equation was the dominant nonlinearity (with adiabatic electrons).

The maximum diffusion coefficient obtained from Eq. (9) occurs for $b = b_0 \equiv (1+\tau)^3 \tau^{-2} \Delta_{PB}^{-1}$, where the nonlinear and linear shear damping become comparable. Here, $\Delta_{PB} \equiv (L_S/L_n)^3 (m_e/m_i)$ is of order unity for tokamaks. The mode width $\Delta x \sim (\tau^3 \Delta_{PB} L_S/L_n)^{1/2}(1+\tau)^{-2} \gtrsim 1$, which justifies the use of the differential Eq. (7). The diffusion coefficient which results from maximizing D_{rr} with respect to b is

$$D_{rr} = 15 \Delta_{PB}^{3/2} \tau^{5/2} [0.5(1+\tau)]^{-9/2} \rho_S^2 c_S / L_S, \qquad (10)$$

where $c_S = (T_e/m_i)^{1/2}$ and $\rho_S^2 = \tau \rho_i^2$. The associated electron thermal conduction coefficient is $\kappa_e \approx \frac{3}{2} D_{rr}$. If $\vec{E} \times \vec{B}$ electrostatic turbulence is the dominant scattering mechanism for these modes, then Eqs. (5) and (10) indicate a density fluctuation level at saturation, $\tilde{n}/n = \Delta_{PB}(\rho_s/L_n)$ for $\tau = 1$, in the strong-turbulence limit $\omega_c \gtrsim \omega'$. The coefficient in Eq. (10) is of the correct order of magnitude to account for electron heat transport in tokamaks outside the $q = 1$ surface, with a fluctuation level of several percent.

In conclusion, destabilization and saturation of the drift mode in a sheared field have been shown to result from a resonance broadening mechanism that dominantly affects electrons. This contrasts with previous turbulence theories in a shearless field,[4] where nonlinear ion damping led to saturation and the electron dynamics were linear. The present theory predicts saturation at modest fluctuation levels.

We wish to thank Dr. D. J. Sigmar, Dr. J. D. Callen, Dr. W. M. Manheimer, Dr. J. A. Krommes, and Professor T. H. Dupree for useful discussions. The numerical results were computed by Dr. J. Whitson and Dr. Julius Smith.

This research was sponsored in part by the Office of Fusion Energy (ETM), U. S. Department of Energy, under contract No. W-7405-eng-26 with the Union Carbide Corporation and the U. S. Energy Research and Development Administration Grant No. EG-77-G-01-4108.

[1]L. D. Pearlstein and H. L. Berk, Phys. Rev. Lett. 23, 220 (1969).

[2]K. T. Tsang, P. J. Catto, J. C. Whitson, and J. Smith, Phys. Rev. Lett. 40, 327 (1978).

[3]D. W. Ross and S. M. Mahajan, Phys. Rev. Lett. 40, 324 (1978).

[4]T. H. Dupree, Phys. Fluids 10, 1049 (1967).

[5]T. H. Dupree and D. J. Tetrault, Phys. Fluids 21, 425 (1978).

[6]K. Molvig and S. P. Hirshman, Massachusetts Institute of Technology Plasma Fusion Center Research Report No. PFC/RR-78-7, 1978 (unpublished); A. A. Galeev, Phys. Fluids 21, 1363 (1978); J. A. Krommes, Princeton Plasma Physics Laboratory Report No. PPPL-1462, 1978 (unpublished).

[7]The nonlinear orthogonality between $\nabla\Phi$ and $d\vec{r}_\perp/dt'$ has been used to separate out the adiabatic response in Eq. (2), which for electrons is the dominant effect of the "β term" of Ref. 5.

[8]P. N. Guzdar, L. Chen, P. K. Kaw, and C. Oberman, Phys. Rev. Lett. 40, 1566 (1978).

[9]G. Hasselberg, A. Rogister, and A. El-Nadi, Phys. Fluids 20, 982 (1977).

Maximum Energy-Confinement Time in Joule-Heated Tokamaks

R. E. Waltz and G. E. Guest
General Atomic Company, San Diego, California 92138
(Received 21 August 1978)

Measurements of the energy confinement time τ_E in the ISX-A (Impurity Study Experiment) tokamak are interpreted theoretically using a one-dimensional time-dependent transport code. The maximum τ_E observed as the plasma density is varied over a wide range occurs at that density above which anomalous electron thermal conductivity leads to a smaller energy flux than neoclassical ion thermal conductivity.

One of the most striking features of recent experiments in the ISX-A (Impurity Study Experiment) tokamak[1] is the apparent saturation of the energy confinement time τ_E with increasing plasma density as shown in Fig. 1. Effective impurity control in the ISX-A, as in the earlier Alcator experiments,[2] permitted operation over a comparatively wide range of plasma density under circumstances such that radiation was not a dominant energy-loss mechanism in the interior of the plasma. Since anomalous heat losses decrease with density, while neoclassical losses

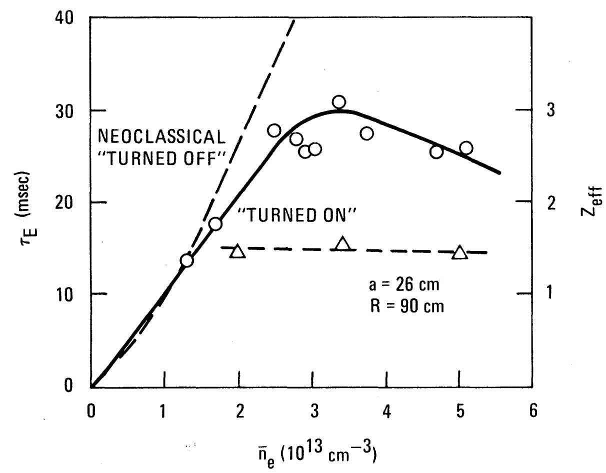

FIG. 1. Density dependence of total energy confinement time for deuterium discharges in ISX-A with $I = 120$ kA and $B_t = 12.8$ kG. The solid line shows full-transport-model predictions, fit to the experimental point at $\bar{n} = 1.5\times10^{13}$ cm^{-3}. The dashed line shows the corresponding results with ion neoclassical thermal conductivity neglected in the transport model.

increase with density, the saturation of τ_E observed in these experiments may indicate a transition from anomalous to neoclassical confinement.[2,3] Indeed, in Ref. 2 it was pointed out that experimental results at the highest plasma densities achieved in Alcator were consistent with a neoclassical transport model. In this note we examine this possible transition in greater detail and seek to identify the density above which neoclassical theory provides an adequate description of energy confinement in tokamaks, thus providing an estimate of the maximum τ_E that can be reached in tokamaks.

We first describe a one-dimensional, time-dependent transport model of the ISX-A tokamak, which uses a simple empirical model of anomalous transport, combined with a standard neoclassical theory of ion thermal conductivity.[4] This model gives a good description of the ISX-A results,[1] particularly the observed saturation of τ_E with increasing plasma density. We then examine the sensitivity of the computed results to uncertainties in the temperature dependence of the anomalous transport rates. Finally, we discuss the dependence of the maximum energy-confinement time on the various plasma parameters, such as the magnetic field strength B_t, the plasma current I, the effective charge Z_{eff}, and the major and minor dimensions of the discharge, R and a.

The basic equations solved by the General Atomic Co. transport code are described, for example, in Ref. 5. Models for the diffusion coefficient and the ion and electron thermal conductivities are discussed below. The electron-ion energy exchange rate is assumed classical. The neutral-particle source and charge-exchange losses were calculated with standard neutral-transport techniques.[6] The power radiated per unit volume is made up of ionization and excitation losses from the working gas and line radiation from any partially ionized impurities which may be present. We assume the latter to be adequately described by coronal equilibrium radiation from oxygen alone. For example, a relative oxygen concentration of 1% at the center of the ISX-A, rising to 2% at the plasma edge, gives a spatially uniform $Z_{\mathrm{eff}} = 1.5$. Note that the assumption of coronal equilibrium is only marginally justified at low densities, since the time to reach coronal equilibrium is then comparable to τ_E. The radiated power could therefore be somewhat greater than is estimated by these equilibrium rates.[7]

The neoclassical ion heat conduction is calculated from Eq. (6.131) of Ref. 4. In the actual cases solved here, the expressions for χ_i must be modified to account for the effects of oxygen impurities,[8] which increase χ_i by roughly a factor of Z_{eff}. Furthermore, the full neoclassical transport matrix is used in the computations, although, with the possible exception of the Ware pinch effect, contributions from elements other than $\eta_\parallel$ and χ_i are generally negligible in Ohmic discharges.

We adopt a simple model of anomalous transport based on empirical confinement results from many different tokamak devices operated at low enough densities that most of the plasma energy is lost through anomalous processes.[9] The model fluxes of plasma and energy are

$$\Gamma = -D^{(A)}\partial n/\partial r$$

and

$$Q_e = \tfrac{5}{2}T_e\Gamma - n\chi_e{}^{(A)}\partial T_e/\partial r,$$

with

$$[nD^{(A)}, n\chi_e{}^{(A)}] = (W_1, W_2)\times(5\times10^{17})\ \mathrm{cm}^{-1}\,\mathrm{sec}^{-1}.$$

The constants W_1 and W_2 are chosen to fit a reference ISX-A discharge at low density: $(W_1, W_2) = (0.08, 0.2)$. Note that unless convective losses are measured experimentally to determine $D^{(A)}$, the presence of an anomalous contribution to the ion conductivity $n\chi_i$ cannot be excluded. Except in large or high-density plasmas, the fits are sensitive only to linear combinations of $D^{(A)}$ and $\chi_e^{(A)}$, and of $D^{(A)}$ and χ_i.

Comparison of the transport-code models and the experimental results of the density scan is shown in Figs. 1 and 2. Clearly, the energy confinement time τ_E, the one-turn voltage V, and the central electron temperature $T_e(0)$ are well described by the transport model. The computed central ion temperature $T_i(0)$ is slightly lower than is observed in the experiment. This difference could indicate an overestimate of anomalous convection losses in the ion channel *or* enhanced power transfer to the ions. However the fit parameters chosen give the best overall description; in view of the experimental uncertainties, we do not regard this difference as significant.

As emphasized in Ref. 9, the temperature dependence of the anomalous transport rates is difficult to determine from Joule-heated tokamak experiments. We can demonstrate the sensitivity of our transport-model results to this particular uncertainty by repeating the density-scan simulation with anomalous transport rates which depend on the average electron temperature as $\langle T_e\rangle^{-\gamma}$. In Fig. 3 we show computed results for $\gamma=0, \pm1$, where $\gamma=0$ is the case shown earlier in Figs. 1 and 2. As noted below, $\tau_E^{\max}$ is sensitive to Z_{eff}. Thus, with a value of $\gamma=-1$, $Z_{\text{eff}}\simeq1.75$ instead of the experimental $Z_{\text{eff}}\simeq1.5$ gives an acceptable fit to the data; with a value of $\gamma=+1$, $Z_{\text{eff}}=1.25$ gives a somewhat poorer fit.

If the transport model assumed here is valid, the maximum value of τ_E observed as the plasma density is varied should increase linearly with plasma current I and be independent of the toroidal magnetic field strength B_t. At the optimal density,

$$\tau_E^{(A)}=\tau_E^{(\text{neoclassical})}\sim f a^2\tau_{ii}\rho_i^{-2}q^{-2},$$

where f is unity in the Pfirsch-Schlüter régime and $\epsilon^{3/2}$ in the banana regime. Thus,

$$(na^2)_{\text{opt}}\sim[n^{-1}a^{-2}f^{-1}Z_{\text{eff}}^{-1}(RI)^2(T_i/M_i)^{1/2}]_{\text{opt}},$$

whence

$$\tau_E^{\max}\sim(na^2)_{\text{opt}}\sim RIf^{-1/2}Z_{\text{eff}}^{-1/2}(T_i/M_i)^{1/4}.$$

If the safety factor, $q=a^2B_t/RI$, is the limiting factor, this can be rewritten using $RI=a^2B_tq^{-1}$:

$$\tau_E^{\max}\sim(na^2)_{\text{opt}}\sim a^2B_tq^{-1}f^{-1/2}Z_{\text{eff}}^{-1/2}(T_i/M_i)^{1/4}.$$

Transport-model density scans were rerun with the same fit parameters for plasma currents of 75 and 150 kA for $B_t=12.8$ kG; the 120-kA case was repeated with $B_t=8$ and 15 kG. The results, computed for deuterium plasmas, are compared with corresponding results from hydrogen experiments, as shown in Fig. 4 (the more detailed confinement results were available only for deuterium plasmas). Despite the difference in absolute value, the linear dependence of $\tau_E^{\max}$ on I and its weak dependence on B_t are clear. No satisfactory explanation has been given as to why deuterium plasmas have superior confinement to hydrogen plasmas. It does not appear to be a neoclassical effect.

The interpretation of $\tau_E^{\max}$ as indicating the

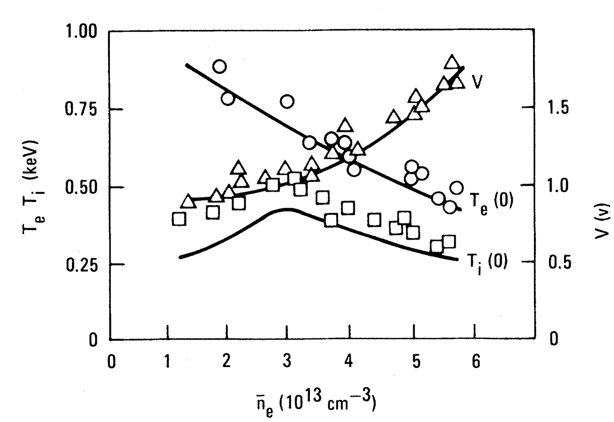

FIG. 2. Density dependence of central ion and electron temperatures and the loop voltage for the cases shown in Fig. 1.

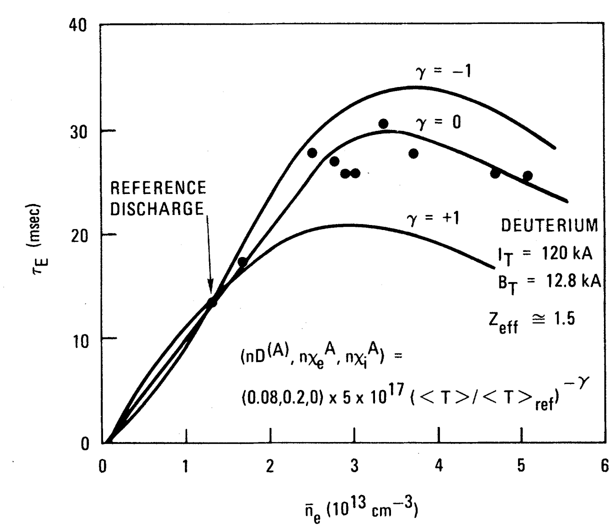

FIG. 3. Full-transport-model predictions with different assumed temperature dependences for the anomalous thermal conductivity (see text).

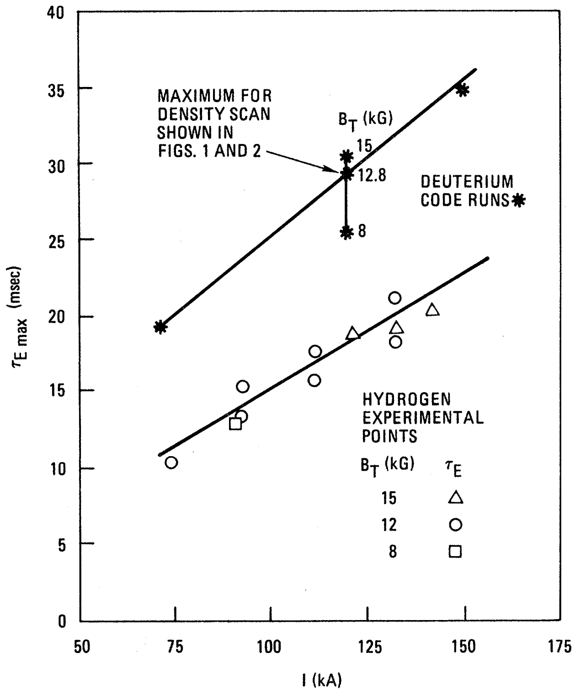

FIG. 4. Dependence of the maximum energy-confinement time on plasma current and toroidal magnetic field strength.

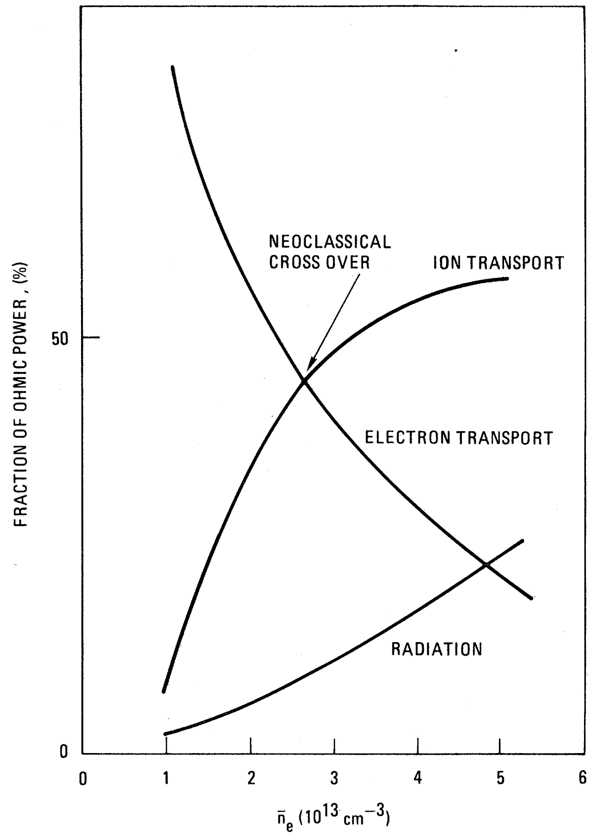

FIG. 5. The density dependence of the fraction of Ohmic power inside the $r = 21$ cm surface dissipated by ionic, electronic, and radiative processes.

density above which neoclassical transport is more rapid than anomalous energy transport can be supported directly by the computed results. Figure 5 shows major heat-loss mechanisms versus line average density at an interior radius. Although radiative cooling becomes important at the highest densities, the anomalous-to-neoclassical transition occurs in the ISX-*A* tokamak below the density at which radiation cooling dominates a significant part of the plasma.

In conclusion, a plausible transport model of ISX-*A* confinement experiments indicates that the maximum energy-confinement time observed over a range of plasma densities marks the density above which neoclassical ion thermal conductivity is a more important energy-loss mechanism than anomalous electron transport. Murakami *et al.*[10] have recently arrived at a conclusion similiar to our own. The optimal density as well as the maximum energy-confinement time vary linearly with B_t if the safety factor is restricted by magnetohydrodynamic stability conditions. Note that vertical elongation of the discharge cross section should enhance $\tau_E^{\max}$ by roughly a factor of $\sqrt{2}\,\kappa^2(1+\kappa^2)^{-1/2}$, where κ is the height-to-width ratio.

We are grateful to Dr. Wayne Pfeiffer for development of the transport code and for numerous discussions of tokamak modeling. We also thank Dr. M. Murakami, Dr. P. H. Edmonds, and Dr. H. C. Howe of the Oak Ridge National Laboratory for extensive discussions of the ISX-*A* data. This work was supported by the U. S. Department of Energy under Contract No. EY-76-C-03-0167, Project Agreement No. 38.

[1]M. Murakami, C. E. Bush, R. J. Colchin, J. L. Dunlap, P. H. Edmonds, R. C. Isler, and G. H. Neilson, in Proceedings of the IEEE International Conference on Plasma Science, Monterey, California, 15–17 May 1978 (to be published), Paper 3C4.

[2]M. Gaudreau, A. Gondhalekar, M. H. Hughes, D. Overskei, D. S. Pappas, R. R. Parker, S. M.Wolf, E. Apgar, H. I. Helava, I. H. Hutchinson, E. S. Marmar, and K. Molvig, Phys. Rev. Lett. 39, 1266 (1977).

[3]B. Basu, B. Coppi, K. Molvig, F. Pegoraro, I. Haber, B. Hui, P. Palmadesso, K. Papadopoulos, and N. Win-

sor, in *Plasma Physics and Controlled Nuclear Fusion Research 1976* (International Atomic Energy Agency, Vienna, 1977), Vol. II, p. 455; see also B. Coppi, Massachusetts Institute of Technology Plasma Research Report No. PRR-7527, 1975 (unpublished).

[4]F. L. Hinton and R. D. Hazeltine, Rev. Mod. Phys. 48, 239 (1976).

[5]For a general discussion of tokamak transport models, see John T. Hogan, in *Methods of Computational Physics*, edited by John Killeen (Academic, New York, 1976), Vol. 16, pp. 131–164.

[6]Keith H. Burrell, General Atomic Company Report No. GA-A14133, 1976 (to be published); see also Ref. 5, above.

[7]G. R. Hopkins and John M. Rawls, Nucl. Technol. 36, 171 (1977), and references contained therein.

[8]P. H. Rutherford, S. P. Hirshman, R. Jensen, D. E. Post, and F. G. P. Seidl, Princeton Plasma Physics Laboratory Report No. PPPL-1297, 1976 (unpublished).

[9]W. W. Pfeiffer and R. E. Waltz, General Atomic Company Report No. GA-A14727, 1978 (to be published).

[10]M. Murakami *et al.*, following Letter [Phys. Rev. Lett. 42, 655 (1979)].

Plasma Confinement Studies in the ISX-*A* Tokamak

M. Murakami, G. H. Neilson, H. C. Howe, T. C. Jernigan, S. C. Bates, C. E. Bush, R. J. Colchin, J. L. Dunlap, P. H. Edmonds, K. W. Hill,[(a)] R. C. Isler, H. E. Ketterer, P. W. King, D. W. McNeill,[(a)] J. T. Mihalczo, R. V. Neidigh, V. K. Pare, M. J. Saltmarsh, J. B. Wilgen, and B. Zurro[(b)]
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830
(Received 27 September 1978)

Gross-energy-confinement times (τ_E) in the ISX-*A* (Impurity Study Experiment) tokamak exceeded predictions of the usual empirical scaling relations. We attribute this performance to reductions of impurity radiation and magnetohydrodynamically driven loss channels. The value of τ_E reached a limit as a function of plasma density. We suggest that this limit is due to a transition from electron- to ion-dominated loss regimes. Maximum attainable values of τ_E increased with discharge current, in agreement with this interpretation.

The ISX-*A* (Impurity Study Experiment) tokamak operated with major radius $R = 92$ cm, minor radius $a = 26$ cm, and relatively low toroidal magnetic field $B_T \leq 15$ kG.[1,2] Only Ohmic heating was applied. Studies of plasma confinement in this device yielded unusually favorable results in comparison with empirical scaling formulas. For example, the gross-energy-confinement times, $\tau_E = \frac{3}{2}k[\int(n_eT_e + n_iT_i)dv]/P_{\mathrm{O.H.}}$, exceeded the values expected from the scaling of Jassby *et al.*[3] by factors of 1–3 (1.6 average) and were larger than the values predicted by the Hugill-Sheffield formula[4] [with scaling 1-1] by factors of 1.5–4.5 (3.1 average). At line-average densities ($\bar{n}_e$) above 10^{13} cm^{-3}, the ISX-*A* data are closest to the scaling proposed by Mirnov,[5] $\tau_E = (3\times10^{-9})a(\mathrm{cm}) \times I(\mathrm{A})\bar{n}_e^{1/2}$ sec ($\bar{n}_e$ is given in units of 10^{13} cm^{-3}), although they still exceed the expectations by an average value of 1.2. Also, the maximum value of $\bar{n}_e$ achieved before a major disruption occurred was 7×10^{13} cm^{-3}, a factor almost 4.5 times larger than that anticipated by B_T/R_0 scaling.[6] The largest values of toroidal beta, $\beta_T(0)$ equal to 2.2% at 12 kG, were comparable to the highest published value for Ohmically heated tokamaks (2.1% at 8 kG in Doublet II[7]).

It is believed that these favorable results from the ISX-*A* were realized because of low impurity content and suppression of magnetohydrodynamic (MHD) oscillations. Metals heavier than iron were excluded from the system and values of $\langle Z_{\mathrm{eff}}\rangle$ (mainly from light impurities) less than 2 were usually achieved by discharge cleaning or by titanium gettering. Radiative losses accounted for less than 30% of the Ohmic heating power for these clean discharges, and the amplitudes of MHD oscillations were relatively low ($\tilde{B}_\theta/B_\theta < 0.1\%$ at the plasma edge). The large values of $\bar{n}_e$ were obtained by programming the gas injection[8] to suppress these oscillations almost completely. Reliable feedback control of plasma position and a relatively wide separation of the limiter and vacuum vessel (6 cm) are also believed to have contributed to low levels of impurity content and MHD activity.

Figure 1 shows τ_E as a function of $\bar{n}_e$ for both

