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Raman Side-Scatter Instability in Nonuniform Plasma

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(Received 1 August 1978)

For the Raman instability in nonuniform plasma, the linear space-time asymptotic response consists of *two* parts, convectively growing wave packets and temporally growing localized side-scattered eigenmodes. Eigenmodes dominate the response only *after* the growth of the side-scattered wave packets is terminated by refraction from their resonance zones. A finite pump-beam diameter typically ends all growth before eigenmodes appear, except near quarter-critical density. These considerations may reconcile simulations, previous theories, and experiments.

The theory of Raman side-scatter instability[1-5] has developed in a confusing way (examples to be given later). This has not seemed to generate much worry, possibly because the instability has not been observed in experiments[6] exceeding the threshold predicted by theory[4] and computer simulation.[5] This should have created even more worry because a potentially violent reflective instability is hiding behind some saturation mechanism that may not operate or may be detrimental under laser-fusion conditions.

In this Letter, our first goal is to present a more complete theory of this instability, in order to resolve the confusion and paradoxes associated with the incomplete nature of the several previous theories. We view this as more than of academic interest or tying up loose ends, because of the necessity to have a clear understanding of the basic features of this potentially important instability. Moreover, the theory presented here provides a possible simple explanation for the lack of experimental evidence for this instability. This linear (i.e., convective) saturation mechanism is not effective for a single incident laser beam of power greater than about 3 TW, and thus could possibly be tested by the new laser systems that are now becoming available. Most important, this mechanism is not very effective for spherically illuminated pellets under typical proposed reactor parameters, and this would necessitate the instability saturating under a nonlinear mechanism that may have detrimental results (e.g., fast-electron generation). This disturbing prediction motivates our second goal, which is to encourage experiments designed specifically to observe and determine the saturation properties of this instability.

Above a threshold intensity, an electromagnetic pump wave (frequency ω_0, wave vector $\vec{k}_0 = k_0\hat{z}$) undergoes stimulated decay into lower-frequency scattered electromagnetic waves ($\omega_1 < \omega_0$, $\vec{k}_1$) and Langmuir waves ($\Omega = \omega_0 - \omega_1$, $\vec{K} = \vec{k}_0 - \vec{k}_1$). For given ω_1, this process is localized to a narrow resonance zone[1] where Ω is near the local plasma frequency $\omega_p(z)$. Thus side-scattered electromagnetic waves (i.e., waves with coincident turning point and resonance position) might grow to a level sufficient to prevent the pump wave from reaching and heating higher-density plasma regions.[2,3]

Indeed, temporally growing eigenmodes have been predicted,[4] and computer simulation[5] (with periodicity assumed perpendicular to the density gradient) has clearly shown strong growth until termination by electron trapping and heating and consequent severe damping of the Langmuir waves. But, as mentioned earlier, this instability has not yet been seen at all in the many laser-fusion experiments[6] apparently exceeding the theoretical threshold intensity. One possible explanation,[7] a linear (i.e., convective) saturation due to wave propagation out of the finite-diameter

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laser beam, is presented here, but first let us review the current state of the theory.

The earliest papers[2] on Raman side scattering considered wave packets which grow convectively while propagating through their resonance zones; sometimes also refraction and diffraction were taken into account.[3] Later work[4] concentrated on finding eigenmodes, which are trapped along the density gradient and grow in time.

Each of these approaches has its limitations. The propagating wave packets alone cannot always give the correct late-time response. Less appreciated is the fact that the eigenmodes alone cannot reproduce known results in the limits of uniform density, of zero pump strength, or of early time.[7] For instance, as the plasma density becomes uniform, there can be no eigenmodes; yet the maximally growing eigenmode approaches a finite growth rate which is almost (but not quite) the known uniform-plasma growth rate.[4] The early-time response, before the waves have time to refract and "feel" the density gradient, should likewise approach the uniform-plasma response (i.e., no eigenmodes). Finally, as the pump strength vanishes, the eigenmodes must vanish; yet there must still be the known response (in terms of Airy functions, for a linear density profile).

We unify these two approaches to give the correct linear z-t response to a δ-function source, through a standard method which, though straightforward in principle, is here quite complicated.[7] We consider a linear density gradient $\hat{z}dn/dz$, a pump $\vec{E}_0$ polarized along x, and a cold-plasma or large-pump limit where the thermal convection of the Langmuir waves can be ignored. The electric field $\vec{E}_1(z, \omega_1, \vec{k}_1)$ of the scattered electromagnetic waves is then described by[4,7]

$$[d^2/dz^2 + Q(z, \omega_1, \vec{k}_{1\perp})]E_1(z, \omega_1, \vec{k}_{1\perp}) = \delta(z - z_s), \qquad (1a)$$

$$Q \approx c^{-2}\{\omega_1^2 - \omega_p^2(z) - c^2k_{1\perp}^2 + D^2[\omega_p^2(z) - (\omega_0 - \omega_1)^2]^{-1}\}. \qquad (1b)$$

Here $D \equiv Kv_0\omega_p(1 - k_{1x}^2/k_1^2)^{1/2}$, $k_1 \equiv (\omega_1^2 - \omega_p^2)^{1/2}/c$, $k_{1x} \equiv \hat{x}\cdot\vec{k}_{1\perp}$, $v_0 \equiv eE_0/m\omega_0$, and D is evaluated at the resonance position given by $\omega_p = \omega_0 - \omega_1$. Equation (1b) is valid only for $\mathrm{Re}(\omega_1) \geq 0$; the full expression for Q is symmetric about the $\mathrm{Im}(\omega_1)$ axis.[7] The right-hand side of Eq. (1a) comes from the model-initiating noise source $\delta(t)\delta(z - z_s)$.

Assuming for the moment that we have somehow obtained the solution to Eq. (1) with the proper boundary conditions, the complete z-t response is then obtained by inverse Fourier transforming in ω_1 the product of $E_1(z, \omega_1, \vec{k}_{1\perp})$ and the appropriate polarization $\hat{e}_1(\omega_1, \vec{k}_{1\perp})$. This integration is carried out above all poles and branch points of the integrand in the complex ω_1 plane and is usually depressed to encircle the poles and branch cuts. The integrals around the poles (z-independent complex frequency) give the eigenmodes.[8] The integrals around the branch cuts (z-dependent complex-frequency branch points) give the wave packets[8]; the wave-packet trajectories are z-t curves of constant saddle-point (wave-packet) frequency ω_1.

The potential Q in Eq. (1) has two roots and one pole (all functions of ω_1) in the complex z plane, assuming a linear density profile. Consequently, we use phase-integral (WKBJ) techniques to trace $E_1(z, \omega_1, \vec{k}_{1\perp})$ across the complex z plane. Unfortunately, most of the calculations and even the results are too complicated and lengthy in form to appear in this Letter. For instance, the wave-packet contribution alone to $\vec{E}_1(z, t, \vec{k}_{1\perp})$ changes form in nine separate regions of the z-t plane.[7] However, we can describe the general qualitative results and a few of the more important quantitative ones. We refer the reader to Ref. 7 and to a later publication for details.

The wave-packet z-t trajectories are labeled by fixed saddle-point frequency. Along each trajectory, the wave packet grows only while it is moving inside its narrow resonance zone centered at $z_0(\omega_1)$, where $\omega_p(z_0) \equiv \omega_0 - \omega_1$. Our WKBJ approximation prevents us from looking at each wave packet while at its turning point, but we can look at it both before and after it has encountered its turning point and check for any changes (e.g., in amplitude). In this way we obtain the exponentiation level Γ_s at which the side-scattered wave packet saturates due to refraction out of the resonance zone. We find[7]

$$\Gamma_s \approx 2\left(\frac{\pi}{2}\right)^{2/3}\left[\frac{5}{4}\cos\left(\frac{\pi}{10}\right)\right]^{5/6}\left(\frac{\omega_1}{\omega_0}\right)^{1/3}\left(\frac{\omega_1}{\omega_p}\right)^{1/12}\Lambda, \qquad (2a)$$

where $\Lambda \equiv D^{3/2}L_n/\omega_p^2c$, $L_n \equiv (d\ln n/dx)^{-1}$, and $\omega_p = \omega_0 - \omega_1$. Note that the side-scattered wave packet also has $\omega_1 = (\omega_p^2 + c^2k_{1\perp}^2)^{1/2}$ and thus a unique frequency $\omega_1 = (\omega_0 + c^2k_{1\perp}^2/\omega_0)/2$ and resonance position $z_0(\omega_1) = z_0(k_{1\perp})$ for given $k_{1\perp}$. For back- or oblique-scattered wave packets, the exponentiation Γ_B is obtained by comparing amplitudes before and after the wave packet has traveled through its resonance zone. This just gives the well-known result[1]

$$\Gamma_B = \tfrac{1}{2}\pi(D/\omega_p c)^2 L_n/k_{1z}, \tag{2b}$$

where $k_{1z} \equiv (k_1^2 - k_{1\perp}^2)^{1/2}$. For a given source position z_s, the side-scattered wave packet travels from z_s to $z_0(k_{1\perp})$ *and* saturates (by refraction) at the exponentiation level Γ_s at a definite time t_s which is minimized by having the source on the resonance zone edge (low-density side). Then[7]

$$(t_s)_{\min} \approx \frac{4}{\omega_1}\left(\frac{\omega_p}{\omega_1}\right)^{1/2}\left(\frac{\omega_1 Z}{c}\right)^2 \Lambda^{-2/3}\Gamma_s, \tag{2c}$$

where $Z \equiv (c^2 L_n/\omega_p^2)^{1/3}$. The wave packets are just straightforward generalizations of the wave packets which make up the *entire* response in the limiting cases $L_n \to \infty$ or $v_0 \to 0$.[7]

The eigenmodes are localized in z near the "side-scatter" resonance position $z_0(k_{1\perp})$ and, above a threshold pump intensity ($\Lambda \approx 0.3$), are temporally growing.[4] However, the coefficient in front of the temporally growing exponential is small (and vanishes as $L_n \to \infty$) so that it takes a finite time before the exponential can overcome this small coefficient.[7] Even at time t_s (arbitrary z_s), the eigenmodes are still negligible compared to the wave packets; the eigenmodes do not dominate until at least approximately[7] time $2(t_s)_{\min}$. The above features of the eigenmodes and the inclusion of the wave packets in the total response therefore ensure the correct limiting behavior.

Thus, for the case considered so far (pump uniform in x and y, and given $\vec{k}_{1\perp}$), the theoretical response always evolves eventually to a nonlinear state where the eigenmodes are dominant over (or in the case of early nonlinear saturation, at least comparable with) the wave packets. This is in agreement with computer simulations[5] where periodicity (discrete $\vec{k}_{1\perp}$) is assumed along x and y, and would tend to support the emphasis[4] that has been placed on the eigenmodes. So why has the instability remained hidden experimentally? There are probably several possible nonlinear saturation mechanisms, but let us focus here on a linear saturation mechanism due to perpendicular convection of the waves out of the finite laser-beam diameter. As shown below, the fact that the eigenmodes can dominate the response only after a certain period of time can drastically reduce the importance of the eigenmodes (relative to the periodic situation) except near quarter-critical density.

To obtain the full three-dimensional space-time response $\vec{E}_1(\vec{x}, t)$ to the source $\delta(t)\delta(\vec{x} - \vec{x}_s)$ one must integrate $\vec{E}_1(z, t, \vec{k}_{1\perp})$ over k_{1x} and k_{1y}. This converts the former wave packets (in z-t) into new wave packets with trajectories now x-y-z-t curves of constant saddle-point frequency ω_1 and saddle-point perpendicular wave vectors k_{1x} and k_{1y}. The former eigenmodes along z are still temporally growing and localized in z near $z_0(k_{1\perp})$ but now act also as propagating "wave packets" along x and y with trajectories given by x-y-t curves of constant saddle-point wave vectors k_{1x} and k_{1y}. The eigenmode frequencies $\omega_{1N}(k_{1x}, k_{1y})$ and the position $z_0(k_{1\perp})$ are also evaluated with the saddle-point wave vectors. While these waves are traveling within their resonance zones, we can use approximately the uniform-plasma result that the response convects (in x and y) with an effective group velocity $\vec{V}_g$ equal to $(\vec{V}_{1\perp} + V_{2\perp})/2$, where $V_{1\perp} \equiv c^2 k_{1\perp}/\omega_1$ and $\vec{V}_{2\perp} \equiv v_{\rm th}^2 \vec{K}/\Omega$ are the group velocities of the uncoupled (unpumped) electromagnetic and Langmuir waves, respectively. Since the electron thermal speed $v_{\rm th}$ is small, we have all waves convecting in x and y with a velocity $\vec{V}_{g\perp} \approx c^2 k_{1\perp}/2\omega_1$. Thus, during the minimum time of approximately $2(t_s)_{\min}$ required for the eigenmodes (along z) to dominate the response, they (as well as the wave packets) will have convected in x and y a distance $d_{\min} \approx (t_s)_{\min} c^2 k_{1\perp}/\omega_1$.

Consider now the realistic situation where the pump (e.g., a focused laser) has a finite beam diameter d_L in the x-y plane. If $d_L < d_{\min}$, all growth will have ended with the eigenmodes still negligible (unless the wave packets nonlinearly saturate early). Observation of the eigenmodes would require $d_L > d_{\min}$ which translates into

$$P_0 \gtrsim 2.3\left(\frac{\Gamma_s}{10}\right)^2\left(\frac{\omega_0^2}{\omega_p\omega_1}\right)\left(\frac{k_{1\perp}}{K}\right)^2, \tag{3a}$$

with the pump power $p_0 \equiv I_0 d_L^2$ expressed in units of 10^{12} W. For fixed Γ_s, this minimum required power vanished at quarter-critical density (where $\omega_p = \omega_0/2$ and $k_{1\perp} = 0$) but diverges as ω_0/ω_p as $\omega_p \to 0$. Taking (e.g.) $\Gamma_s \sim 10$ and $\omega_p = \omega_0/3$ gives $P_0 \gtrsim 3\times 10^{12}$ W, which has only recently become available. The intensity must also be consistent with Eq. (2a) to give the chosen value of Γ_s; i.e.,

$$\Gamma_s \approx 1.0\left(\frac{\omega_1}{\omega_0}\right)^{5/12}\left(\frac{\omega_p}{\omega_0}\right)^{11/12}\left(\frac{Kc}{\omega_p}\right)^{3/2} \times L_n \lambda_0^{1/2} I_0^{3/4}\left(1 - \frac{k_{1x}^2}{k_1^2}\right)^{3/4}, \tag{3b}$$

where L_n and the pump wavelength λ_0 are in microns, and I_0 is in units of 10^{17} W/cm^2. It is interesting to note that, for fixed Γ_s and ω_p/ω_0, P_0 is independent of L_n and λ_0.

For given pump intensity I_0, a finite beam diameter d_L or power P_0 thus limits the density range over which the eigenmodes can grow to dominance and require a nonlinear saturation mechanism (e.g., electron trapping and heating, resulting in suprathermal electrons[9]). With a spherically illuminated pellet, the size of a fixed polarization region can be used for d_L provided it is much smaller than the pellet radius. Typical proposed reactor parameters of $I_0 = 10^{16}$ W/cm^2, $L_n = 100$ μm, $\lambda_0 = 1\,\mu$m, and taking $\omega_p = \omega_0/3$, give such large values of Γ_s (≈ 32) that, even if $d_L \ll d_{\rm min}$, the wave packets (and possibly the eigenmodes) would probably saturate nonlinearly rather than convectively.

In conclusion, we believe that many of the paradoxes that confused interpretation of early theories of the Raman instability, and comparisons with simulation and experiment, have now been resolved. These paradoxes are primarily due to the inability of wave packets alone or eigenmodes alone to give the correct response in all physical limits (e.g., of t, L_n, and v_0). In cases where the pump finite beam diameter d_L is not an important constraint, theory and simulation apparently agree that the eigenmodes eventually become comparable with or dominant over the wave packets and saturate nonlinearly. For typical experimental values of d_L, however, the eigenmodes may not have sufficient room (before convecting out of the pump) to form except near the quarter-critical-density position where the waves have vanishing group velocity. This reduced region of automatically required nonlinear saturation (which generates detrimental fast electrons) may offer one explanation (others are possible) for the lack of experimental evidence for Raman side scattering. For laser-fusion reactors, even if the eigenmodes are reduced in importance by this effect, the wave packets can be expected to require nonlinear saturation over most of the plasma below quarter-critical density.

We thank B. Cohen, V. Fuchs, G. Johnston, S. Johnston, A. B. Langdon, C. S. Liu, C. Max, H. Mynick, D. Nicholson, and G. Smith for many helpful discussions. This work was supported by the U. S. Energy Research and Development Administration.

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