Quenching of Allowed Gamow-Teller β Transitions in Mirror Nuclei

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Consideration of core polarization, meson exchange, and relativistic corrections gives a satisfactory, quantitative understanding of mirror Gamow-Teller β-transitions in A=3, 15, 17, 39, and 41 nuclei.

Gamow-Teller β-transition rates in simple "doubly closed LS shell ± 1 nucleon" mirror nuclei ($A=3$, 15, 17, 39, and 41) are slower[1-4] than those predicted with zero-order shell-model wave functions and an axial-vector coupling constant g_A ($\sim 1.23 g_V$) derived from neutron β-decay data. This slowness (quenching) is attributed to an interplay among (a) core polarization, i.e., improvements in the wave functions used to calculate the nuclear β-decay matrix elements,[5,6] (b) corrections due to mesonic exchange effects,[7-9] and (c) allowance for relativistic effects[10] in the hadron wave functions. A consideration of these three corrections by Barroso and Blin-Stoyle[8] led to the conclusion that for $A=17$ and $A=41$ an understanding of the quenching was well in hand, while for $A=15$ and 39 there were strong discrepancies indicating a need for further quenching contributions of the order -10% to -20%, that is, of the same size as the entire effect.

It should be remarked that the theoretical calculations are sensitive to the treatment of short-range correlations, the length parameter of the oscillator wave function, and so on. Furthermore there is considerable cancellation between core-polarization and meson-exchange effects, the extent of which can only truly be assessed if both contributions are calculated consistently and on the same footing.[11] In this Letter, we present results from such a consistent calculation,[12] which for the first time give a quantitative understanding of quenching in Gamow-Teller β transitions. A different but related description based on Landau-Migdal theory for the quenching, which is also in quantitative agreement with experiment, is given in the accompanying paper by Oset and Rho.[13]

Our calculation differs from earlier ones in the treatment of the $\Delta(1236)$ resonance. We use a nonrelativistic model[14] in which the Δ is described as a baryon bound in a harmonic-oscillator potential ($\hbar\omega = 41A^{-1/3}$ MeV) of the same characteristic frequency as that used for nucleons. The single-particle energy for the Δ is that of the oscillator plus 294 MeV. Two-body interactions between nucleons and between nucleons and isobars are described by one-boson–exchange transition potentials as displayed in Fig. 1, the exchanged bosons being the π, ρ, and ω mesons. The coordinate-space representation for the case of one-pion exchange, in the static limit with the δ-function term removed, is

$$W_1(r) = \tfrac{1}{3} m_\pi f_{\pi NN}^2 (\vec{\tau}_1 \cdot \vec{\tau}_2) \times [(\vec{\sigma}_1 \cdot \vec{\sigma}_2) Y_0(x_\pi) + S_{12} Y_2(x_\pi)],$$

where $S_{12} = 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$; $Y_0(x) = e^{-x}/x$; $Y_2(x) = (1 + 3/x + 3/x^2) Y_0(x)$; and $x_\pi = m_\pi r$. The transition potentials W_2 and W_3 are simply obtained from W_1 by substituting[15] $f_{\pi NN} \to f_{\pi N\Delta}$, $\vec{\sigma} \to \vec{S}$, and $\vec{\tau} \to \vec{T}$ at the appropriate vertices where $\vec{S}$ and $\vec{T}$ are generalizations of the ordinary spin and isospin matrices acting as transition operators between $\frac{1}{2}$ and $\frac{3}{2}$ spinors. The coupling constants are $f_{\pi NN}^2 = 0.08$ and $f_{\pi N\Delta}/f_{\pi NN} = 6\sqrt{2}/5$, the latter result being a prediction from the quark model.[15] No form factors are introduced at the pion-nucleon vertices; rather the whole question of short-range behavior and wave-function correlations will be avoided by introducing, purely phenomenologically, a sharp cutoff at $0.5\hbar/m_\pi c$ (0.71 fm) as the lower limit for all radial integrals.[6]

The one-ρ- and -ω-exchange potentials are given by similar expressions[16] with

$$f_{BNN}^2 = \frac{1}{4\pi}\left(\frac{g_{BNN} m_B}{2M}\right)^2 (1+K)^2,$$

where m_B and M are the boson and nucleon mass-

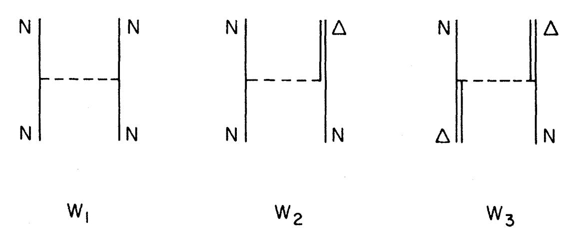

FIG. 1. One-boson–exchange contributions to the nucleon-nucleon interaction, W_1; to the transition potential for $NN \to N\Delta$, W_2; and for $\Delta N \to N\Delta$, W_3.

es. The coupling constants, $g_{\rho NN}$ and $g_{\omega NN}$, can be deduced from analyses of exclusive 2π and 3π production in e^+e^- annihilation[17] and vector-meson dominance: $g_{\rho NN}=2.84$ and $g_{\omega NN}=7.60$. Strict SU(3) would require $g_{\omega NN}=3g_{\rho NN}$. The vector-meson-dominance model identifies K, for ρ exchange, with the isovector anomalous moment, $K=3.7$, and for ω exchange with the isoscalar anomalous moment, $K=-0.12$. Recent developments[18] seem to suggest instead that for ρ exchange $K=6.6$ (and $g_{\rho NN}=2.63$), and we adopt these values. For the transition potentials, the quark model again gives the relation $g_{\rho N\Delta}/g_{\rho NN}=6\sqrt{2}/5$.

Lastly, the Gamow-Teller matrix element between a Δ and a nucleon is expressed in terms of the transition spin operators: $\langle\Delta|\frac{1}{2}G_A\vec{S}\vec{T}|N\rangle$, this being of the same form as the usual Gamow-Teller matrix element between nucleons: $\langle N|\frac{1}{2}g_A\vec{\sigma}\vec{\tau}|N\rangle$. Note that the same replacements apply here, $g_A\rightarrow G_A$, $\vec{\sigma}\rightarrow\vec{S}$, and $\vec{\tau}\rightarrow\vec{T}$, as in the case of the one-boson-exchange potential and the coupling constants are related analogously: $G_A/g_A=f_{\pi N\Delta}/f_{\pi NN}=6\sqrt{2}/5$.

The calculation now proceeds in the traditional shell-model way. For "doubly closed LS shell ± 1 nucleon" nuclei, the first-order core-polarization contribution (from nucleons only) to the Gamow-Teller matrix element is zero. In second order, we need only consider what is known as the number-conserving sets[19] (NCS), a sample of which is shown in the first row of Fig. 2. These are the diagrams previously calculated by Shimizu, Ichimura, and Arima.[6] Extending the calculation to include just one isobar in an intermediate state reduces the contribution to the Gamow-Teller matrix element from the NCS, as can be seen in the third row in Table I. Intermediate states of energy up to $12\hbar\omega$ were explicitly calculated and a geometric extrapolation to infinite energy yields the quoted results.

With the introduction of the isobar, a first-order core-polarization contribution emerges, as shown in the second row of Fig. 2, arising from Δ-particle, nucleon-hole (Δ-h) intermediate states. This contribution has also been considered before,[7-9] but as a meson-exchange graph and evaluated using partial conservation of axial-vector current and low-energy theorems. The advantage of our procedure is that the calculation is easily extended to higher orders by summing the random-phase-approximation (RPA) series. Note, as shown in Fig. 2, that, starting with second order, nucleon particle-hole (p-h) states are mixed in the chain with Δ-h states. In row four

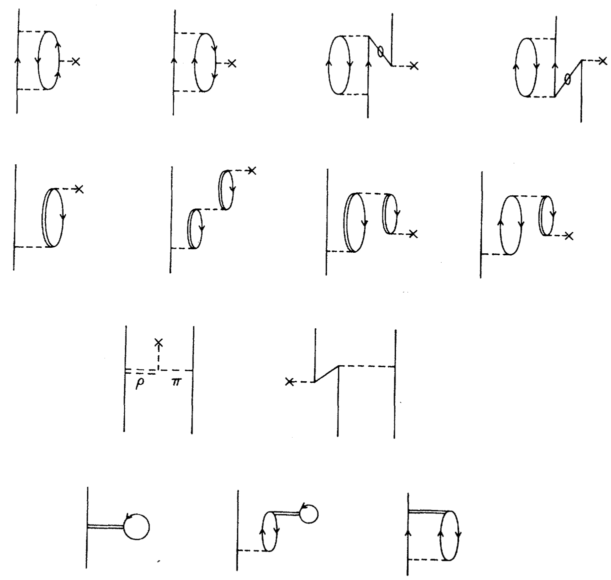

FIG. 2. Sample core-polarization and meson-exchange diagrams. The first and second rows display a number-conserving set involving 2p-1h (two-particle, two-hole) intermediate states and the start of the RPA series for Δ-particle, nucleon-hole states. The third row shows two Feynman diagrams depicting ρ-π and pair-excitation meson-exchange processes. Nonrelativistic reduction of the ρ-π diagram leads to a two-body transition operator represented in the fourth row as two horizontal lines.

of Table I, we show the contribution to the Gamow-Teller matrix element from the RPA chain arising from Δ-h intermediate states only and from keeping only the direct matrix element at the first vertex. The contribution from the exchange matrix element and from p-h states is then shown in row five. This separation facilitates a comparison with Oset and Rho[13] who list the direct RPA contribution. The antisymmetry at the first vertex leads to a considerable cancellation, which, for example, is complete in a first-order calculation in $A=3$.

Two additional meson-exchange diagrams, illustrated in the third row of Fig. 2, arise from ρ-π and pair excitations. The pair-excitation diagram gives a very small contribution and has not been included since in the low-energy limit it is canceled by the seagull term.[20] The ρ-π diagram is evaluated with the proper q^2 dependence of the ρ propagator retained,[7] and its contribution to the Gamow-Teller matrix element is listed in row six of Table I.

The nonrelativistic reduction of the ρ-π diagram leads to a transition operator that is two-

TABLE I. Contribution to the Gamow-Teller matrix element from various core-polarization and meson-exchange corrections expressed as a percentage of the single-particle matrix element.

		A = 3	A = 15	A = 17	A = 39	A = 41
1.	NCS ($2\hbar\omega$) nucleons	-5.7	-7.3	-5.5	-8.9	-5.7
2.	NCS ($4\to\infty\hbar\omega$) nucleons	-6.3	-11.6	-8.4	-16.1	-11.4
3.	NCS ($0\to\infty\hbar\omega$) isobars	2.2	4.5	2.1	3.6	2.4
4.	RPA (Δ-h) direct	-11.6	-5.7	-8.3	-11.8	-9.8
5.	RPA exchange	12.8	10.0	5.8	8.7	5.7
6.	ρ-π	0.9	-8.0	-1.4	-3.7	-1.4
7.	ρ-π; RPA	1.1	2.3	0.1	0.4	-0.1
8.	ρ-π; NCS	5.5	10.8	4.7	7.4	5.4
9.	Relativistic[a]	-1.5	-4.4	-2.7	-3.7	-3.4
10.	Sum	-2.6	-9.4	-13.7	-24.0	-18.3
11.	Expt. (ref. 4)	-2.5	-11.5	-12.2	-32.3	-24.8
		±0.7	±0.6	±0.6	±1.0	±0.6

[a] From Bell and Blin-Stoyle (Ref. 10) for $A=3$; as quoted by Barroso and Blin-Stoyle (Ref. 8) for $A=15$, 17, 39, and 41. Results obtained by using nonrelativistic reduction of the axial-vector operator to order M^{-2} for these nuclei are quite similar to those obtained by solving the Dirac equation for nucleons with a parametrized form for a one-body potential.

body in character. This operator may now be used in perturbation theory to give in first and higher orders a core-polarization correction. Sample diagrams are shown in the fourth row of Fig. 2, where the two horizontal lines represent the two-body transition operator. Intermediate states up to $12\hbar\omega$ were included and a geometric extrapolation to infinite energy yields the results listed in row seven for the RPA iterated series and in row eight of Table I for the NCS diagram.

Lastly the nonrelativistic reduction of the axial-vector current, which in lowest-order impulse approximation leads to the usual one-body Gamow-Teller operator, produces an extension to order $1/M^2$ of the relativistic correction[10] listed in row nine of Table I.

Summing all these contributions, we obtain the total theoretical correction to Gamow-Teller matrix elements, which, considering the high degree of cancellation involved and the inherent model dependence, is in good agreement with experiment.[4]

Finally, as an addendum, we comment further on the cancellations involved. It has been suggested by Rho[21] and others[22] that the contributions from the NCS from (a) p-h states with excitation $\geq 4\hbar\omega$, (b) Δ-h states, and (c) iterated ρ-π diagram should cancel each other. This cancellation, which for the π-tensor force in triton β decay is almost complete,[21] is expected to persist in heavier nuclei and to be unaffected by the introduction of ρ exchange. The validity of this conjecture can be assessed from the entries in Table I, where the sums of rows two, three, and eight are predicted to be small. These sums divided by the sums of their moduli are 0.10, 0.14, −0.11, −0.19, and −0.19 for $A=3$, 15, 17, 39, and 41, respectively, indicating that cancellation is better than 80%.

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Nuclear Size Comparison of Even Titanium Isotopes Using 140-MeV α-Particle Scattering

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Systematic variations in nuclear-matter distributions have been determined by analyzing the measured elastic scattering of 140-MeV alpha particles from ^{46,48,50}Ti. The "unique" optical potentials obtained ($J_R/4A \simeq 300$ MeV fm^3, $J_I/4A \simeq 100$ MeV fm^3) indicate that isotopic differences occur primarily in the strength of the imaginary potential. The rms matter radii increase with A, in contrast to those of the charge distributions. The matter-radius results are in agreement with Hartree-Fock calculations.

We have measured differential cross sections for elastic and inelastic scattering of 140-MeV α particles from the even isotopes of titanium in order to study the isotopic dependence of the scattering interaction (which relates nuclear properties to measured cross sections) and of relative nuclear structure properties. The sensitivity of such measurements to the radial shape of the effective scattering potentials has been demonstrated.[1-3] The detailed sensitivity of our data to the potential over a broad range of radii has been achieved by the use of α particles of sufficiently high energy to obtain "unique" optical-model potentials.[4] This sensitivity is used to determine isotopic shifts of nuclear-matter radii.

The experimental apparatus used in the present experiment has been described elsewhere.[5] The angular step size and resolution were 0.75° and 0.4° in the diffraction region (small angles) and 3° to 6° and 1.2° in the nuclear-rainbow region (large angles), respectively. The uncertainty in the scattering angle was ± 0.02°. The beam energy was determined to be 140.1 ± 0.5 MeV by two independent techniques. The targets were 5-mg/cm^2-thick self-supporting foils, isotopically enriched to 83.8%, 99.1%, and 83.2% for the