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Coriolis Interaction

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A possible explanation of the intriguing problem of the so-called Coriolis attenuation factors, in the framework of the particle-plus-rotor model, is suggested. The explicit inclusion of the recoil term renders unnecessary the assumption of an attenuation factor for the Coriolis interaction and leads to a hitherto unsuspected parallelism with the cranking model.

For several years it has been generally accepted[1,2] that the particle-plus-rotor model[2] (PRM) is able to reproduce Coriolis-distorted bands quantitatively only if the strength of the Coriolis interaction is considerably reduced; this has remained as an outstanding puzzle in these calculations. In order to compensate for the supposedly too large Coriolis matrix elements purely *ad hoc* attenuation factors have been introduced.[1] It has also been asserted[3,4] that such a problem does not appear if the cranking model (CM) is applied. In this Letter I show that a complete treatment of the PRM gives indeed a satisfactory answer.

The Hamiltonian to be used is given by[2]

$$\begin{aligned} H &= (\hbar^2/2\theta)\vec{R}^2 + h_0 \\ &= (\hbar^2/2\theta)[I(I+1) - I_3{}^2 - 2\vec{I}^\perp\circ\vec{j}^\perp + \vec{j}^{\perp 2}] + h_0 . \end{aligned} \tag{1}$$

The superscript $\perp$ denotes those components lying in the plane perpendicular to the symmetry axis. h_0 comprises a one-body operator for the mean field (a Nilsson Hamiltonian) and a two-body term which accounts for the pairing correlations (which has been treated here in the BCS approximation). Recalling that $I_3 = j_3$, Eq. (1) may be rewritten in the following way:

$$H = (\hbar^2/2\theta)[I(I+1) - j^2 - 2\vec{j}^\perp\circ(\vec{I}^\perp - \vec{j}^\perp)] + h_0 . \tag{2}$$

The true Coriolis interaction is

$$\begin{aligned} H_C &= -(\hbar^2/\theta)\vec{j}^\perp\circ(\vec{I}^\perp - \vec{j}^\perp) \\ &= -\hbar\vec{j}\cdot(\hbar\vec{R}/\theta) = -\hbar\vec{j}\circ\vec{\omega} . \end{aligned} \tag{3}$$

Indeed, the field producing the Coriolis effect is the angular velocity associated with the collective rotation and the expectation value of the Coriolis interaction [calculated with the eigenstates of the full Hamiltonian of Eq. (1)] becomes

$$\langle H_C\rangle = -\frac{\hbar^2}{\theta}\langle\vec{j}^\perp\cdot\vec{I}^\perp\rangle\left(1 - \frac{\langle\vec{j}^\perp\cdot\vec{j}^\perp\rangle}{\langle\vec{j}^\perp\cdot\vec{I}^\perp\rangle}\right). \tag{4}$$

As compared to the situation in which the recoil term ($j^{\perp 2}$) is neglected (which amounts to considering the interaction with the total angular momentum rather than R) it is seen that the explicit inclusion of this term naturally leads to an attenuation of the particle-rotation coupling $\langle\vec{j}^\perp\cdot\vec{I}^\perp\rangle$.

As we restrict ourselves to a basis of one-quasiparticle-plus-rotor states for the description of odd-A nuclei, only the one-quasiparticle part of the recoil operator has to be considered. The importance of this term in connection with the location of bandhead energies has already been stressed by Osnes, Rekstad, and Gjøtterud.[5] Indeed, this term mainly produces a significant renormalization of the single-quasiparticle spectrum thus affecting in a major way the effect of the nondiagonal part of the Coriolis interaction. In cases where the quasiparticle states strongly coupled by the Coriolis interaction have the same high-j unique-parity parentage essentially only H_C (and h_0) determines the wave functions because the j^2 operator is nearly constant.

We come now to the comparison between the

PRM and the cranking model (CM) in its most simple form, the so-called particle-plus-cranked-rotor model (PCRM).[3] More subtle effects, like the angular momentum dependence of the self-consistent fields and state-dependent variation of the moment of inertia cannot be studied with the PCRM but are, nevertheless, negligible for relatively low spin values.[3] The PCRM treats the core as rigid [which as stated, is borne out by the fully self-consistent constrained Hartree-Fock-Bogoliubov (HFB) calculation[3]] and is therefore especially suited for a comparison with the PRM. The cranked Hamiltonian is

$$H' = h_0 - \omega\hbar j_1, \tag{5}$$

where the angular velocity is determined for each spin from the subsidiary condition

$$[I(I+1) - \langle j_3^2\rangle]^{1/2} = \theta\omega/\hbar + \langle j_1\rangle. \tag{6}$$

With ω obtained from Eq. (6) the Coriolis interaction is rewritten[3] as

$$-\frac{\hbar^2}{\theta} j_1[I(I+1) - \langle j_3^2\rangle]^{1/2} \times \left(1 - \frac{\langle j_1\rangle}{[I(I+1) - \langle j_3^2\rangle]^{1/2}}\right). \tag{7}$$

In fact, both "attenuations" implied by Eqs. (4) and (7) become similar for I larger than a certain value (see example below). The similarity between the two procedures can be exhibited even further by writing Eq. (6) as

$$I(I+1) - \langle j_3^2\rangle = (\theta\omega/\hbar)^2 + (2\theta\omega/\hbar)\langle j_1\rangle + \langle j_1\rangle^2 \tag{8}$$

and its equivalent in the PRM

$$I(I+1) - \langle j_3^2\rangle = \langle \vec{R}^2\rangle + 2\langle \vec{R}\cdot\vec{j}^{\perp}\rangle + \langle \vec{j}^{\perp 2}\rangle. \tag{9}$$

I now discuss how far one can go in the description of experiments with the PRM through its application to the concrete case of positive-parity high-spin bands of $\nu i_{13/2}$ parentage in rare-earth nuclei where the Coriolis effects are known to be very large. Then the results obtained with the PCRM for the same case are presented and the differences discussed. In this case the only core parameter is the moment of inertia which is known to be normally larger by $\simeq 20\%$ than that of the neighboring even-even (e-e) nucleus.[2] Two different procedures have been adopted: *Theory 1.*—θ of the rotor for the odd system has been taken as the average value of the two e-e neighbors ($\bar{\theta}_{ee}$). This amounts to neglecting the influence of the odd particle on the e-e core associated with the rotor. *Theory 2.*—The moment of inertia has been considered adjustable and allowed to vary in the range $\bar{\theta}_{ee} \leq \theta \leq 1.2\bar{\theta}_{ee}$ (best values are $\hbar^2/2\theta$ = 16.76, 14.54, and 13.40 keV for ^{157,159,161}Dy, respectively). The other type of parameters refer to the single-quasiparticle properties. The quadrupole deformation is taken from the measured half-lives of the 2_1^+ states of the $A-1$ e-e cores.[6] The single-particle energies and wave functions at these deformations were obtained using the Nilsson parameters from Ref. 7. The strength of the pairing interaction was fixed so as to reproduce a gap corresponding to the value (Δ = 660 keV) given by the blocked HFB equations.[4] Eventual uncertainties in the single-particle spectrum are expected to be absorbed by the variation of the Fermi level (λ) within reasonable limits. These values referred to the energy of the $\Omega = \frac{5}{2}$ state are given in Table I. As may be seen in Fig. 1 the agreement is satisfactory if one has in mind that theory 1 has one adjustable parameter (λ) and theory 2, two parameters (λ and θ).

In order to compare the PRM and PCRM reliably, a cranking calculation has been done for ^{159}Dy using exactly the same parameters as in theory 2. (All states originating in the spherical $i_{13/2}$ state are used throughout.) The resulting spectrum is displayed in Fig. 1 and labeled as theory 3. The differences between the two procedures are not too large but it is evident that the Coriolis effects are somewhat smaller for the lower-spin states in the PCRM. This is more clearly seen in Fig. 2 where the amplitudes for all the basis states are displayed as a function of Ω for $\frac{5}{2} \leq I \leq \frac{33}{2}$ for theory 2 (full lines) and theory 3 (dashed lines). We see that the cranking wave functions, especially for the favored states $I = \frac{9}{2}, \frac{13}{2}, \ldots$, are more concentrated on the $\Omega = \frac{5}{2}$ state. For $I \gtrsim \frac{19}{2}$ both descriptions merge into each other.

To gain even more insight into the differences between the two models Fig. 3 shows the behavior of two quantities (as functions of I) related to the collective rotation, corresponding to the wave

TABLE I. Strength of the pairing interaction in D_y from theory 1 and theory 2.

Dy mass	Theory 1 (keV)	Theory 2 (keV)
157	−521	−630
159	−377	−481
161	−90	−224

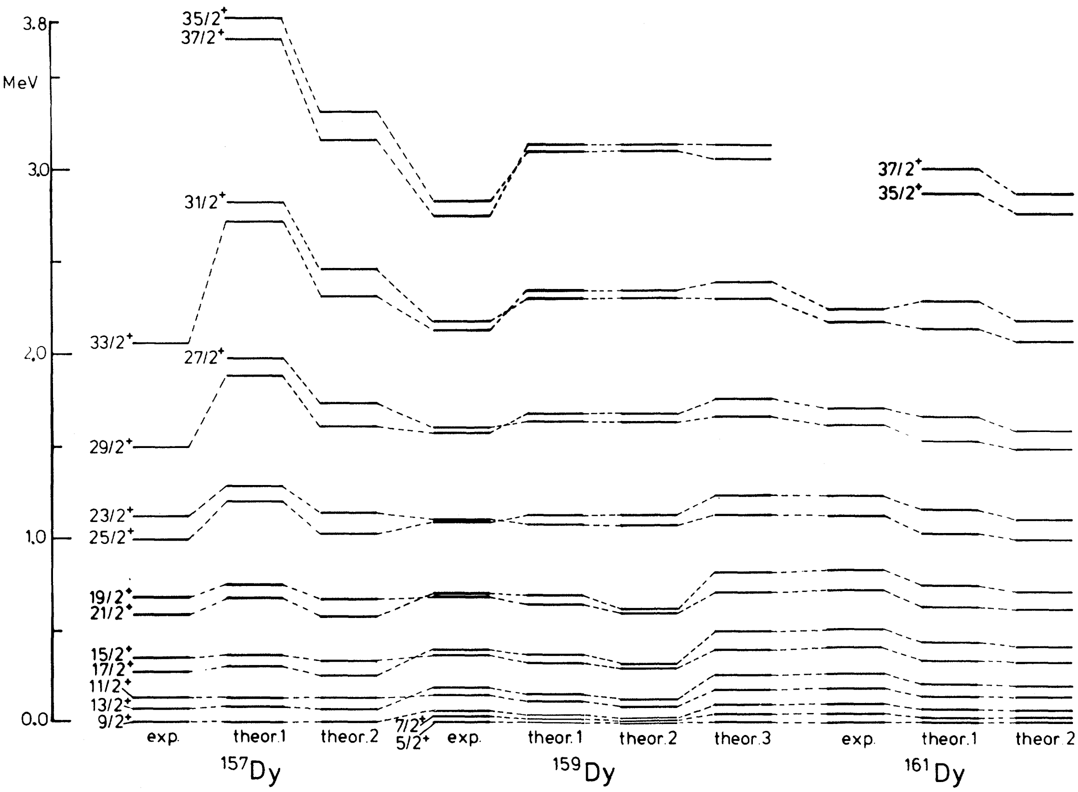

FIG. 1. Experimental and calculated (see text) $\nu i_{13/2}$ bands in three different odd Dy isotopes (data from Ref. 1).

functions of Fig. 2. For the PRM it is $[R(R+1)]^{1/2}$ defined as $(\langle R^2\rangle)^{1/2}=[R(R+1)]^{1/2}$ and for the PCRM it is $\xi=\theta\omega/\hbar$ (the classical core angular momentum in units of $\hbar$). As anticipated in the discussion of the wave functions one sees that the major differences appear at low spin values.

While the $[R(R+1)]^{1/2}$ values lie more or less on a parabola (or better, two parabolas, one for each

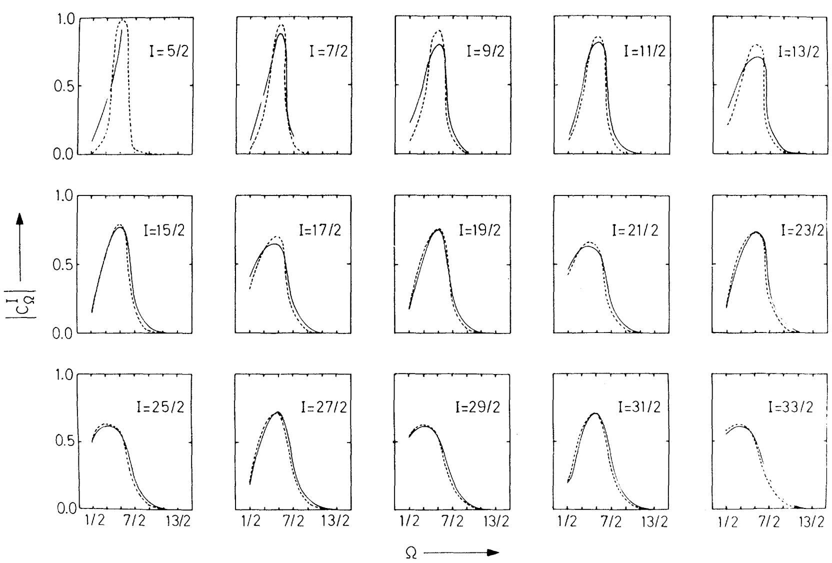

FIG. 2. Amplitudes over the seven Nilsson basis states as functions of total angular momentum $I=\frac{5}{2}-\frac{33}{2}$ for ^{159}Dy. Full lines correspond to the PRM (theory 2) solutions, dashed lines to the PCRM (theory 3) solutions.

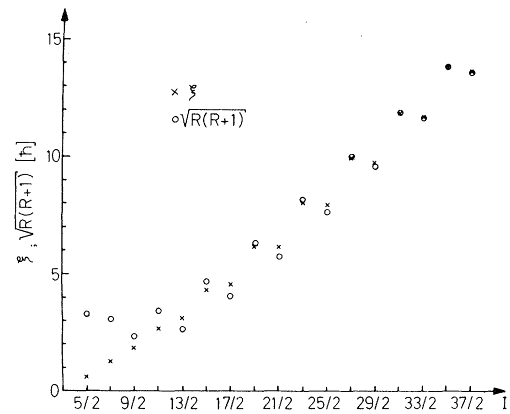

FIG. 3. PCRM (ξ) and PRM $\{[R(R+1)]^{1/2}\}$ angular momenta corresponding to the wave functions of Fig. 2.

family of states pertaining to a definite signature[2]) the ξ values first increase in an approximately monotonic way. In the PRM case the motion of the rotor is described in a fully quantal manner. $\vec{R}$ is a vector operator, free to move in the plane perpendicular to the 3 axis, and R^2 has a spectrum of eigenvalues restricted to even integers and its quantal nature is especially important at low spin values. For total spin values near $\frac{13}{2}$, the tendency of the system is to try to minimize the rotational energy since $\vec{R}=\vec{I}-\vec{j}$. On the other hand, the angular velocity is a classical vector directed along the 1 axis in the PCRM just restricted by the constraint (6) which allows ω to take smaller values thus giving purer wave functions. Both models become equivalent as fluctuations become more unimportant.

To summarize, the PRM is shown to provide a fair reproduction of strongly Coriolis-distorted bands in the deformed rare-earth region. The need of introducing *ad hoc* attenuation factors appears to stem from the neglect of the recoil term in the PRM Hamiltonian. The latter transforms the conventionally called Coriolis interaction $-\hbar^2 \times (\vec{I}^\perp \cdot \vec{j}^\perp)/\theta$ into the true one $-\hbar\vec{j}\cdot\vec{\omega}$. While the Nilsson model describes an independent-particle picture in the intrinsic reference frame, the recoil term brings in an aspect of the many-body problem through the moment of inertia of the core. The recoil term cannot be absorbed into the mean field in a universal form and thus has to be considered explicitly.

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Influence of Retardation on the Angular Distribution of Radiative Electron Capture

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Angular distributions and forward-backward intensities of photons emitted during electron capture have been measured in collisions between fast projectile ions (93-MeV oxygen, 110–123-MeV sulfur) and target atoms. In contrast to previous expectations, the radiation pattern in the laboratory system is not strongly shifted in forward direction but turned out to exhibit forward-backward symmetry independent of the projectile velocity. We attribute these findings to a cancellation between the Doppler-shift and retardation effects.

Radiative electron capture (REC) is a collision process in which an ion captures an electron and emits a photon. REC of free electrons has been known for a long time, mainly from work in astrophysics and plasma physics where it is an important recombination process. More recently,