. The major change in the present model from that of Dennison is the use of deformed α clusters.

For a nucleus with a cluster distribution yielding an intrinsic deformation with tetrahedral symmetry, one obtains[2,3] a rotational band with the spin and parity sequence

$$J^\pi = 0^+, 3^-, 4^+, 6^+, 7^-, 8^+, \ldots .$$

The relative energy of these states in lowest order is given by

$$E_0^{J\pi} = (\hbar^2/2I)J(J+1),$$

with

$$I \approx I_0 = \tfrac{8}{3} M_\alpha R^2$$

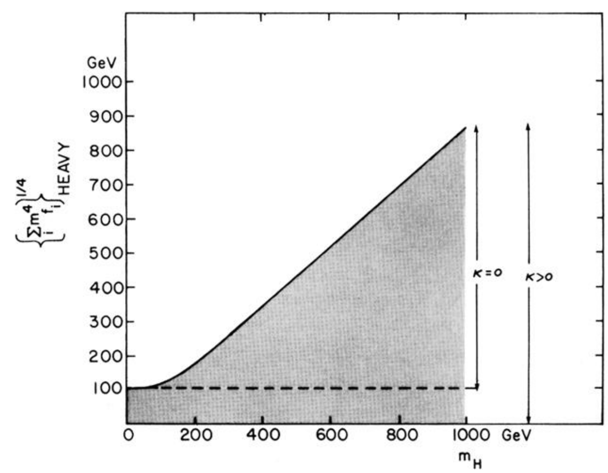

FIG. 1. The allowed region (indicated by the shaded area) of $\{\Sigma_i m_{f_i}{}^4\}^{1/4}_{\text{heavy}}$ as a function of $m_{\rm H}$ (1.0 GeV is the absolute upper bound on $m_{\rm H}$) for $\kappa \geq 0$ and for $\sin^2\theta_{\rm W} = 0.2$.