enable the testing of specific theoretical models for reaction mechanisms which introduce random angular momentum in strongly damped collisions. The simple quantitative analysis which we have outlined suggests both the importance of these mechanisms and their role in explaining the large variance of the $\gamma$-ray multiplicity.

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# Radiative Capture of Intermediate-Energy Protons to High-Lying States in Light Nuclei

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Observations are reported from the first systematic studies of proton radiative capture at intermediate energies. In addition to captures to the ground and first few excited states, the reactions $^{11}B(p,\gamma)^{12}C$ and $^{27}Al(p,\gamma)^{28}Si$ reveal unexpectedly strong transitions to isolated high-lying states. These latter transitions could take place via "second-harmonic" giant resonances. Protons on $^{12}C$ produce a rich spectrum of $\gamma$ rays which may arise from captures to high-lying single-particle states.

Proton radiative capture reactions at intermediate energies comprise a virtually unexplored territory. With the exception of a few studies at a single energy,[1] the only relevant information thus far has been obtained through studies of the inverse, photonuclear reactions.[2,3] Recent theoretical work[4,5] underscores the importance of such measurements for an understanding of the role of meson exchange excitation in nuclear reaction mechanisms. Information about the nucleon momentum distributions of nuclear states may also be obtained through such studies.[2,3] In initial intermediate-energy radiative capture measurements, we have observed strong primary capture $\gamma$ rays not only to ground and low-lying excited states of the nuclei investigated, but to several isolated high-lying excitations as well. In this Letter, we present the evidence for this latter unexpected phenomenon, together with a simple picture of how it may be understood. Detailed descriptions of the results for low-lying states (including ground-state capture cross sections for direct comparison with existing calculations) will be presented elsewhere.

In our initial experiments, performed at the Indiana University Cyclotron Facility, we recorded $\gamma$-ray spectra arising from proton bombardment of $^{11}B$ and $^{12}C$ targets at energies from 40 to 100 MeV, at a fixed angle of 60°; less extensive measurements were made with $^{27}Al$, $^{89}Y$, and $^{3}H$ targets. The spectra presented below were taken using targets with the following characteristics: $^{11}B$, $31.5 \pm 0.5$ mg/cm$^2$, and $^{12}C$, $27.1 \pm 2.5$ mg/cm$^2$, both prepared from separated isotopes by pressing from powder form; and $^{27}Al$, $41.0 \pm 2.0$ mg/cm$^2$, from commercial foil stock. Beam currents employed in these experiments were in the 20–50-nA range. $\gamma$ rays were detected with a new plastic-scintillator shielded NaI(Tl) system specially designed for intermediate energies,[6] which has an energy resolution of ~ 3.8% for $E_\gamma = 50$ MeV. The linearity and zero-energy intercept of the spectrometer were checked with LED (light-emitting diode) pulses, and the absolute energy scale was calibrated with the 2.61-MeV line from a $^{228}Th$ source and the 15.11-MeV $\gamma$ ray from the reaction $^{12}C(p,p'\gamma)$. After observation of the $^{11}B(p,\gamma)^{12}C$ spectra, capture transitions to the ground and first excited states of $^{12}C$ were used as secondary standards for the 48 MeV $\leq E_\gamma \leq$ 108 MeV region.

Figure 1 shows the high-energy portion of the $\gamma$-ray spectra obtained from 40-, 60-, and 80-MeV protons on $^{11}B$. Neutrons have been eliminated by a time-of-flight requirement. The flight path from target to detector face was 1.1 m. The inset to Fig. 1 shows the time-of-flight spectrum recorded at $E_p = 60$ MeV. Time resolution was typically 2.1 ns FWHM (full width at half maximum). Cosmic-ray–induced background is rejected with high efficiency by the plastic-scintillator shield. All the spectra exhibit peaks corresponding to capture reactions to the ground and first excited states of $^{12}C$. The absolute energies, the energy difference, and the dependence of $\gamma$-ray energy on bombarding energy unambiguously identify these as originating in the reaction $^{11}B(p,\gamma)^{12}C$. In addition, another strong feature is apparent in the spectrum at lower $E_\gamma$; this feature takes the form of a distinct peak in the $E_p = 40$ and 60 MeV spectra and becomes a shoulder at $E_p = 80$ MeV. The peak in the 40-MeV spec-

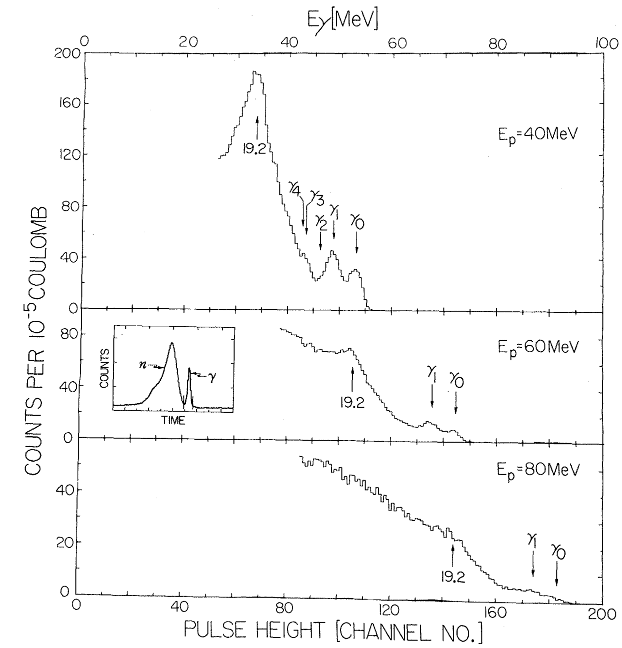


FIG. 1. $\gamma$-ray spectra from the bombardment of $^{11}B$ with 40-, 60-, and 80-MeV protons, observed at $\theta_{lab} = 60°$. Labeled arrows indicate the expected positions of captures to the ground state ($\gamma_0$) and low-lying excited states of $^{12}C$. The peak labeled "19.2" is identified as radiative capture to a state at ~ 19.2-MeV excitation energy. *Inset:* time-of-flight spectrum, at $E_p = 60$ MeV, for events leaving more than 30 MeV in the NaI(Tl) crystal. The time interval between the peaks is 12 ns. Following beam bursts provide the "stop" signal.

trum lies close to the lower-energy cutoff set by the electronics in the run shown in Fig. 1; however, a shorter run taken with a lower-energy discrimination threshold shows an isolated peak at the indicated position. This peak moves, with bombarding energy, as expected for radiative capture to a state, or narrow group of states, at an excitation energy of $19.2 \pm 0.6$ MeV in $^{12}C$. Although $\gamma$ rays following competing reactions, such as $^{11}B(p,p')^{11}B$, $^{11}B(p,n)^{11}C$, and $^{11}B(p,\alpha)^{8}Be$, are energetically allowed, the energy dependence of this $\gamma$-ray peak rules out its origin as being from such reactions. (Except for relativistic and recoil corrections, $E_\gamma$ for a capture $\gamma$ ray is a linear function of $E_p$, while the other reactions, in general, produce fixed-energy $\gamma$ rays.) The observations can be simply and consistently interpreted only in terms of radiative capture; the peak is therefore identified as arising from $^{11}B(p,\gamma)^{12}C^*(19.2$ MeV).

$\gamma$-ray spectra from $^{27}Al + p$ are shown in Fig. 2. The detector resolution is insufficient to sep-

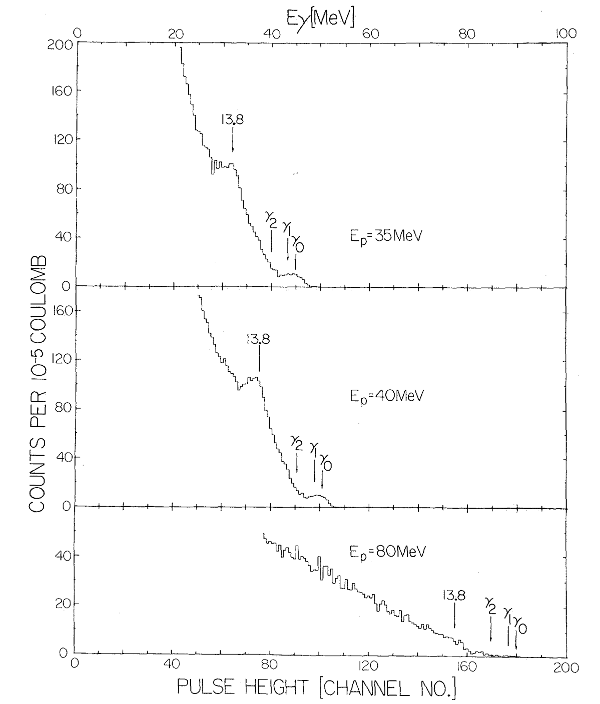


FIG. 2. $\gamma$-ray spectra from 35-, 40-, and 60-MeV protons on $^{27}$Al observed at 60°. See Fig. 1 for notation.

arate individual transitions to the low-lying states of $^{28}$Si, but there is a bump at the high-energy end which corresponds to $^{27}$Al$(p,\gamma)$ captures to the ground and first excited states of $^{28}$Si. As in the $^{11}$B$+p$ case, a more intense peak is visible in these spectra at a lower $\gamma$-ray energy. Again, the peak moves with bombarding energy as expected for a capture $\gamma$ ray, in this case to a state or group of states at $13.8 \pm 0.6$ MeV in $^{28}$Si. The short run at 35 MeV was taken to confirm this energy dependence for a smaller bombarding-energy step, and the result again agrees with the hypothesis of a $(p,\gamma)$ origin for this peak.

The 60° differential cross section for the $^{11}$B$(p,\gamma)^{12}$C*(19.2 MeV) peak, estimated by subtracting a smooth background from the peak, is $0.96 \pm 0.3$ $\mu$b/sr at 40 MeV and $0.12 \pm 0.04$ $\mu$b/sr at 60 MeV. The $^{27}$Al$(p,\gamma)^{28}$Si*(13.8 MeV) peak is produced with a cross section of $\sim 1.0 \pm 0.3$ $\mu$b/sr at 40 MeV. The large uncertainties arise almost entirely from a lack of knowledge of the shape of the continuum of $\gamma$ rays in the vicinity of the peaks of interest.

States most strongly populated in a $(p,\gamma)$ reaction on a target with a single-proton hole in its outermost subshell, which is the case for both $^{11}$B and $^{27}$Al, would likely be ones of a simple one-particle, one-hole nature, where the incoming proton simply radiates away enough energy to allow it to drop into the appropriate particle orbit. The most interesting candidates for the high-lying $^{12}$C and $^{28}$Si states reported here are the "stretched" configuration states described by Donnelly and Walker[7] and experimentally observed in inelastic electron scattering,[8] inelastic proton scattering,[9] and the $(\alpha,t)$ reaction.[10] These states are thought to have, in $^{12}$C, a $(p_{3/2}^{-1}, d_{5/2})$ configuration summing to $J^\pi = 4^-$, and in $^{28}$Si, $(d_{5/2}^{-1}, f_{7/2})$ summing to $6^-$. The $(e,e')$ experiments place the $4^-$ state in $^{12}$C at 19.6 MeV, compared with our observed final-state energy of $19.2 \pm 0.6$ MeV. In $^{28}$Si the $6^-$ state has been located at 14.3 MeV, while our peak appears to arise from captures to a state or group of states at $13.8 \pm 0.6$ MeV. In both cases, the inelastic scattering studies indicate states narrower than our detector resolution; in our experiment, the observed $^{28}$Si transition shows a nearly unbroadened line shape, while the $^{12}$C line appears to be broadened somewhat beyond the $\sim 200$ keV width reported from $(e,e')$ although background uncertainties do not allow definite widths to be assigned in the present experiment.

It is not unreasonable to suppose that other members of the same particle-hole configurations can also participate, to some extent, as final states, thus producing the observed broadening.

If we do identify the final states observed here as stretched configurations, we can expect some further interesting effects to show up in radiative proton capture experiments. The $6^-$ state in $^{28}$Si is a $T=1$ state which $\gamma$ decays 100% by $M1$ radiation to the $6^-$, $T=0$ state at 11.6 MeV; the decays of the $4^-$ state in $^{12}$C are not yet known. With little or no $E1$ strength for decays to lower states, these configurations would have all their electric dipole sum-rule strength concentrated in transitions to or from states of *higher* energy. Very strong $E1$ transitions could then take place between higher-lying states and the states of interest. For example, $f$-wave protons on $^{11}$B could produce a $(p_{3/2}^{-1}, f_{7/2})$ giant dipole resonance for transitions to the $4^-$ state in $^{12}$C; likewise, a $(d_{5/2}^{-1}, g_{9/2})$ resonance would be coupled, by $E1$ transitions, to the $6^-$ state in $^{28}$Si. Such "second-harmonic" resonant capture could be sought in $(p,\gamma)$ experiments in the 20–40-MeV range of bombarding energies, and, if observed, would provide strong support for the simple picture presented here, namely, that we are seeing

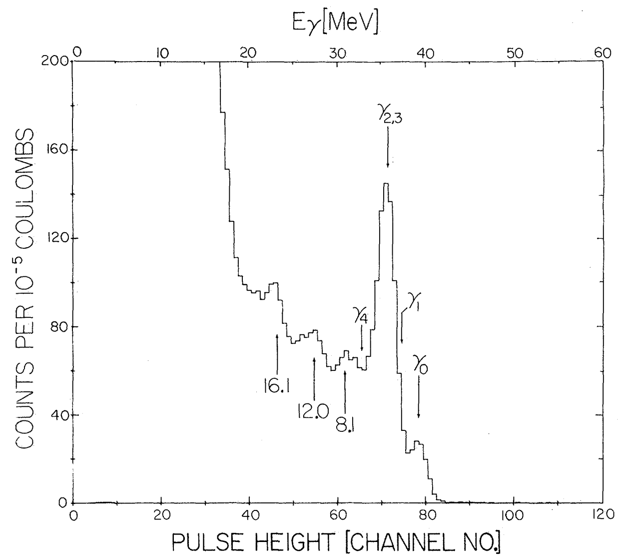


FIG. 3. The spectrum from $^{12}C(p,\gamma)$ at 40-MeV bombarding energy, $\theta_{lab} = 60°$.

the first evidence for $2\hbar\omega$ to $1\hbar\omega$ radiative transitions.

The spectrum of $\gamma$ rays from $^{12}C+p$ is qualitatively different from the other two reactions. The dominant peak at the high-energy end of the spectra measured for 20–100-MeV protons is observed to be from $^{12}C(p,\gamma_{2,3})^{13}N$, where the final state is the unresolved second and/or third excited state of $^{13}N$.[11] The lower-energy portion of the 40-MeV spectrum, shown in Fig. 3, is characterized by three broad peaks. If these peaks arise from $^{12}C(p,\gamma)$, the final state groups in $^{13}N$ would be centered at 16.1, 12.0, and 8.1 MeV. Unfortunately, in this case, the peaks cannot yet be assigned with certainty to the capture reaction, because the structure in the spectrum becomes washed out at the other measured energies. Measurements at smaller energy increments in the vicinity of 40 MeV should clarify this identification. As in the other two cases reported, final states seen most strongly in $^{12}C(p,\gamma)^{13}N$ should be of a simple single-particle nature. The $\frac{5}{2}^+$ third excited state, for example, has a substantial $d_{5/2}$ spectroscopic factor, while the nearby second excited state appears to be a much less pure single-particle state.[12] Thus the strong peak indicated as $\gamma_{2,3}$ is likely to consist mostly of $\gamma$ rays to the $\frac{5}{2}^+$ state. The bumps in the spectrum appearing at lower $\gamma$-ray energies do not seem to have a one-to-one correspondence to reported levels in $^{13}N$ and further work is necessary to clarify matters.

The first results reported here indicate that measurements of $(p,\gamma)$ reactions at intermediate energies open an interesting new window on the nucleus. Most of the final states observed in this manner cannot be probed at all in the inverse $(\gamma,p)$ studies, but further information on their special configurations, the momentum distributions of protons, and other characteristics of these states, as well as additional clues to the mechanisms involved, should be obtainable from more detailed capture measurements. Further studies, including angular distributions and measurements with polarized beams, are in the planning stages.

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# Bender-Wu Formula and the Stark Effect in Hydrogen

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We discuss a close connection between the formula of Banks, Bender, and Wu for the asymptotics of the Rayleigh-Schrödinger coefficients of the two-dimensional rotationally symmetric anharmonic oscillator and the behavior of resonances of the hydrogen Stark problem in *two* regimes: small field (Oppenheimer's formula) and large field (where we obtain the new results $\arg E \to -\pi/3$, $|E| \sim \alpha[F(\ln F)]^{2/3}$ for $F$, the electric field strength, going to infinity). We also announce a rigorous proof of Bender-Wu–type formulas.

In this note, we wish to discuss various aspects of the hydrogen Stark Hamiltonian (in atomic units)

$$H = -\tfrac{1}{2}\Delta - Zr^{-1} + Fx_3, \qquad (1)$$

and, in particular, the celebrated formula of Oppenheimer[1] for the width, Γ, of the ground state in small positive field ($Z=1$),

$$\Gamma = -2\,\mathrm{Im}E(F) = 4F^{-1}\exp(-\tfrac{2}{3}F^{-1})[1+O(F)]. \qquad (2)$$

We will link this closely to certain ideas developed by Bender and Wu during the past ten years[2] involving the large-$n$ behavior of the Rayleigh-Schrödinger coefficients of the ground state of an anharmonic oscillator. Equation (1) is connected[3] to the Hamiltonian

$$h = -d^2/dx^2 + (m^2 - \tfrac{1}{4})x^{-2} + \alpha x^2 + \beta x^4 \qquad (3)$$

with the boundary conditions[4]

$$u(0)=0 \;(m=1,2,\ldots),\quad u(x)\sim x^{1/2}\;(m=0), \qquad (4)$$

which is the Hamiltonian of the two-dimensional oscillator $-\Delta + \alpha r^2 + \beta r^4$ reduced to the subspace $\{f(r)e^{\mathrm{Im}\Phi}\}$. We will let $\mu_n^{(m)}(\alpha,\beta)$ denote the $(n+1)$th eigenvalue of (3). Using the Bender-Wu methods, $h$ was studied by Banks, Bender, and Wu,[5] who found, in particular, that the ground-state Rayleigh-Schrödinger coefficients $a_n$, defined by

$$u_0^{(0)}(1,\beta) \sim \sum a_n \beta^n, \qquad (5)$$

obey ($n\sim\infty$)

$$a_n \sim (8/\pi)(\tfrac{3}{2})^{n+1}(-1)^n n!\,[1+O(n^{-1})]. \qquad (6)$$

Here we will discuss three results whose technical details will appear elsewhere[6,7]:

(A) The equivalence of Eqs. (2) and (6) modulo technical conditions[6,7];

(B) a rigorous proof of formulas of Bender-Wu–type including the one-dimensional anharmonic oscillator, formula (6) and formula (2) (Ref. 6);

(C) an analysis of the relation between (6) and the *large*-$F$ behavior of $E(F)$ (Ref. 7); in particular, for one state that we consider here (which is *probably*[8] the continuation of the ground state) for $F\sim\infty$,

$$\arg E \sim -\pi/3 + O((\ln F)^{-1}), \qquad (7)$$

$$|E| \sim \alpha F^{2/3}(\ln F)^{2/3} + \beta F^{2/3}(\ln\ln F)^{-1/3} + O(F^{2/3}(\ln F)^{-1/3}), \qquad (8)$$

 