Octupole-Octupole Residual Interactions and Low-Lying 0^+ Excited States in the Isotopes ^{232}U, ^{234}U, and ^{236}U

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Solutions applicable for all interaction strengths are obtained for a schematic Hamiltonian with pairing forces and particle-hole octupole-octupole forces. The method is applied to the light U isotopes. Adjusting the particle-hole interaction strength to the measured $K^\pi = 0^-$ excitation energies leads to an immediate explanation for the low-lying 0^+ excited states in this region.

The even nuclides in the vicinity of mass 230 are noteworthy because of the extremely low-lying 0^- (K^π) intrinsic states and also because of the low-lying 0^+ excited states. In this work, I examine the connection between these two groups of low-lying states.

The appearance of 0^- intrinsic states at energies well below the 2-quasiparticle gap implies that 0^- correlations may play a very important role in determining the properties of ground states and excited states; a role that is much larger than would be inferred from calculations based on a BCS vacuum, such as quasi-random-phase-approximation calculations. Higher-order effects have been taken into account through second order in the particle-hole interaction and have been found[1] to be quite important. There remains a question as to whether or not still-higher-order diagrams are important, as well as the possibility of stable octupole deformations. In an early calculation looking into this latter possibility, it was concluded[2] that there are no nonzero octupole deformations for ground states in this region. In a later calculation,[3] the suggestion was raised, for the Ra and Rn isotones, that there is a nonzero octupole deformation associated with the 0^- excited states, but not with the ground-state, 0^+ band. In addition to these two possibilities, there is the alternative of the octupole particle-hole interaction being in the "transition" regime. This regime of interaction strengths is not accessible to either of the two types of calculations mentioned above. In this work, I consider solutions of a Hamiltonian with both pairing and octupole-octupole interactions that treat the pairing and octupole correlations on an equal footing in both the ground state and in excited states; solutions that are applicable in the vibrational, transitional, and rotational regions of the octupole interaction.

The Hamiltonian I use here is

$$H = \sum_{\nu=1,2} \sum_{\kappa} \epsilon_{\kappa\nu} N_{\kappa\nu} - \sum_{\nu=1,2} G_\nu \sum_{\kappa>0} a_{\kappa\nu}^\dagger a_{-\kappa\nu}^\dagger \sum_{l>0} a_{-l\nu} a_{l\nu} - V \sum_{\nu=1,2} \sum_{\kappa,l} \langle \kappa_\nu | r^3 Y_3^0 | l_\nu \rangle a_{\kappa\nu}^\dagger a_{l\nu} \sum_{\nu' \geq \nu} \sum_{m,n} \langle m_{\nu'} | r^3 Y_3^0 | n_{\nu'} \rangle a_{m\nu'}^\dagger a_{n\nu'}, \quad (1)$$

where the indices ν and ν' denote the type of nucleon (proton or neutron); ϵ, a single-particle energy in an appropriately deformed Woods-Saxon potential; N, a number operator; and $a_\kappa^\dagger$ and a_κ are creation and annihilation operators. The pairing term is conventional; the octupole-octupole partilce-hole term includes n-n, p-p, and n-p contributions. I use deformation parameters[4] $\nu_2 = 0.19$, $\nu_4 = -0.04$ and pairing interaction strengths $G_n = 21/A$ MeV, $G_p = 30/A$ MeV. I have treated the octupole interaction strength, V, as an adjustable parameter in my calculations, using it to fit approximately the observed excitation energy of the 0^- band. The 0^- band head lies ~ 10 keV below the 1^- level because of zero-point rotational effects. Note that there is a 3% increase in V as I go from ^{232}U to ^{236}U. As the changes in this parameter must compensate for the simple form of the particle-hole interaction, possible changes in deformation, and possible deviation of pairing matrix elements from constancy,[5-7] I feel that this is quite satisfactory.

The first set of solutions that I consider for this problem is of the product form suggested[8] in a previous work. I set

$$\psi_i = \left[\prod_{\alpha,\beta}^{n} \varphi_{\alpha,\beta}^{i} \prod_{\gamma,\delta,\epsilon}^{n} \xi_{\gamma,\delta,\epsilon}^{i} \right]_n \left[\prod_{\alpha',\beta'}^{p} \varphi_{\alpha',\beta'}^{i} \prod_{\gamma',\delta',\epsilon'}^{p} \xi_{\gamma',\delta',\epsilon'}^{i} \right]_p |0\rangle, \quad (2)$$

where $|0\rangle$ denotes the physical vacuum and subscripts n and p denote neutrons and protons. The indi-

ces α and β denote deformed orbitals with the same values of Ω and opposite parities. The indices γ, δ, and ϵ indicate three orbitals having the same value of Ω, one of which is of parity opposite to the other two. In general, the distribution of the active orbitals near the Fermi level is such that a group of three levels appears to be adequate for a given value of Ω. In each group $\varphi_{\alpha,\beta}$ I include all configurations of the doubly degenerate orbitals α and β with Ω^π of 0^+ and 0^-. For $\varphi_{\alpha,\beta}$ there are six such possibilities; for $\zeta_{\gamma,\delta,\epsilon}$ there are twenty possibilities. For purposes of illustration, I note that

$$\varphi_{\alpha,\beta} \equiv U_1(\alpha,\beta) + U_2(\alpha,\beta)a_\alpha^\dagger a_{-\alpha}^\dagger + U_3(\alpha,\beta)a_\beta^\dagger a_{-\beta}^\dagger + U_4(\alpha,\beta)a_\alpha^\dagger a_{-\alpha}^\dagger a_\beta^\dagger a_{-\beta}^\dagger + U_5(\alpha,\beta)a_\alpha^\dagger a_{-\beta}^\dagger + U_6(\alpha,\beta)a_\beta^\dagger a_{-\alpha}^\dagger. \quad (3)$$

Note that $\varphi_{\alpha,\beta}$ conserves neither parity nor particle number. Previously, I have discussed how these conservation rules can be taken into account by introducing correlations[8] between terms in a product wave function. Here, because of the delicate balance between pairing and octupole correlations, I have chosen to minimize the quantities $U_i(\alpha,\beta)$ and $U_i(\alpha,\beta,\gamma)$ for fully projected wave functions, i.e., wave functions with sharp values of neutron number, proton number, and parity. These amplitudes are obtained by solving, iteratively, the set of equations

$$\frac{\partial\langle\psi_i(N,P,\pi)|H|\psi_i(N,P,\pi)\rangle}{\partial U_i(\alpha,\beta)} = 0. \quad (4)$$

I carry out a separate minimization for positive- and negative-parity solutions. There are some problems in the solutions of Eq. (4). I have found that solutions can be obtained easily for small and large values of V. Such solutions are then used as first approximations to the solutions for the desired value of V (incrementing V in small steps). I denote the positive- and negative-parity solutions of Eq. (4) as ψ_1^+ and ψ_2^- respectively. I also consider the opposite-parity projections of these solutions, denoted as ψ_1^- and ψ_2^+.

In examining the solutions of Eq. (4), I find that pairing correlations are dominant for the light U isotopes in the state ψ_1^+. In the states ψ_2^-, however, the situation is quite different and the octupole correlations play an important role, particularly for the neutron orbitals. I consider ^{234}U in some detail. In ^{234}U, neutron pairing gives a contribution to the energy of -1.64 MeV for the state ψ_1^+ and neutron-neutron octupole correlations give -0.53 MeV. In the state ψ_1^-, the two contributions are -1.55 and -1.14 MeV respectively. This increase in the octupole correlation energy is completely due to projection effects. In the state ψ_2^- these two contributions are -1.75 and -3.05 MeV, respectively. Comparing these values with those obtained for ψ_1^- immediately shows that there is a substantial difference in the underlying states ψ_1 and ψ_2 and that there is a nonperturbative change arising from the octupole correlations. Direct examination of ψ_2 shows 0^- amplitudes comparable in magnitude to 0^+ amplitudes.

I next consider the energies of the states $\psi_1^\pm, \psi_2^\pm$. Taking the energy of the state ψ_1^+ as zero of energy, I calculate $\langle\psi_1^-|H|\psi_1^-\rangle = 2.08$ MeV, $\langle\psi_2^-|H|\psi_2^-\rangle = 0.85$ MeV, and $\langle\psi_2^+|H|\psi_2^+\rangle = 0.79$ MeV. The fact that ψ_2^+ and ψ_2^- are not degenerate in energy indicates that the solution ψ_2 does not correspond to a region of permanent octupole deformation, but rather to the transition region for the octupole interaction. An increase of V of $\simeq 10\%$ gives a state ψ_2' with degenerate energies.

The low energy that I obtain for the state ψ_2^+ is of considerable interest. It is in good agreement with the experimentally observed value of 0.81 MeV and seems to offer an explanation for the long-standing mystery[5-7,9] of this low-lying 0^+ excited state. The problem is not quite so simple because the states ψ_1^+ and ψ_2^+ are not orthogonal ($\langle\psi_1^+|\psi_2^+\rangle \simeq 0.2$). Also, the projected product wave functions are only approximate solutions of the Hamiltonian of Eq. (1); so we have $\langle\psi_1^+|H|\psi_2^+\rangle \neq 0$. Diagonalizing the interaction for these two nonorthogonal states gives an excitation of 0.90 MeV for the 0^+ excitation energy. I have carried out a more elaborate calculation of the 0^+ excitation energy using additional basis states. The additional basis states are of two types. The first type involves the projection of a component from the states ψ_1 or ψ_2, i.e.,

$$\theta_i^\pm(\alpha,\beta) = [\mathfrak{N}_i(\alpha,\beta) - \langle\mathfrak{N}_i(\alpha,\beta)]\psi_j^\pm, \quad (5)$$

where, e.g., the symbol $\mathfrak{N}_2(\alpha,\beta)$ represents a product of number operators [cf. Eq. (3)],

$$\mathfrak{N}_2(\alpha,\beta) = N_\alpha N_{-\alpha}(1-N_\beta)(1-N_{-\beta}). \quad (6)$$

I have used all such states with excitation energies below 2.5 MeV. I have also used G and V as generator coordinates[10,11] to get some additional basis states. I calculate overlaps and interaction matrix elements exactly, however. Carrying out

this larger diagonalization gives an excitation energy of 0.87 MeV, which is in quite good agreement with the experimental value. An equivalent diagonalization procedure gives the 0^- excited-state energy at 0.79 MeV. If we use only a pairing force as the residual interaction, I note that the 0^+ excited-state energy is ~ 1.3 MeV. Using a density-dependent δ-function force to calculate pairing force matrix elements, I got[7] 1.35 MeV for this excitation energy.

I have applied the same calculation to ^{232}U, and the results are equally encouraging. Adjusting V to get the 0^- excitation at 0.55 MeV, I calculate a 0^+ excited state at 0.76 MeV. This agrees well with the measured value of 0.69 MeV.

In ^{236}U, I adjust V to get a 0^- band at 0.68 MeV and calculate a 0^+ excited state at 0.90 MeV, which is in extremely good agreement with the known excitation energy of 0.92 MeV. However, the 0^+ state calculated to be at 0.90 MeV has little relation to the octupole interaction. I obtain another 0^+ excited state at 1.16 MeV in my calculation, which is more closely associated with the octupole interaction. This state has not been identified as yet in ^{236}U.

In a previous work,[7] I pointed out that the neutron pairing matrix elements associated with the $j_{15/2}$ orbitals appear to be somewhat larger than other pairing matrix elements. I have carried out another set of calculations with enhanced $j_{15/2}$ neutron pairing matrix elements and find essentially no change in the results.

The best experimental probe of these octupole correlations is the measurement of $E3$ transition probabilities between the 0^+ and 0^- bands in these U isotopes (and the corresponding Th isotones). My calculations suggest that there are several large $B(E3)$ values, although I have not included Coriolis interaction effects which may modify[1] the results somewhat. In Table I, I give the calculated $B(E3)$ values and available measured values[12] for relevant Th and U isotopes. Rather than making use of effective charges, I have normalized the calculated results to the ground-state transition in ^{232}U and the measured results to the equivalent transition in ^{230}Th. I note that the calculated ratios do not change very much if I use just the proton $B(E3)$ rather than the sum of proton and neutron $E3$ amplitudes. An experimental program is now underway at Argonne National Laboratory to measure the $B(E3)$ values involving excited 0^+ states.

Another problematic aspect of nuclides in this mass region is the magnitude of pair transfer cross sections to excited 0^+ states relative to the ground-state cross section. Experimentally, this ratio is found to be ~ 15% in (p,t) reaction[13] studies and ~ 0% in (t,p) reaction[14] studies. This asymmetry has been ascribed to differences in the specific orbitals populated in the two reactions. As I do not have a code for calculating reaction mechanism effects, I cannot test the wave functions directly. I have evaluated the pair transfer operator, $T_{\alpha,\beta}(\langle\alpha|a_{\kappa_n}{}^\dagger a_{-\kappa_n}{}^\dagger|\beta\rangle^2)$, for transitions to both ground and excited 0^+ states. Using just a pairing force to generate wave functions gives rise to rather small values of $T_{\alpha,\beta}$ to the excited state; and the problem, as I have noted, has been to find ways of explaining the (p,t) enhancement. Using the wave functions calculated with the octupole interaction, I find that T_{α,β^*} (where β^* denotes the excited 0^+ state) is typically ~ 20% of $T_{\alpha,\beta}$. This seems to be generally true in this region for pair removal. Unfortunately, it is equally true for pair addition. It appears that the problem is now one of explaining the reduction in the

TABLE I. $B(E3)$ values. Measured values[a] are for the corresponding transitions in U and Th isotones.

Nuclide	0^+ (MeV) calc.	0^- (MeV) calc.	$B(E3)$ calc.	$B(E3)$ Th	$B(E3)$ U
^{232}U	0	0.55	(1)	(1)	...
^{232}U	0.76	0.55	1.25	...	...
^{234}U	0	0.79	0.6	$< 0.7^b \pm 0.15$	$< 0.95^b \pm 0.2$
^{234}U	0.87	0.79	1.77	...	...
^{236}U	0	0.68	0.83	...	0.85 ± 0.2
^{236}U	0.90	0.68	0.15	...	...
^{236}U	1.16	0.68	0.80	...	...

[a]See Ref. 12.

[b]Additional uncertainties due to near degeneracy of 2^+ and 3^- levels.

(t,p) reaction.

In summary, I find that a general treatment of the octupole-octupole particle-hole residual interaction provides a natural, unforced explanation for the low-lying 0^+ excited states in the light U isotopes (and Th isotones).

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Structure in Backward-Angle Excitation Functions for Strongly Absorbed Particles

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A strong, isolated pole gives rise to periodic structure in backward-angle excitation functions. A moderate pole interfering with a nonresonant diffraction amplitude produces similar structure. The $^{16}O+^{28}Si$ backward-angle elastic excitation function is analyzed from these points of view.

Recent measurements by Braun-Munzinger *et al.*[1] revealed a large, oscillatory angular distribution in the backward direction for $^{16}O+^{28}Si$ elastic scattering. This discovery generated considerable interest.[2] A number of papers investigating the experimental and theoretical ramifications of this phenomenon have subsequently appeared.[2-11]

Barrette *et al.*[4] measured excitation functions for $^{16}O+^{28}Si$ and $^{12}C+^{28}Si$ at extreme backward angles and observed some gross structure. Additional backward-angle excitation measurements are reported by Clover *et al.*[5] and Renner *et al.*[9] Dehnhard *et al.*[6] fitted the elastic $^{16}O+^{28}Si$ excitation function by introducing a parity dependence into the optical potential. On the other hand Lee,[8] based on a more conventional optical-model calculation, has interpreted the structure of the excitation function in terms of interference between internal and surface-barrier waves—as it was done for the case of α scattering.[12] Here I discuss a simple, rather general interpretation of structures in backward-angle excitation functions for strongly absorbed particles. At present, I consider only elastic scattering and focus on the $^{16}O+^{28}Si$ data.

The amplitude for the elastic scattering of spinless, nonidentical nuclei is given by

$$f(\theta)=\frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)[\exp(i2\sigma_l)S_l-1]P_l(\cos\theta), \tag{1}$$

where S_l is the nuclear partial-wave S matrix and

$$\exp(i2\sigma_l)=\Gamma(l+1+i\eta)/\Gamma(l+1-i\eta) \tag{2}$$

is the Coulomb partial S matrix, η being the Coulomb strength parameter. Now at $\theta=\pi$, $P_l(\cos\pi)=(-1)^l$ so that Eq. (1) becomes a sum of alternating terms. It can then be expressed in terms of a contour in-

