mined quite accurately as $\pm\Omega x_s(1/\Omega - 1 - k_y^2\rho_s^2)^{1/2}$. If we ignore the outer turning points, then, since the wave is propagating inside the inner turning points, the eigenvalue is determined by the quantization conditions

$$\int_{-E}^{E} Q^{1/2}(\Omega, \eta)\, d\eta = (n + \tfrac{1}{2})\pi, \quad n = 0, 1, 2 \ldots, \qquad (15)$$

where $Q(\Omega, \eta)$ is the real potential. Because of the presence of outer turning points, the wave cannot remain trapped and, therefore, no real-Ω eigensolution is possible. However, if the tunneling factor $\exp(-\int_E^P Q^{1/2}\, d\eta)$ is small, then it is possible to consider the eigenvalue Ω as determined by Eq. (15) to be a resonance. For real Ω sufficiently far from resonance, $r(\Omega) + i = O(\exp(-\int_E^P Q^{1/2}\, d\eta))$ and so $R(\Omega) = O(1)$; in this case there is no significant convective amplification. On the other hand, for Ω very close to resonance, $|r(\Omega) + i| = O(1)$ and both the reflection and transmission coefficients are of order $\exp(\pi a)$ which is assumed to be large. Thus, we can identify $\exp(\pi a)$ as the convective amplification factor, which can be estimated using the data of Ref. 8. Taking $m_i/m_e = 1837$, $L_s/L_n = 50$, and $k_y^2\rho_s^2 = 0.5$, we find the resonance frequency to be $\Omega \simeq 0.55$. We thus have $\pi a \simeq 13.74$ and the amplification factor is exponentially large, $O(10^6)$![12]

In conclusion we have shown that there are no unstable drift modes and drift-Alfvén modes of either parity and that the convective amplification of the electrostatic drift modes can become exponentially large. The obvious implication of the latter result is that we may expect anomalous transport of a plasma confined in a sheared magnetic field due to the convectively unstable drift modes.

This work was supported by the National Science Foundation Grant No. PHY-77-12873, and the U. S. Department of Energy, Contracts No. EY-76-C-02-3073, No. EY-76-S-03-0034-PA128, and No. W-7405-ENG-48.

[1]D. W. Ross and S. M. Mahajan, Phys. Rev. Lett. 40, 325 (1978).

[2]K. T. Tsang, P. J. Catto, J. C. Whitson, and T. Smith, Phys. Rev. Lett. 40, 327 (1978).

[3]T. M. Antonsen, Phys. Rev. Lett. 41, 33 (1978).

[4]K. T. Tsang, J. C. Whitson, J. D. Callen, and J. Smith, Phys. Rev. Lett. 41, 557 (1978).

[5]P. J. Catto, J. A. El Nadi, C. S. Liu, and M. N. Rosenbluth, Nucl. Fusion 14, 405 (1974).

[6]L. Chen, J. Hsu, P. K. Kaw, and P. H. Rutherford, Nucl. Fusion 18, 1371 (1978).

[7]J. F. Drake and Y. C. Lee, Phys. Fluids 20, 1341 (1977); L. Chen, P. H. Rutherford and W. M. Tang, Phys. Rev. Lett. 39, 460 (1977).

[8]L. Chen, P. N. Guzdar, R. B. White, P. K. Kaw, and C. Oberman, Phys. Rev. Lett. 41, 649 (1978).

[9]Y. C. Lee, Bull. Am. Phys. Soc. 23, 798 (1978).

[10]It is necessary to consider two branches of eigenmodes with frequencies $\Omega_1(\lambda_i)$ and $\Omega_2(\lambda_i)$ such that, as $\lambda_i \to 0^+$, $\Omega_1(\lambda_i) \to O(1)$ and $\Omega_2(\lambda_i) \to O(\lambda_i)$. A detailed presentation of this more general s-matrix technique will be published elsewhere.

[11]F. W. J. Olver, Proc. Roy. Soc. London, Ser. A 250, 479 (1958), and private communications.

[12]This enormous amplification indicates that the system has eigenmodes with negligibly small damping rates, which cannot be distinguished from marginally stable modes by numerical and WKBJ analyses.

Effect of Beam Scattering on Plasma Heating with Relativistic Electron Beams

J. D. Sethian[a] and C. A. Ekdahl[b]
Laboratory of Plasma Studies, Cornell University, Ithaca, New York 14850
(Received 20 December 1978)

A fully ionized plasma column was heated with a relativistic electron beam. Experiments in which plasma heating was dominated by the beam-plasma two-stream interaction demonstrated a scaling of energy deposition with the beam-to-plasma density ratio and the beam angular scatter in accordance with a theorectial model of the two-stream instability. Beam-to-plasma energy-transfer efficiencies exceeding 25% and plasma electron temperatures of 600 eV were observed.

The availability of high-power relativistic-electron-beam (REB) generators has stimulated interest in their use for the rapid heating of plasma targets.[1,2] Because energy transfer from the REB to the plasma by means of Coulomb collisions is negligible for the plasma densities and dimensions of interest, it is necessary to rely upon collective processes to provide significant energy deposition. One of the most promising of these processes is the two-stream interaction between the relativistic beam and the target plasma electrons.[3] Experiments to investigate this instability have injected the REB through an anode consisting of a thin metal foil. The beam is scattered in passing through this foil, and, consequently, acquires an angular spread in velocity, even though the actual spread in energy is small. However, because the two-stream instability is a longitudinal interaction, only those beam electrons with parallel velocity components greater than the phase velocity of the most unstable wave can "coherently pump" the instability and hence make a significant contribution to the plasma heating. Thus, this angular spread can seriously degrade the strength of the two-stream interaction.[4] In this Letter, results of experiments that verify this theoretically predicted effect are reported. This was accomplished by first establishing a set of beam and plasma parameters for which the energy deposition was dominated by the two-stream interaction, and then varying the beam angular spread by changing the anode foil thickness.

The experimental apparatus used in this study has been described in detail elsewhere,[5,6] and is shown in Fig. 1. The hydrogen target plasma was created by injecting a z-pinch gun plasmoid through a curved hexapole guide field into a solenoidal 1.8 : 1 magnetic-mirror trap ($B_0 = 2.6$ kG at the midplane, mirror spacing was 150 cm). The initial plasma column was 100% ionized, free of impurities, had a midplane radius of 3.5 cm, electron density in excess of 6×10^{13} cm^{-3}, ion temperature of 10 eV, and was separated from the 20-cm-radius stainless-steel flux-conserving interaction-chamber wall by a hard vacuum ($P \leq 2\times10^{-6}$ Torr). The roughly triangular radial density profile gradually expanded, and plasma was lost out of the mirror ends with a particle loss time of ~400 μs. The plasma density, therefore, could be controlled by varying the delay between plasma formation and beam injection. An initial electron temperature of ~5 eV is consistent with both the column expansion rate and the value expected from a simple model of cooling by electron thermal conduction to the ends.[7]

The relativistic electron beam was produced by an oil-insulated Marx generator driving a 5.8-Ω pulse line terminated with a cold-cathode field-emission diode. The diode was located near the magnetic-mirror throat, and the extracted beam expanded along field lines from an initial radius of 2.5 cm in the diode to a radius of ~3.4 cm at the midplane. Throughout these experiments typical beam parameters included a beam energy of 350 keV, beam current of 16 kA, and a 60-ns pulse width [full width at half maximum (FWHM)].

Both the collisional and collisionless skin depths in this experiment were much less than either the beam or plasma radius, and, consequently, large return currents were induced by the beam.[8] As a result, it was possible to heat the plasma in two different ways, either directly by the two-stream interaction, or, depending on the beam and plasma parameters, by Ohmic dissipation of the return current. (In the Ohmic case, collective interactions contributed to the

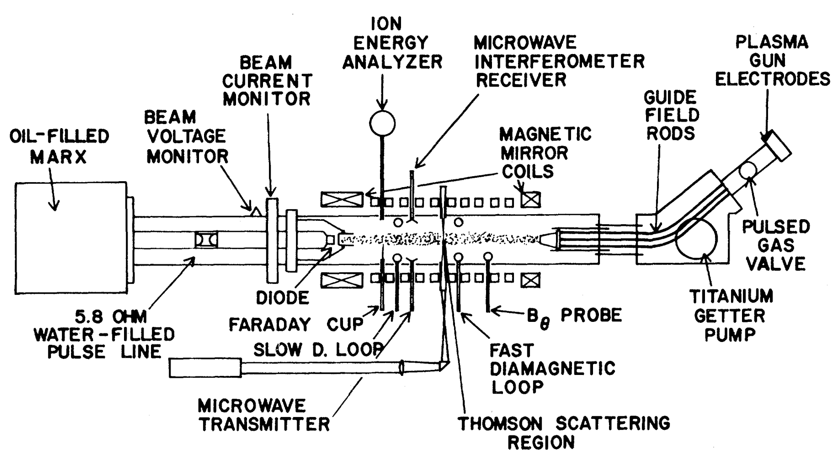

FIG. 1. The Cornell University relativistic-electron-beam plasma-heating experimental facility.

heating by providing an anomalously large plasma resistivity.) Because miniature Faraday-cup measurements of the REB current distribution showed a uniform radial profile,[9] the instantaneous return-current heating power can be expressed as $P_r = I_r V$. Here I_r is the induced plasma return current, and V is the induction voltage given by $V = -L\, dI_n/dt$. $I_n = I_b + I_r$ is the net current, measured with magnetic probes in the interaction chamber, L is the system inductance, and I_b is the beam current, measured with magnetic probes in the diode. The total energy transferred to the plasma by dissipation of the return current, W_r, can then be calculated by integrating P_r; i.e., $W_r = \int_0^\infty P_r\, dt$. Assuming that W_r is isotropic, two-thirds of W_r contributes to the diamagnetic-loop[10] measurement of the total perpendicular plasma energy, $W_\perp$. These measurements established that at high plasma densities, $n_p > 5\times 10^{12}$ cm^{-3}, return-current heating contributed less than 10% to the total observed plasma heating, and thereby suggest that the two-stream interaction was the dominant plasma-heating mechanism.

A one-dimensional model of the two-stream interaction,[4,11] relevant for the conditions of these experiments, predicts that ΔW, the energy deposited in the plasma (i.e., that lost by the REB), scales as

$$\Delta W \propto (\Delta n/n_b) 1.5S(1+1.5S)^{-2.5} \gamma n_b m_c^2. \qquad (1)$$

Here m is the electron mass, βc is the beam electron velocity, γmc^2 is the beam electron energy, and n_b/n_p is the beam-to-plasma density ratio. Evidently, ΔW is characterized by a strength parameter, $S = \beta^2\gamma(n_b/2n_p)^{1/3}$, and a scattering parameter, $\Delta n/n_b$. Physically, $\Delta n/n_b$ is the fraction of beam electrons with parallel velocity components, v_z, that exceed the phase velocity of, and hence can resonate with, the fastest-growing two-stream wave. This phase velocity is given by[4]

$$\frac{\omega_0}{k_0} = \beta c\left[1 - \frac{1}{2\gamma}\left(\frac{n_b}{2n_p}\right)^{1/3} - \frac{3}{4\gamma^2}\left(\frac{n_b}{2n_p}\right)^{2/3}\right]. \qquad (2)$$

By defining a maximum angle, $\theta_{\max}$, from $v_0 \times \cos\theta_{\max} = \omega_0/k_0$ (v_0 is the beam velocity and θ is the angle through which beam electron is scattered), $\Delta n/n_b$ can be calculated from

$$\Delta n/n_b = \int_0^{\theta_{\max}} f(\theta,\tau)\, d\theta, \qquad (3)$$

where τ is the foil thickness, and $f(\theta,\tau)$ is the beam angular-distribution function resulting from scattering in the anode foil. Several multiple-scattering theories were used to derive $f(\theta,\tau)$. For scattering in very thin foils, the screening-angle theory of Moliere was used.[12,13] The distribution for foils so thick that the Moliere formulation was inaccurate was calculated using the Goudsmit-Saunderson theory,[14,15] which requires an expansion in Legendre polynomials. Finally, for very thick foils the age diffusion distribution derived by Bethe, Rose, and Smith[16] was used.

Figure 2 shows the experimentally observed perpendicular plasma energy as a function of the beam-to-plasma density ratio, n_b/n_p. This ratio was determined by dividing the number of beam electrons per unit length (obtained from the beam current and voltage) by the number of plasma electrons in the beam channel per unit length (obtained with 4- and 8-mm microwave interferometry and Langmuir probes). The solid lines represent the scaling based on Eq. (1). The agreement between data and theory supports the supposition that the observed plasma heating was a result of the two-stream interaction. Thomson-scattering measurements of the perpendicular plasma electron energy[17] give electron temperatures in excess of 600 eV, and exhibit the same scaling with n_b/n_p as the diamagnetic-loop measurements of total perpendicular plasma energy, but can only account for 25–30% of the observed plasma heating at $t = 200$ ns after beam injection.

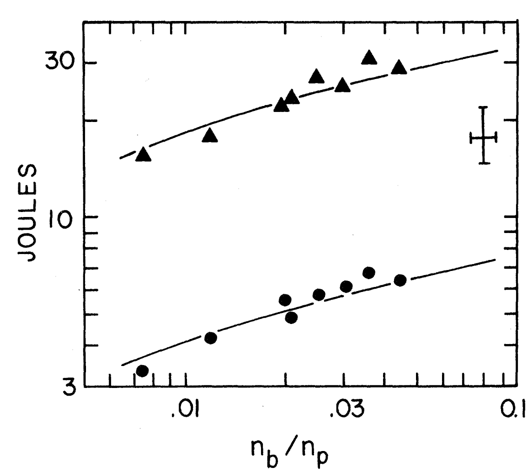

FIG. 2. Scaling of plasma heating as determined by diamagnetic-loop measurements (triangles) and Thomson scattering (circles), with beam-to-plasma density ratio, n_b/n_p. The theoretical scaling law for the two-stream interaction has been adjusted by a coupling factor to fit these data (solid curve). Error bars represent typical scatter, and each datum point represents the average of three or more shots.

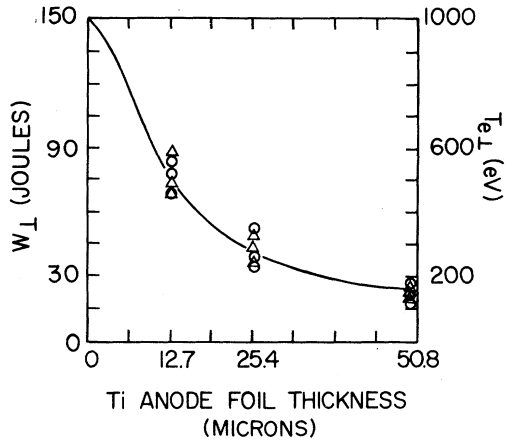

FIG. 3. Variation of energy transferred to plasma with foil thickness. The theoretical prediction (solid curve) is proportional to $\Delta n/n_b$ and has been normalized to the data for 25.4-μm Ti anode foils. Circles represent measurements of total perpendicular plasma energy with diamagnetic loops, and triangles represent measurements of plasma electron energy with Thomson scattering.

This discrepancy is a result of the contribution of heated ions and an observed high-energy nonthermal plasma electron component of the total plasma perpendicular energy.[18]

Figure 3 shows the energy deposition as a function of foil thickness for $n_b/n_p = 0.012$. The solid curve was derived from Eq. (3), and normalized to the $W_\perp$ data obtained with the 25.4-μm foil. Again, the theoretical scaling for the two-stream interaction was closely followed by the experimental data. It is observed that by decreasing the foil scattering, the efficiency of the energy deposition was increased by over a factor of 4, and with a total diode energy of 340 J, the maximum energy transfer efficiencies were in excess of 25%.

In conclusion, these experiments, performed in a range of parameters for which the plasma heating by the return current was weak, demonstrated an experimental scaling of energy deposition in agreement with the theory of the two-stream interaction for variations of both n_b/n_p and anode foil scattering. Plasma electron temperatures of more than 600 eV were directly measured with Thomson scattering. Finally, by decreasing the anode foil thickness, the beam energy transfer efficiency was increased to over 25% compared with the 6% achieved in earlier experiments.

The authors wish to thank C. B. Wharton, L. E. Thode, and M. A. Greenspan for stimulating discussions and encouragement, and Mr. P. Brown for his technical assistance. This work was performed under the auspices of the U. S. Department of Energy.

[a] Present address: U. S. Naval Research Laboratory, Washington, D. C. 20375.

[b] Present address: Los Alamos Scientific Laboratory, University of California, Los Alamos, N. Mex. 87545.

[1] See, for example, D. A. Hammer *et al.*, Phys. Fluids 21, 483 (1978), and references therein.

[2] See, for example, B. N. Brejzman and D. D. Ryutov, Nucl. Fusion 14, 873 (1974), and references therein.

[3] L. E. Thode and R. N. Sudan, Phys. Rev. Lett. 30, 732 (1973), and Phys. Fluids 18, 1552 (1975).

[4] L. E. Thode, Phys. Fluids 19, 831 (1976).

[5] T. H. Jensen and F. R. Scott, Phys. Fluids 11, 1809 (1968).

[6] C. Ekdahl, M. Greenspan, R. E. Kribel, J. Sethian, and C. B. Wharton, Phys. Rev. Lett. 33, 346 (1974).

[7] R. Morse, Phys. Fluids 16, 545 (1973).

[8] R. Lee and R. N. Sudan, Phys. Fluids 14, 1213 (1971).

[9] M. A. Greenspan, Ph.D. thesis, Cornell University, 1976 (unpublished).

[10] T. S. Green, Nucl. Fusion 2, 92 (1962).

[11] L. E. Thode, Phys. Fluids 20, 2121 (1977).

[12] G. Moliere, Z. Naturforsch. (a) 3, 78 (1948).

[13] H. A. Bethe, Phys. Rev. 89, 1256 (1953).

[14] S. A. Goudsmit and J. L. Saunderson, Phys. Rev. 57, 24 (1940).

[15] S. A. Goudsmit and J. L. Saunderson, Phys. Rev. 58, 36 (1940).

[16] H. A. Bethe, M. E. Rose, and L. P. Smith, Proc. Am. Philos. Soc. 78, 573 (1938).

[17] J. D. Sethian and C. A. Ekdahl, Rev. Sci. Instrum. 49, 729 (1978).

[18] J. D. Sethian, Ph.D. thesis, Cornell University, 1976 (unpublished).