structures at $E_x(^{40}\text{Ca}) = 44.2$ MeV overlapping in backward (present work) and forward[8] measurements [Figs. 3(c1) and 3(c2)]. The same $L = 21$ value was found in both cases but with different absolute cross sections. The differential cross section is written as $\sigma(\theta) = |f_{\text{dir}} + f_{\text{res}}|^2$. From the background in Fig. 3(c2) we estimate $|f_{\text{dir}}(0°)|^2 \sim 2$ mb/sr. The value $|f_{\text{res}}(176°)|^2 \sim 0.1$ mb/sr taken from Fig. 3(c1) and the $L = 21$ dominant L value allows as to extrapolate 0.4 mb/sr $\sim |f_{\text{res}}(180°)|^2 = |f_{\text{res}}(0°)|^2$. Then $\sigma(0°) \sim 4$ mb/sr which is consistent with the measured value of Ref. 8 and indicates a common origin for these gross structures.

The width of the structures appearing in the ^{12}C channels are nearly equal to the energy loss of the ^{24}Mg beam in the Ta_2O_5 target. At present, it is hard to make a conclusion as to whether they are intermediate structures or compound states in ^{40}Ca.

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Narrow $I^\pi = 10^+$ Resonance for ^{12}C + ^{16}O in the Region of Strong Absorption

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A 10^+ resonance with a width of about 700 keV (c.m.) for ^{12}C + ^{16}O has been observed in the reaction $^{12}\text{C}(^{16}\text{O}, {}^8\text{Be}_{\text{g.s.}})^{20}\text{Ne}$ at $E_{\text{c.m.}} = 18.8$ MeV leading to the ground state and several excited states in ^{20}Ne. The spin value of $I = 10$ is four units of angular momentum below the grazing value of $l = 14$ obtained from elastic scattering, and is still two units below the strong-absorption l value obtained from ^{12}C + ^{16}O total fusion cross sections.

The ^{12}C + ^{16}O system has been under intense study throughout recent years and various resonances have been reported.[1] For some of these structures spin values were assigned and found to be close to the grazing angular momentum in the entrance channels. This has lead to the assumption of surface transparent complex potentials.

In this Letter for the first time evidence is reported that the concept of surface transparency probably needs to be extended even farther into the nuclear interior: The novel phenomenon observed is a *narrow* resonance ($\Gamma_{\text{c.m.}} \approx 700$ keV) for ^{12}C + ^{16}O with a spin value four units below the grazing l value of the entrance channel and well inside the region where strong absorption is expected. This resonance has been observed through the reactions $^{12}C(^{16}O, {}^{8}Be_{g.s.})^{20}Ne$ and $^{12}C(^{16}O, \alpha)^{24}Mg$ at 18.8 MeV (c.m.).

The data were taken with the FN tandem Van de Graaff accelerator of the University of Washington. The ^{8}Be events were measured through the coincident detection of the two α particles from the ground-state decay of ^{8}Be; α particles from the reaction $^{12}C(^{16}O, \alpha)^{24}Mg$ were detected simultaneously. The target thickness was 47.2 μg/cm^2 corresponding to an energy loss of about 120 keV (c.m.) of the incident ^{16}O beam. A complete set of data will be published in a forthcoming paper.

For an overall-view of the energy-dependence of the reaction $^{12}C(^{16}O, {}^{8}Be_{g.s.})^{20}Ne$ in the vicinity of the resonance energy of 18.8 MeV we show in Fig. 1 angle-integrated cross sections (integrated between angles at which excitation functions were measured: $\theta_{1ab} = 7.5°$, 12.5°, 17.5°, 22.5°, 27.5°, and 32.5°) for the ground-state reaction and for the transitions to the 2^+ and 4^+ rotational states at 1.63 and 4.25 MeV in ^{20}Ne and to groups of states (not resolved here) centered at 5.7 and 7.1 MeV. The deviation function

$$D(E) = \frac{1}{5}\sum_{i=1}^{5} \frac{\sigma_{\rm int}{}^i(E) - \langle \sigma_{\rm int}{}^i(E)\rangle}{\langle \sigma_{\rm int}{}^i(E)\rangle}$$

of these excitation functions (denoted by i) is shown on the bottom of the figure. The energy averaging interval $\langle\ \rangle$ was taken over 2.7 MeV (c.m.). The deviation function reveals a resonance at an energy of 18.8 MeV, indicated by the dashed line. The width is about 700 keV; from inspection of the excitation functions in the upper part of the figure, it seems to be indicative that this resonance is either split or is actually a doublet or triplet with somewhat smaller individual widths. The occurrence of such a behavior has been observed before at other energy regions and is also expected theoretically.[2] It is gratifying to note that a previously reported resonance[3] (with $I^\pi = 12^+$) at 19.9 MeV is confirmed by our deviation function in Fig. 1; it is, however, less pronounced than the one reported here.

A spin of $I^\pi = 10^+$ for the resonance at 18.8 MeV can be assigned from the ground-state angular distribution shown in Fig. 2. The angular distribution closely follows the square of the $l = 10$ Legendre polynomial: As indicated by the vertical arrows five maxima are observed between 0° and 90° in accordance with a $l = 10$ partial wave. For $l = 8$ or $l = 12$ the number of maxima would be expected to be 4 or 6, respectively; odd partial waves are ruled out by the pronounced maximum

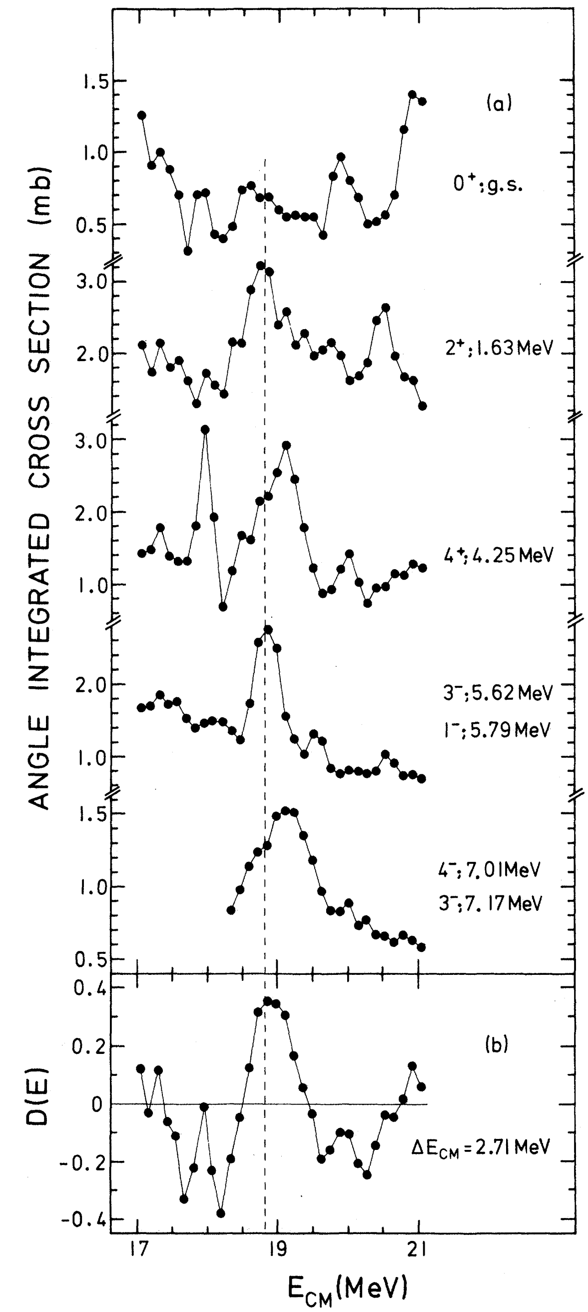

FIG. 1. (a) Angle-integrated cross section for $^{12}C(^{16}O, {}^{8}Be_{g.s.})^{20}Ne$ to various states in ^{20}Ne as indicated. (b) Deviation function as described in the text.

at 90°. The $P_{l=10}{}^2$ structure is also displayed by the $^{12}C(^{16}O, \alpha)^{24}Mg_{g.s.}$ angular distribution (not shown here) within the angular range of 10° to 60° (c.m.) through the observation of the first three minima (maxima) at about 13° (21°), 31° (40°), and 47° (56°) (c.m.). Predictions of the expected compound background from a Hauser-

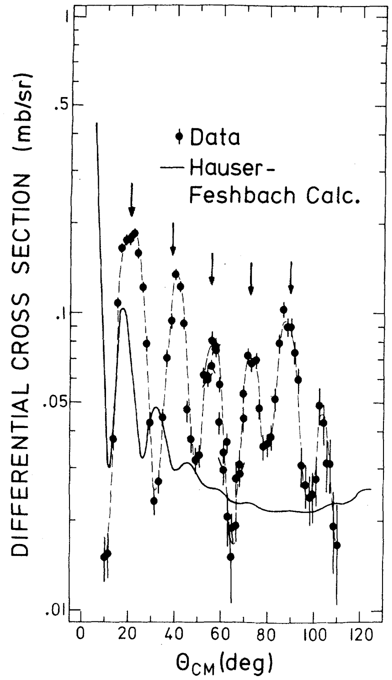

FIG. 2. Angular distribution for $^{12}C(^{16}O, ^{8}Be_{g.s.})^{20}Ne_{g.s.}$ at 18.8 MeV (c.m.). The vertical arrows indicate the positions of the maxima for a $P_{l=10}{}^2$ structure. The solid line shows the prediction of a Hauser-Feshbach calculation (see text). The dashed line serves to guide the eye.

Feshbach calculation, shown by the solid line in Fig. 2, clearly fail to reproduce the experimentally observed $P_{l=10}{}^2$ structure. Angular distributions measured at 18.2 and 19.4 MeV (i.e., 600 keV below and above the 18.8 MeV resonance) do not show a pronounced structure and are close to the Hauser-Feshbach predictions both in shape and magnitude.

We have estimated the grazing partial waves at 18.8 MeV (c.m.) for the entrance channel $^{12}C + ^{16}O$ and for the exit channels $^{8}Be_{g.s.} + ^{20}Ne_{g.s.}$ and $\alpha + ^{24}Mg_{g.s.}$ from optical-model calculations using the $^{12}C + ^{16}O$ potential of Malmin[4] for the entrance channel and also (radius-scaled) for $^{8}Be + ^{20}Ne$ and the potential of Singh *et al.*[5] for $\alpha + ^{24}Mg$. From the transmission coefficients, listed in Table I, grazing values of $l \approx 14$ for $^{12}C + ^{16}O$, $l \approx 11.5$ for $^{8}Be + ^{20}Ne$, and $l \approx 12$ for $\alpha + ^{24}Mg$ are found. It is therefore expected that these l values, i.e., $l \approx 11$–12, dominate the $^{12}C(^{16}O, \alpha)^{24}Mg_{g.s.}$ and $^{12}C(^{16}O, ^{8}Be_{g.s.})^{20}Ne_{g.s.}$ angular distributions and not $l = 10$ as is observed experimentally. In this context it is interesting to note that for the $^{12}C + ^{12}C$ system the angular distributions for the

TABLE I. Optical-model transmission coefficients.

	$l=10$	11	12	13	14
$^{12}C + ^{16}O$	0.99	0.91	0.84	0.90	0.38
$^{8}Be + ^{20}Ne$	0.81	0.89	0.25	0.06	0.02
$\alpha + ^{24}Mg$	0.97	0.88	0.52	0.13	0.02

$\alpha + ^{20}Ne_{g.s.}$ and $^{8}Be_{g.s.} + ^{16}O_{g.s.}$ exit channels do follow the l_{gr} systematics by showing (at 18.5 MeV c.m.) a $P_{l=10}{}^2$ structure for $\alpha + ^{20}Ne$ and a $P_{l=12}{}^2$ structure for $^{8}Be + ^{16}O$.[6] We thus conclude that the observed $P_{l=10}{}^2$ structure in our case for both the α and ^{8}Be exit channels, which is two units of angular momentum lower than expected from the l_{gr} values, further supports the fact that the observed structure is a resonance indeed.

Inspection of elastic and inelastic scattering data around 18.8 MeV yields the following picture: For elastic scattering the observation of this relatively narrow resonance is complicated by strong Ericson-type fluctuations.[4] Therefore spin assignments based solely on the analysis of elastic scattering data have to be taken with some caution. For inelastic scattering an indication of resonantlike structure around 19 MeV is seen in the back-angle particle data of Malmin, Harris, and Paul,[7] most evident in the scattering to the unresolved 3^-, 0^+ states at 6.1-MeV excitation energy in ^{16}O. A peak at ~18.8 MeV (c.m.) is also clearly visible in the γ-ray excitation function for the 6.13-MeV ($3^- \to 0^+$) transition in ^{16}O.[8] However, no such structure is evident from the angle-integrated excitation function for the inelastic scattering to the first excited 2^+ state at 4.43 MeV in ^{12}C.[9]

Figure 3 summarizes in the form of an energy versus spin plot resonances for $^{12}C + ^{16}O$ which have been reported in the literature and for which spin assignments were made.[10] These spin assignments result from analyses of angular distributions for spin-0 particles in the various exit channels and are indicated by the crosses in Fig. 3. The open circles are l_{max} values extracted from total fusion cross sections[11] for which the highest absolute accuracies have been claimed. Within the sharp cutoff limit one obtains

$$(l_{max} + 1)^2 = \sigma_{fus}/\pi \lambda\!\!\!^{-2}.$$

These l_{max} values are connected by the solid line (to guide the eye) inside of which the region of strong absorption is expected (shaded area in Fig. 3). At bombarding energies above about

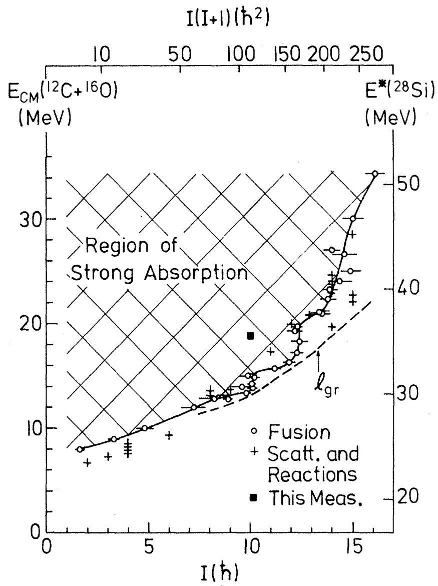

FIG. 3. Energy vs spin plot of known $^{12}C+^{16}O$ resonances along with "strong-absorption" l values from total fusion and the grazing angular momenta (l_{gr}) in the $^{12}C+^{16}O$ entrance channel. For details see text.

$E_{c.m.}=20$ MeV it is suggested by Tabor *et al.* (see Ref. 11) that the total fusion cross section is even larger due to increasing 3α emission out of the compound nucleus. This would shift the l_{max} values to larger angular momenta, thus increasing the area of strong absorption shown in Fig. 3. The dashed line shows the orbital angular momentum for grazing collisions, calculated from the energy-dependent optical-model potential of Ref. 4 which was obtained from fitting the elastic scattering data. The corresponding l_{gr} values were taken where the transmission coefficients equal 0.5. Except for the low energies (where the determination of the grazing l values is somewhat uncertain because of large Coulomb scattering) most of the resonances observed are located between the grazing curve and the strong absorption curve with a tendency of being closer to the latter. Only the $I^\pi=11^-$ resonance at 17.29 MeV (c.m.) (Charles *et al.*, see Ref. 10) is located inside of the strong-absorption curve. The spin assignment for this resonance, however, is based on one elastic-scattering angular distribution only, and is therefore somewhat uncertain due to large statistical fluctuations. As can be seen from Fig. 3 the 10^+ resonance at 18.8 MeV (c.m.) reported in this Letter lies well inside the region of strong absorption. Such a behavior has not been observed before for the $^{12}C+^{16}O$—or any other system.

Along with the previously reported resonances at 19.7 MeV ($I^\pi=14^+$) from elastic scattering[4] and at 19.9 MeV ($I^\pi=12^+$) in the reaction[3] $^{12}C(^{16}O,^{8}Be)^{20}Ne$ (confirmed by the present study) a situation emerges for the $^{12}C+^{16}O$ system, that within an energy interval of only about 1 MeV (c.m.) three resonances are observed with spin values of $I^\pi=10^+$ at 18.8 MeV (present case), $I^\pi=14^+$ at 19.7 MeV, and $I^\pi=12^+$ at 19.9 MeV (c.m.). Most striking among these resonances is the occurrence of the narrow 10^+ resonance, since it lies well inside the region of strong absorption. That such a resonance can be observed seems to require the assumption that either the potential is more transparent for low partial waves than is currently thought (*surface* transparent only) or that the configuration of this resonance is such that it does not couple to the other continuum states (at an excitation energy in the compound system of 35.6 MeV). Both assumptions are not expected from our present understanding of heavy-ion collisions.

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Microscopic Description of the Nuclear Triaxial Rotor at High Angular Momentum near the Yrast Line

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We describe a derivation of a microscopic theory of the wobbling motion of a triaxial nuclear rotor at high angular momentum as a possible phase for the yrast line. The solution of a rotationally invariant approximation to the Heisenberg equations of motion (the Kerman-Klein equations) is outlined as a systematic expansion in reciprocal powers of the total angular momentum and of the number of particles.

A standard elementary result in the theory of the asymmetric rigid rotor in classical mechanics[1] is that the system can rotate with fixed angular velocity about any of its three principal axes. The rotations about the axes with minimum and maximum moments of inertia are stable against small perturbations and the latter case represents the state of minimum energy for a prescribed angular momentum. Motion near the minimum can be represented as a small oscillation and is referred to as the wobbling motion of the triaxial rotor.

It has been conjectured[2] that the rotational motion of even nuclei at sufficiently high angular momentum may exhibit such a phase, and that this will probably occur above the value of angular momentum at which pairing correlations, responsible for nuclear superfluidity near the ground state, disappear. Though the physical characteristics[3] of the nuclear wobbling motion have yet to be identified experimentally, it would appear to be of considerable interest to establish the theoretical tools for describing the changing nuclear density and rotational parameters under such circumstances.

It may come as no surprise that a suitable version of the cranking formalism emerges as a sensible first approximation for this domain. However, *theoretical* justification[4-7] for this formalism, starting from a description which respects the rotational invariance of the nuclear Hamiltonian, exists only for *nearly axially symmetric systems at low angular momentum.*[8,9] In this Letter we give a new derivation with the following features: (i) It is tailored specifically to apply to the wobbling motion along or near the line of states of minimum energy for a prescribed angular momentum (yrast line). (ii) The collective Hamiltonian described can be more general than that of the usual triaxial rotor (quadratic in the angular momentum operators.) (iii) The cranking limit is derived from a rotationally invariant theory as the first terms of a *systematic expansion.* (iv) The basic nuclear interaction may be quite general.

Phenomenological model.—We associate a set of nuclear states $|IM\alpha\rangle$ (where I, M identify total angular momentum and one of its components and α distinguishes states with the same value of I, M) with states of a triaxial rotor. By triaxial rotor we mean a purely quantum mechanical system specified as follows: We are given three operators I_i, $i = 1, 2, 3$, satisfying the commutation relations of the *body-fixed* components of the angular momentum

$$[I_i, I_j] = -i\epsilon_{ijk} I_k, \tag{1}$$

and a Hamiltonian operator which is a symmetrized polynomial

$$\mathcal{H} = \sum_i \tfrac{1}{2} a_i I_i^2 + \sum_{i,j} a_{ij}\{I_i^2, I_j^2\} + \cdots \tag{2}$$

in the I_i. The eigenfunctions and eigenvalues of $\mathcal{H}$,

$$\mathcal{H} F_{M\alpha}{}^I(\Omega) = E(I\alpha) F_{M\alpha}{}^I(\Omega), \tag{3}$$