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Second-Order Excitation in Nucleus-Nucleus Interaction Potential

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The second-order interaction potential has been calculated for two colliding nuclei where target excitation is induced by the interaction between a nucleon and the incoming self-consistent one-body field of the projectile. The real part of the second-order term is insignificant for distances larger than the touching radius, but significant for smaller distances. The imaginary potential agrees favorably with phenomenological results.

In recent years the first-order nucleus-nucleus interaction potential has been derived either by double folding the nuclear density distributions of the nuclei with a suitable two-body effective interaction[1] or by single folding a nucleon-nucleus potential with the target/projectile density distribution.[2] In the first instance the nuclear saturation effect can be incorporated effectively by using a suitable two-body interaction.[1,3] In the single-folding case, the phenomenological free-nucleon–nucleus potential is folded in with the nucleon density distribution to derive the nucleus-nucleus interaction potential.[2]

It is apparent that in neither of the cases are all the Pauli exchange effects included. Using a reasonable effective interaction might guarantee nuclear saturation but the model does not incorporate exchange effects. The single-folding procedure has the further drawback of ignoring saturation. Pauli exchange corrections are primarily of two kinds—the antisymmetrization of the interaction matrix element and the Pauli distortion effect, which necessarily tends to increase the internal kinetic energy of the system at the expense of the relative-motion kinetic energy. The blocking of the intrinsic states of the colliding nuclei leads to the excitation of the nuclei. Mosel[4] has shown the importance of including the distortion effect for a correct evaluation of the interaction potential.

The two exchange effects just mentioned have been estimated recently by various authors, employing a range of techniques and models.[4-6] Recently the exchange effects were estimated by folding in the two density matrices of the interacting nuclei,[7] whereas the direct part was computed by folding in the diagonal part of the density matrices—essentially in the spirit of the sudden approximation. The generalized folding procedure just described has an added advantage: One can self-consistently extract a nucleon-nucleus potential which when folded in with the target/projectile density distributions reproduces by definition the direct plus exchange total interaction potential. The self-consistent nucleon-nucleus potential in this method has all the Pauli effects built into it. It was noted that such a potential deviates from a free-nucleon–nucleus potential considerably, as expected,[7] becoming weaker with increasing overlap of the two densities.

One inescapable manifestation of Pauli exchange is a shallow potential, which turns repulsive at short intranuclear distance, a fact not of great practical importance since the very concept of a

potential breaks down long before such an overlap of nuclear densities takes place. The fact remains, however, that the exchange effects linger on even for large intranuclear distances.

The preceding discussion ignores second-order effects. It is well known that the principal value of the second-order term gives rise to a contribution to the real part of the interaction potential. The phenomenon refers to the virtual excitation of target states. The pole term, on the other hand, leads to the imaginary potential. There exist already in literature several attempts to calculate the second-order term. Love, Terasawa, and Satchler[8] and Baltz *et al.*[9] have computed the second-order term for the long-range Coulomb potential as well as for the nuclear potential; these calculations are carried out with the motivation of investigating the effects on the interaction potential due to the coupling to specific inelastic channels. Mosel[4] has indicated the effect of the second-order term in an adiabatic approximation. The spirit of the present work is somewhat different: Rather than investigating the consequences for coupling to specific deformation degrees of freedom, the calculation presented in the following relies upon a suitable yet simple method of taking into account all possible intermediate channels without any reference to specific channels. In collisions between two nuclei many channels are excited; a simple scheme which exhausts a complete set might therefore be rather useful.

The formulation is essentially in the sudden approximation which one hopes would enable us to estimate the second-order corrections to the first-order interaction potential, also calculated using the sudden approximation.

The contention of this work is to compute the second-order terms, both the principal value and the pole term; the driving potential used for the excitation of the target states is the one-body nucleon-nucleus potential extracted in deriving the first-order potential, which now incorporates Pauli effects. In this model, target excitation is induced by the interaction between a nucleon in the target and the one-body field, representing the incoming projectile. As will be shown in what follows, the so-called "shallow" potential turns somewhat "deep" with the contribution of the second-order term. The pole term has features which seem to agree with phenomenology.

The second-order potential can be written in general as[10]

$$U_0 = \sum_{m\neq 0} \lim \int d\epsilon_n \langle \psi_0 | \sum_i U_{12}(i) | \psi_m \varphi_n \rangle \frac{\langle \varphi_n \psi_m | \sum_j U_{12}(j) | \psi_0 \rangle}{\epsilon_0 - \epsilon_n - \epsilon_m + i\eta}, \tag{1}$$

where $U_{12}(i)$ is the one-body interaction potential between (say) a target nucleon and the projectile as a whole, ψ_0 is the ground-state wave function of the target, ψ_m and ϵ_m are the wave function and excitation energy of its excited states, φ_n and ϵ_n are the wave functions and energies of the projectile in the intermediate states n, and ϵ_0 is the center-of-mass energy of the incident projectile. The computation of Eq. (1) is similar to the work of Vinh Mau.[11] I shall assume that (a) the wave function φ_n of the projectile in the field of the target nucleus can be approximated by a plane wave with a wave number $k_n^2 = 2\mu(\epsilon_n - U_R)/\hbar^2$, where μ is the reduced mass and U_R is the interaction potential. In principle, U_R should be the sum of both the first- and the second-order terms. For simplicity we shall neglect the contribution from the second-order term in computing U_2. I shall also assume that (b) the energy ϵ_0 is high enough so that all the energies would be such that $\epsilon_m < \epsilon_0$. This is justified for a nucleus-nucleus collision because the average energy of excitation in the entrance phase is around $\epsilon_m \sim 16.0$ MeV,[12] which is small compared to the incident energy of the nucleus, for laboratory energy somewhat higher than the Coulomb barrier. This approximation would fail for very low incident energies. One can now write Eq. (1) as

$$U_2(\vec{r}_0, \vec{r}_0') = \sum_{m\neq 0} F(K, S_0) \langle \psi_0 | \sum_i U_{12}(i) | \psi_m \rangle \langle \psi_m | \sum_j U_{12}(j) | \psi_0 \rangle, \tag{2}$$

where

$$F(K, S_0) = (-\mu/2\pi\hbar^2)[\cos(KS_0) + i\sin(KS_0)]\, S_0^{-1} k/K, \tag{3a}$$

$$K^2 = (2\mu/\hbar^2)(\epsilon_0 - U_R - \epsilon_m), \quad k^2 = (2\mu/\hbar^2)(\epsilon_0 - \epsilon_m), \tag{3b}$$

and where $\vec{r}_0$ and $\vec{r}_0'$ are the coordinates of the centers of the interacting nuclei such that $\vec{R}_0 = \frac{1}{2}(\vec{r}_0 + \vec{r}_0')$ and $\vec{S}_0 = (\vec{r}_0 - \vec{r}_0')$.

Since one is primarily interested in the surface region of interaction where $U_R \sim -5.0$ MeV, it is expected that k and K are not very different. Further, we assume that ϵ_m is replaced by an average value of $\langle \epsilon_m \rangle \sim 16.0$ MeV—there are good reasons to believe that giant resonance states around 16.0 MeV are probably the states most likely to be excited in the entrance phase of nucleus-nucleus interaction.[12]

With the average value assumed for $\langle \epsilon_m \rangle$ and the two assumptions mentioned above we can now add and subtract the second-order term corresponding to the ground state of the target, i.e., for $\psi_m \equiv \psi_0$. Collecting the first two terms we can now apply "closure" for the intermediate state $|m\rangle$, exhausting a complete set, and obtain the second-order potential as

$$U_2(R_0, S_0) = F(K, S_0)[\langle \psi_0 | \textstyle\sum_i U_{12}(i) \sum_j U_{12}(j) | \psi_0 \rangle - \langle \psi_0 | \sum_i U_{12}(i) | \psi_0 \rangle \langle \psi_0 | \sum_j U_{12}(j) | \psi_0 \rangle], \tag{4}$$

where the second term is obtained for $|m\rangle \equiv |0\rangle$. In configuration space,

$$U_2(R_0, S_0) = F(K, S_0)[U_{12}(|\vec{R}_0 - \vec{r} + \tfrac{1}{2}\vec{S}_0|)\, U_{12}(|\vec{R}_0 - \vec{r} - \tfrac{1}{2}\vec{S}_0|)\rho_t(r) d^3r$$
$$- \tfrac{1}{4} \int U_{12}(|\vec{R}_0 - \vec{R} + \tfrac{1}{2}\vec{S}_0 - \tfrac{1}{2}\vec{S}|) U_{12}(|\vec{R}_0 - \vec{R} - \tfrac{1}{2}\vec{S}_0 + \tfrac{1}{2}\vec{S}|) \rho_t^2(R,S)\, d^3R\, d^3S] \tag{5}$$

and the equivalent local potential is obtained by employing the Perey-Saxon method.[11] The density matrix $\rho_t(R, S)$ corresponds to the target, the diagonal part being given by $\rho_t(r)$. For the density matrix we use the Negele-Vautherin-Campi[13] approximation.

It is imperative now, as mentioned before, to use the self-consistent U_{12} which leads on to the total first-order interaction potential when folded in with the target/projectile density distribution. The first-order interaction potential, including Pauli effects mentioned before,[7] has been calculated in Ref. 7 using a Skyrme type of momentum- and density-dependent two-body effective interaction. The self-consistent U_{12}'s are given by

$$U_{12} = [\tfrac{3}{4} t_0 \rho_p + \tfrac{3}{8}(3t_1 + 5t_2) 0.6 k_F^2 \rho_p + \tfrac{3}{64}(5t_2 - t_1)(\nabla^2 \rho_t + \rho_p \nabla^2 \rho_p / \rho_t) + \alpha E \rho_p + \tfrac{3}{16} t_3 \rho_L \rho_p][1 + \alpha(\rho_p + \rho_t)]^{-1} \tag{6}$$

such that the first-order interaction potential is given by folding in the target density distribution. In Eq. (6) ρ_p and ρ_t are the projectile and target density distribution functions, respectively; the local density ρ_L has to be evaluated taking into account the contribution of both the target and the projectile density distributions[7,14] and $k_F^3 = 1.5\pi^2 \rho_L$ where k_F is the local Fermi momentum. The first-order potential U is shown in Fig. 1. The parameters t_0, t_1, t_2, and t_3 are Skyrme parameters, $\alpha = 1/8(3t_1 + 5t_2)(2m/\hbar^2)$, and E is the incident energy per nucleon.

In Fig. 1, the results obtained for the real part of Eqs. (5) is presented for a center-of-mass energy (ECM) of 50.0 MeV for two colliding ^{16}O nuclei. The second-order term has a significant contribution up to ~ 5.0 fm but is reduced to negligible values for larger intranuclear distance. The first-order potential without Pauli exchange is also presented in Fig. 1. It is clear that beyond the touching radius ~ 5.2 fm the Pauli exchange is still primarily responsible for the reduction in the depth of the potential; but for intranuclear distance less than the touching radius, Pauli exchange to a great extent is canceled by the polarization effect, making the interaction potential turn "deep" from being "shallow." The increase in binding due to virtual excitation tends to relax the repulsion due to Pauli principle; in the surface region however, Pauli exchange lingers on whereas polarization gets switched off.

The interesting speculation is, of course, the importance of these effects for the ion-ion scattering data or for the data on heavily damped collisions. It is known[15] that for light ion systems,

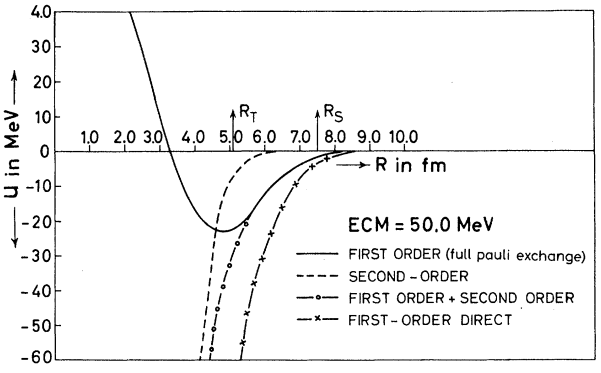

FIG. 1. The first- and second-order real parts of the interaction potential as a function of R, the intranuclear distance; the touching radius is R_t and the strong-absorption radius is R_s.

especially $^{12}C + {}^{28}Si$ for example, the potential around $\sim 0.6(A_1^{1/3} + A_2^{1/3})$ is well determined, and if that be the case then polarization is certainly going to be important. Secondly, the raging debate about the sensitivity of the restoring force, gradient of the interaction potential, in analysis of data on heavily damped collision throws open an interesting speculation about the sensitivity of the polarization term, especially at small distances.

The pole term of the second-order potential gives rise to the imaginary part of the optical potential. In Fig. 2 the results obtained are compared with phenomenology.[16] In terms of the form factor and the general characteristics, the deep energy-independent phenomenological form factor looks quite similar to the results obtained theoretically. It is significant that for an ECM of 30.0 MeV the theoretical prediction agrees remarkably well with the phenomenological form factor around ~ 7.5 fm, the strong absorption radius for the $^{16}O + {}^{16}O$ system.

Both the real and the imaginary second-order potentials tend to decrease with the increase in the incident energy, for the real term turning repulsive around an ECM of 500.0 MeV. The energy dependence could very well be the result of transforming a nonlocal potential to an equivalent energy-dependent local potential.

A detailed application of the results obtained for analysis of scattering data and data on heavily damped collisions is postponed for a future publication. I would like to conclude by noting that for small intranuclear distance, less than the touching radius, the polarization term could be significant, but for larger distances it has virtually no importance. The imaginary potentials calculated self-consistently agree rather well with phenomenological results.

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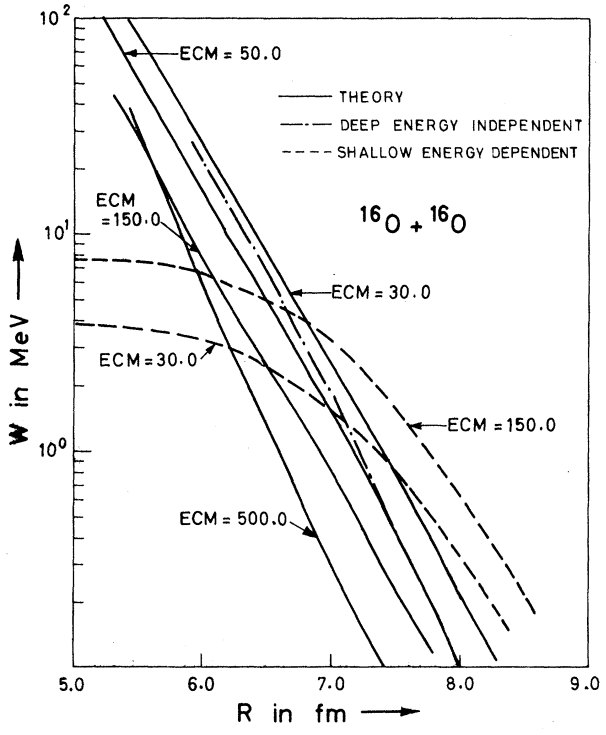

FIG. 2. The imaginary part of the interaction potential compared with phenomenological predictions.

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