Formation and Coalescence of Electron Solitary Holes

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Electron solitary holes were observed in a magnetized collisionless plasma. These holes were identified as Bernstein-Green-Kruskal equilibria, thus being purely kinetic phenomena. The electron hole does not damp even though its velocity is close to the electron thermal velocity. Two holes attract each other like particles of negative mass, and coalesce when their relative velocity is small.

The formation and coalescence of electron solitary holes were observed in a magnetized collisionless plasma surrounded by a wave guide. The experiment was carried out in a setup similar to that described by Saeki[1] and by Ikezi *et al.*[2] for investigating collisionless electron shocks. This experimental setup is particularly well suited for exciting large-amplitude, strongly nonlinear pulses. The observed electron solitary holes are a kind of stable (or at least weakly unstable) Bernstein-Green-Kruskal (BGK) equilibria[3] that appear as vortexes in electron phase space. They were anticipated by Saeki.[1] Similar, almost stationary structures have been observed in one-dimensional computer simulations of the electron two-stream instability,[4] while the stability of vortexlike configurations in phase space has been investigated Kako, Taniuti, and Watanabe.[5] One-dimensional systems were observed to be most favorable for the stability of such vortices. The mutual interaction of the electron holes was also investigated in our experiment, and we found that two holes, which are close enough and have a small relative velocity, will attract each other and coalesce.

The experiment was conducted in a single-ended Q machine, Fig. 1(a). A cesium plasma was produced by surface ionization on a hot (~2000 K) tantalum cathode 3 cm in diameter. A homogeneous magnetic field up to 0.4 T confined the plasma radially. The length of the entire plasma column was 120 cm. Electron temperatures were ~0.2 eV determined by the hot plate. Plasma densities were in the range 10^6–10^7 cm^{-3} and the neutral background pressure was 10^{-6} Torr. Collisions are thus entirely unimportant for pulse propagation. We note that $\omega_p \ll \omega_c$ (ω_p and ω_c are the electron plasma and cyclotron frequencies, respectively). The plasma was surrounded by a grounded cylindrical brass tube, with a 4-cm inner diameter, which acted as a waveguide. Pulses or waves were excited by applying a potential to the terminating brass tube of 30 cm in length (see Ref. 1 for details). Potential variations in the plasma were detected by a Langmuir probe connected directly to a capacitive amplifier (~2 pF). A slot in the waveguide surrounding the plasma allowed 85-cm axial movement of the detecting probe. We measured the dispersion relation for electron waves and they were found to be well described by the Trivelpiece-Gould mode[6] including a thermal correction term, $\omega^2 = \omega_p^2(ka)^2/[1+(ka)^2] + 3v_e^2k^2$, where $2.4a$ is approximated by the plasma radius and $v_e = (T_e/m)^{1/2}$ ($\simeq 2\times 10^5$ m/s). Applying a short negative pulse with time duration $\tau_p \approx 2\pi/\omega_p$ to the exciter, we obtained a temporal plasma response as shown on Fig. 2. Two distinct structures are recognized; a fast negative compressive pulse (i.e., an increase in electron density) and a slower positive pulse indicating a deficit of electrons in the following denoted an "electron hole," or simply a "hole." The former part of the signal is readily identified as the Korteweg–de Vries (KdV) soliton,[2] propagating with

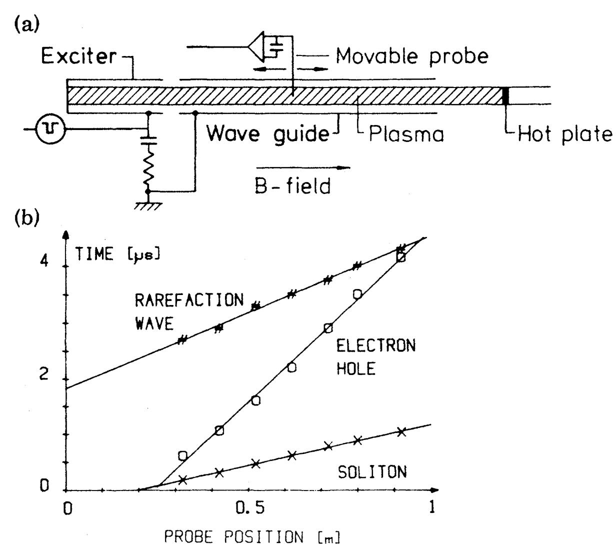

FIG. 1. (a) Experimental setup. (b) Space-time diagram of the observed signals.

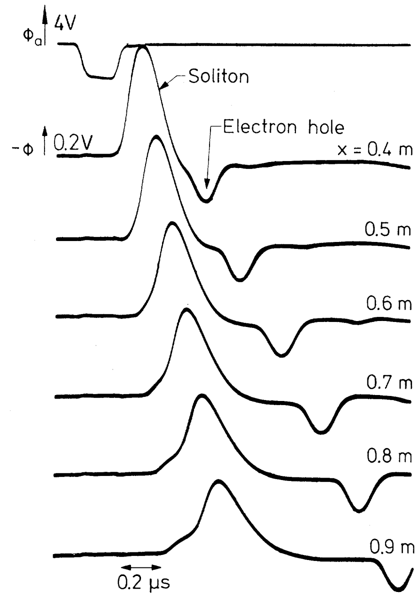

FIG. 2. Time pictures of a soliton and an electron hole at different distances. Applied potential, φ_a, and measured potential φ.

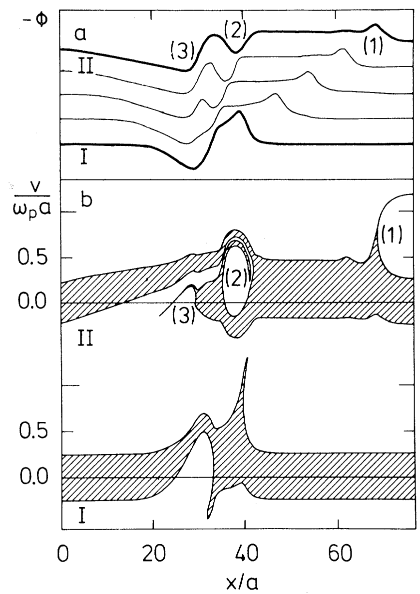

FIG. 3. Waterbag simulation. (a) Potential versus position for increasing time ($t/\tau_p = 0.8$, 1.6, 2.4, 3.2, and 4). (b) Waterbag distribution for two corresponding times.

a velocity slightly above the maximum phase velocity for the Trivelpiece-Gould mode, $\omega_p a$ ($\simeq 1.2\times 10^6$ m/s), and has already been examined in detail in Ref. 2. Here we concentrate on the slow positive pulse, i.e., the hole. This can clearly be distinguished from the rarefactive pulse by tracing it on an (x, t) diagram [Fig. 1(b)]. The rarefactive pulse (outside the time region of Fig. 2) travels downstream into the exciter tube and is reflected from the terminating plate. While the soliton and the rarefactive wave can be traced from the linear regime, the electron hole is only excited when the excitation potential exceeds a critical value φ_c. The hole travels with a velocity of approximately $v_e \ll \omega_p{}^a$, which shows that it cannot be a simple Trivelpiece-Gould mode. Experimentally, we find $\varphi_c \simeq \frac{1}{2}(m/e)(\omega_p a)^2$. We note that the soliton is (Landau) damped as expected at this plasma density, while the electron hole, when fully developed, propagates virtually without change of shape. The soliton, as described by the KdV equation,[2] is a fluid phenomenon except for weak collective damping. If the electron hole could be explained in a similar way, we would expect it to be damped more strongly because of its much lower velocity, in contradiction to observations.

To unambiguously demonstrate that an electron hole can indeed be excited in the present setup, we performed a computer simulation based on a simple waterbag model. The calculation method was similar to that described in Ref. 4, but instead of using periodic boundary conditions, reflecting boundary conditions were assumed, these being the most appropriate for simulating the experimental situation. A leap-frog scheme was applied for the movement of the waterbag boundaries. At each step, the electric potential was calculated from Poisson's equation in the form

$$\frac{\partial^2 \varphi}{\partial x^2} - \frac{\varphi}{a^2} = \frac{e}{\epsilon_0}(n - n_0)$$

appropriate for a magnetized plasma in a waveguide, where only the lowest radial eigenmode is considered. Only the simple case with a single waterbag as initial condition was considered. To represent the excitation, we applied an external charge of the form $\rho(x, t\simeq 0) = -\rho_0/\{1+\exp[(x-L)/a]\}$ during the first plasma period, where L is the length of the exciter tube.

In Fig. 3 the results from such a calculation are shown. We cannot expect an exact agreement between the calculations and the experiment, but the main structures observed experimentally also are found in the computer simulation. From the spatial potential variations, Fig. 3(a), we can identify the soliton [the fastest pulse (1)] as well as the hole (2) and the rarefactive wave (3) (moving to the left). The hole, however, only appeared when the applied potential step was above a certain critical value φ_c, that in both the experiment and the simulation was found to be $\varphi_c \simeq \frac{1}{2}m(\omega_p a)^2/e$. This indicates that the hole develops when an electron beam traveling faster than the characteristic velocity $\omega_p a$ is injected in the main plasma by the excitation.[1] By following the development of the waterbag [Fig. 3(b)], it is observed that the hole is formed because of an "arm," growing in an opposite direction to the soliton, which creates a vortex in phase space. This is quite similar to the creation of a vortex found in computer simulation of the ion-acoustic beam shock,[7] and also to the formation of holes in the two-stream instability.[4] The most important result of our calculation is that it shows our experimentally observed electron hole to be caused by this phase-space vortex effect, i.e., a trapping of electrons.

Both in the experiment and in the computer simulation, we find that the hole moves with a velocity that is close to the electron thermal velocity. The electron hole is thus an entirely kinetic effect characterized by a *strong* coupling between the pulse and the particles. Although the hole is not a stable BGK equilibrium,[8,9] theory predicts that its deformation is a slow process,[5] and we have been unable to detect any significant change of shape within the time it takes it to pass the full length of our waveguide. The hole represents an equilibrium that is very sensitive to perturbations of the electron orbits in phase space, e.g., electron-neutral collisions. These will destroy the refined structure of the electron velocity distribution and tend to restore the original unperturbed Maxwellian distribution, whereafter the pulse will be rapidly Landau damped since its velocity is approximately v_e. By slowly increasing the neutral pressure with helium, we can entirely destroy the hole structure at even moderate neutral pressures; see Fig. 4. The damping length for ordinary collisional damping[10] of a Trivelpiece-Gould mode at the hole velocity is ~1.5 m at 5×10^{-4} mm Hg, thus ruling out that the damping is explained by simple collisional damping. Note also that the front of the soliton is only weakly affected by collisions, as expected.

Although theoretical investigations of hole equilibria are sparse, their existence is well known from computer studies of one-dimensional, two-stream, or bump-on-tail instabilities (in our experiment the one-dimensionality is brought about by the strong magnetic field). One important observation from our study is that almost any form of the excitation will develop into one or more electron holes provided that the amplitude exceeds φ_c. From the results of this and previous studies, it is obvious that the solitary hole should be considered as a quasiparticle just like the soliton. Solitons are known to preserve their identity during collisions and they may be accelerated by, e.g., density gradients. Hence, a soliton may be considered as a particle with negative charge and

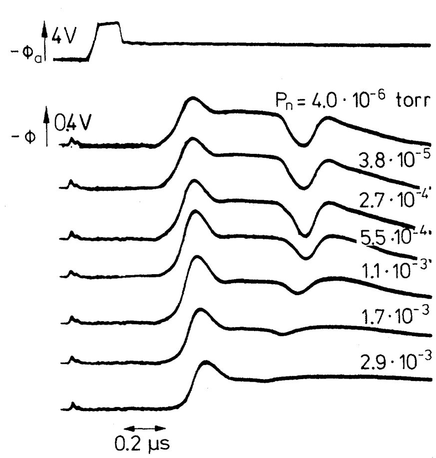

FIG. 4. Damping of electron hold due to electron-neutral collisions, $x = 0.7$ m. P_n is the pressure of neutral helium.

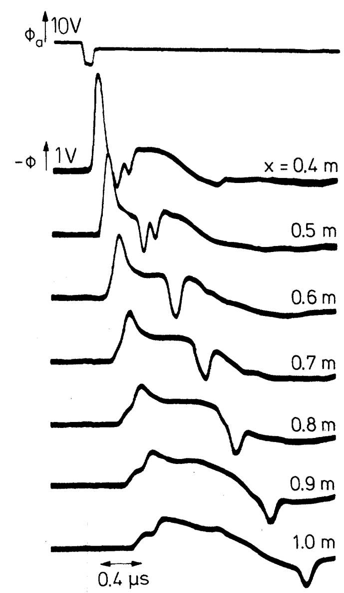

FIG. 5. Hole-hole coalescence. Time pictures at various distances.

positive mass. By analogy we assign a positive charge to the solitary hole, and consistency then requires its effective mass to be negative.[4] This in turn implies that the Coulomb repulsion between two holes gives rise to an attraction. In order to demonstrate the quasiparticle behavior of the solitary electron hole, we modified the exciting pulse so that two holes close to each other and having almost equal velocity were excited; see Fig. 5. Following their trajectory we observed an attraction and a subsequent coalescence that prevailed throughout the entire plasma column. The collision thus appears to be inelastic. Such a coalescence has already been observed in numerical investigations.[4] If, however, the initial velocity difference is large, two holes are observed to pass through each other. We may thus conclude that the observed properties of the solitary electron holes are indeed compatible with those expected for quasiparticles, at least within the limits of space and time in our experimental setup.

We are aware that the solitary holes described in this Letter may have some relevance for describing one-dimensional strong Langmuir turbulence.[11]

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Phase Transitions in Two-Dimensional Superfluid ^{3}He

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A discussion of phase transitions in two-dimensional ^{3}He is presented. A new type of phase transition is proposed, in which "islands of reversed $\vec{l}$" spontaneously nucleate, leading to an Ising-like transition. It is shown that such transitions may need to be taken into account, possibly competing with the well-know Kosterlitz-Thouless transition. Consequences of both types of transition are discussed.

Phase transitions in two-dimensional systems have attracted considerable attention over the past few years since the discovery of Kosterlitz and Thouless[1] that a new kind of ordering ("topological ordering") may exist in these systems when conventional order is absent. This new type of ordering is destroyed upon the unbinding of vortex-antivortex pairs (or their analogs) in the system under consideration. The phase diagram then exhibits a line of critical points below T_c with continuously variable critical exponents.[2] Renormalization-group analyses based on these ideas have recently been applied to two-dimensional superfluid ^{4}He and X-Y ferromagnets,[2] two-dimensional solids,[3] and two-dimensional liquid crystals.[4,5]

Experimental attention is just now beginning to be focused on two-dimensional superfluid ^{3}He

