315 (1969) [JETP Lett. 9, 184 (1969)]; G. Steigman, D. N. Schramm, and J. E. Gunn, Phys. Lett. 66B, 202 (1977).

[18]It is straightforward to verify that any additional U(1) gauge symmetry leads to anomalies.

[19]There also exists the awkward possibility of allowing two $SU_V(2)$ multiplets to transform as an $SU_H(2)$ doublet and the third $SU_V(2)$ multiplet to transform as an $SU_H(2)$ singlet.

[20]This includes the possibility of having $SU_H(2)$ for leptons and $SU_H(3)$ for quarks at the price of a more complicated Higgs structure.

[21]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1975).

# Isospin Decomposition of Nuclear Multipole Matrix Elements from $\gamma$ Decay Rates of Mirror Transitions: Test of Values Obtained with Hadronic Probes

Aron M. Bernstein
*Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

and

V. R. Brown and V. A. Madsen[(a)]
*Lawrence Livermore Laboratory, Livermore, California 94550*

(Received 25 September 1978)

The electromagnetic decay rates of mirror states are used to decompose the multipole electromagnetic matrix elements into isoscalar and isovector or equivalently proton and neutron matrix elements. This method is sensitive to the neutron matrix element because it is measured directly, and it is accurate since it depends only on electromagnetic interactions; it therefore can be used to test methods using inelastic hadron scattering. There is agreement with the results for $(\alpha,\alpha')$ on $^{26}$Mg and $^{42}$Ca but disagreement with the recent $^{18}O(\pi,\pi')$ experiments.

An incisive method to study the dynamical properties of nuclear states is to observe their electromagnetic (EM) transition rates, which is the square of the proton transition matrix element $M_p$. In the long-wavelength limit one can write the electric multipole operator of order $\lambda$ as a sum over protons. One can also define a similar sum for neutrons. These operators can be written in one equation as[1]

$$O_{n\,(p)}{}^{\lambda}=\sum_{k=1}^{A}\left[\frac{1\pm\tau_3(k)}{2}\right]r_k{}^{\lambda}Y_{\lambda\mu}(\hat{r}_k). \qquad (1)$$

The matrix elements of $O_p{}^\lambda$ and $O_n{}^\lambda$ between nuclear states are defined to be $M_p$ and $M_n$, respectively. The apparent lack of a physical process to make *precision* measurements of $M_n$ has left a gap in our arsenal of nuclear probes. In this paper we utilize, for the first time, a purely EM emthod to obtain $M_n$ from the EM decay rates of mirror transitions [see Eq. (5) below]. This method has both sensitivity and precision and, to the accuracy of the data, solves this outstanding problem for the case of light nuclei. At present there are approximately 25 measured $E2$ mirror transitions for which this method is applicable.[2] In this paper a subset of the known $E2$ data is presented and compared to the predictions of the schematic model of core polarization. In addition, these results can be used to test other methods of obtaining $M_n$.

An alternative method to measure $M_n$ has been to use the inelastic scattering of hadrons (which interact with both neutrons and protons); this can then be combined with EM data (or another hadronic-probe experiment) to obtain both $M_n$ and $M_p$. This procedure usually depends on several features. First, $O_p{}^\lambda$ and $O_n{}^\lambda$ have $r^{\lambda+2}$ in the radial matrix elements which weights the surface region, an effect similar to the surface weighting that occurs for strongly absorbed hadronic projectiles.[3] In addition, the effective interaction between the probe and the bound nucleons must be known, and be of the form of a sum of single-body operators. The validity of this latter condition is far from obvious.

There have been several papers which utilize the results of inelastic hadron scattering experiments such as $\alpha$ particles,[3-6] protons,[7] and, more recently, pions,[8,9] to obtain $M_n$ and $M_p$. Unfortunately, there have been very few tests of the basic foundations of the proposed methods or of the results which have been obtained by them. An early test using low-energy $(\alpha, \alpha')$ reactions was reported.[3] For the transitions $T=0$ to $T=0$ only the isoscalar part of $O_p{}^\lambda$ contributes, and the transition rates obtained from $(\alpha, \alpha')$ and EM experiments should be approximately equal; the data revealed equality to about 30% for $E2$ and $E3$ transitions.[3] More recent tests using ~100-MeV $\alpha$ particles have indicated about the same level of agreement provided that the folding model and not Woods-Saxon potentials are used to analyze the data.[4,5] To date no extensive tests for other hadronic probes have been performed.

In this paper we shall show that the same information is available from comparison of corresponding EM transitions of mirror nuclei assuming only charge independence. This type of experimental data has been used previously[2] to study the isovector strengths for low-lying $E2$ transitions in nuclei with $A=21$ to 34. Here we use the data to obtain multipole matrix elements as a test of hadron-scattering measurements. To demonstrate this we define the reduced angular-momentum neutron and proton $\lambda$-pole matrix elements $M_p$ and $M_n$ as

$$M_{n\,(p)}(T_z) = \langle J_f T T_z \| O_{n\,(p)}{}^\lambda \| J_i T T_z \rangle = \tfrac{1}{2}[M_0(T_z) \pm M_1(T_z)], \qquad (2)$$

where $M_0(T_z)$ and $M_1(T_z)$ are the matrix elements in an isospin representation. The $B(E\lambda)$ value is given by

$$B(E\lambda; J_i \to J_f) = |M_p(T_z)|^2/(2J_i+1). \qquad (3)$$

With the assumption of isospin conservation, the EM matrix elements in different isobars are simply related:

$$M_0(T_z') = M_0(T_z), \qquad (4a)$$

$$M_1(T_z') = M_1(T_z)T_z'/T_z, \qquad (4b)$$

so that in the nucleus with $T_z' = -T_z$, it follows from Eqs. (2) and (4) that

$$M_p(-T_z) = M_n(T_z). \qquad (5)$$

This important new result tells us that $M_n$ in a particular nucleus (e.g., $^{42}$Ca) may be obtained by measuring the electromagnetic transition for the corresponding transition in the mirror nucleus $^{42}$Ti. Furthermore, for a $T_z=0$ nucleus ($^{42}$Sc) Eq. (4) states that $M_p(T_z=0) = \frac{1}{2}M_0(T=1)$ so that the $M_0$ in $^{42}$Ca can be obtained directly from the corresponding transition in $^{42}$Sc. Equation (5) requires only charge symmetry but (4) requires charge independence, both of which are expected to be reasonably well fulfilled in nature as evidenced by the high degree of correspondence of levels in mirror nuclei. In addition, for EM decays a detailed test has been made of the prediction that the mixing ratios of mirror nuclei are equal and opposite.[10]

The data for $E2$ transitions in selected mirror nuclei[2,11] have been used to obtain $M_0(T_z)$ and $M_1(T_z)$ or $M_p(T_z)$ and $M_n(T_z)$. The results are presented in Table I. In the case of a $T=1$ isotope there are three corresponding transitions. If the partial lifetimes were all accurately measured this would give an experimental check of isospin purity. In practice this is difficult because for the $T_z=0$ odd-odd nucleus there are often several ways that the $2^+$ state can decay which complicates the experimental situation. Nevertheless, there are three cases in which these measurements have been performed. For $A=30$ the two values of $M_0$ do not agree while for $A=34$ and 42 they are in agreement within the experimental errors. Before any conclusions about lack of isospin purity in $A=30$ are drawn, it would probably be wise to check the experimental data. More accurate data would be extremely useful.

Also included in Table I for the single-closed-shell (SCS) nuclei are calculations based on the no-parameter (NPSM) and one-parameter schematic model (OPSM).[12] In the absence of systematic $E2$ giant-resonance energies in light nuclei, empirical corrections related to the $A$ dependence of the position of the $E1$ giant resonance was made. This arbitrary procedure gives better agreement with effective charges obtained using measured matrix elements[13] than using proportionality to $A^{-1/3}$ for both giant-resonance and shell-model energies.

As can be seen in Table I, $M_1$ is generally smaller than $M_0$ and fluctuates in sign (note that only the relative sign is significant). A positive $M_1/M_0$ means $M_n/M_p > 1$ while a negative $M_1/M_0$ means $M_n/M_p < 1$. The only negative $M_1/M_0$ value in Table I is for $A=26$. From previous work on SCS nuclei one expects $M_n/M_p > 1$ for neutron valence SCS nuclei and $M_n/M_p < 1$ for proton valence SCS nuclei.[6,12] The case of $^{18}$O and $^{42}$Ca fits into this general observation. The case of $^{26}$Mg is more puzzling. For a uniform rigid rotor one expects $M_n/M_p = N/Z$, which is 1.17 for $^{26}$Mg. The experimental result is $0.72 \pm 0.15$. The fact that $M_n/M_p < 1$ seems consistent with the following simple picture: $^{26}$Mg consists of a $^{24}$Mg prolate

TABLE I. $E2$ transition rates and matrix elements.[a,b]

| Nucleus | $J_i \to J_f$ | G | $M_p$ | $\|M_0\|$ | $M_1/M_0$ | $M_n/M_p$ EM | $M_n/M_p$ Hadron | $M_n/M_p$ Theory |
|---|---|---|---|---|---|---|---|---|
| $^{17}$O | $\frac{1}{2}^+ \to \frac{5}{2}^+$ [c] | 2.41(0.03) | 1.55(0.01) | 6.60(0.06) | 0.53(0.01) | 2.63(0.04)[e] | | 1.81[f] |
| $^{17}$F | | 25.5 (0.6 ) | 5.05(0.06) | | | | | 2.44[g] |
| $^{17}$O | $\frac{5}{2}^+$ [d] | | −2.65(0.05)[d] | 12.7 (2.0 ) | 0.58(0.18) | 3.31(0.76)[e] | | |
| $^{17}$F | | | −10(2)[d] | | | | | |
| $^{18}$O | $2^+ \to 0^+$ [h] | 3.00(0.25) | 1.73(0.07) | 6.04(0.22) | 0.43(0.04) | 2.26(0.16)[e] | 1.75(0.10)[i] | 1.85[f] |
| $^{18}$Ne | | 18.6 (1.8 ) | 4.31(0.21) | | | | 1.27(0.13)[j] | 2.40[g] |
| | | | | | | | 1.5 (0.5 )[k] | |
| $^{18}$O | $4^+ \to 2^+$ | 1.22(0.06) | 1.10(0.03) | 4.10(0.20) | 0.46(0.05) | 2.42(0.20)[e] | | |
| $^{18}$Ne | | 9.0 (1.2 ) | 3.00(0.20) | | | | | |
| $^{26}$Mg | | 13.1 (0.06) | 3.62(0.08) | 6.23(0.56) | −0.16(0.09) | 0.72(0.15) | 0.80(0.17)[k] | |
| $^{26}$Al | $2^+ \to 0^+$ | 19.7 (6.8 ) | 4.44(0.77) | 8.88(1.54) | | | | |
| $^{26}$Si | | 6.8 (2.9 ) | 2.61(0.56) | | | | | |
| $^{30}$Si | | 7.23(0.4 ) | 2.70(0.07) | 6.59(0.38) | 0.18(0.06) | 1.44(0.14) | | |
| $^{30}$P | $2^+ \to 0^+$ | 7.6 (1.5 ) | 2.76(0.28) | 5.52(0.54) | | | | |
| $^{30}$S | | 15.1 (2.9 ) | 3.89(0.37) | | | | | |
| $^{34}$S | | 5.95(0.25) | 2.44(0.05) | 6.11(0.56) | 0.20(0.09) | 1.50(0.23) | | |
| $^{34}$Cl | $2^+ \to 0^+$ | 7.4 (1.6 ) | 2.72(0.29) | 5.44(0.58) | | | | |
| $^{34}$Ar | | 13.5 (4.1 ) | 3.67(0.56) | | | | | |
| $^{42}$Ca | | 9.58(0.32) | 3.10(0.05) | 7.32(0.40) | 0.15(0.06) | 1.36(0.13) | 1.27(0.18)[k] | 1.69[f] |
| $^{42}$Sc | $2^+ \to 0^+$ | 8.4 (3.1 ) | 2.90(0.53) | 5.80(1.06) | | | 1.28(0.13)[l] | 1.27[g] |
| $^{42}$Ti | | 17.8 (3.4 ) | 4.22(0.40) | | | | | |

[a] The number $a \pm b$ is listed as $a(b)$. The transition rates $G$ and matrix elements $M$ are in single-particle units $B_{sp}(E2)\downarrow = 0.0594 A^{4/3}$ fm$^4$.

[b] Except as noted for $A = 17$ and 18 on the data are from Ref. 11 and do not differ greatly from Ref. 2.

[c] Data from F. Ajzenberg-Selove, Nucl. Phys. A166, 1 (1971).

[d] Quadropole moment of ground state given in squared fermis. Data are from Ref. 15.

[e] Using the Coulomb correction factors of Ref. 15 which are 1.24, 1.14, 1.10, and 1.13, respectively.

[f] No-parameter schematic model (NPSM); see Ref. 12.

[g] One-paremeter schematic model (OPSM); see Ref. 12.

[h] Data for $^{18}$O from P. B. Vold *et al.* [Phys. Rev. Lett. 39, 325 (1977)] and C. Flaum *et al.* [Phys. Rev. Lett. 39, 446 (1977)] and the results tabulated there. For $^{18}$Ne the weighted average of previous measurements is given by A. B. McDonald *et al.*, Nucl. Phys. A258, 152 (1976).

[i] $(\pi, \pi')$ from LAMPF, Ref. 8. Average of 164- and 230-MeV data.

[j] $(\pi, \pi')$ from SIN, Ref. 9. 180-MeV data.

[k] $(\alpha, \alpha')$ from Ref. 3. See text for discussion of $^{18}$O.

[l] $(p, p')$ at 1 GeV; see Ref. 7.

core plus two neutrons in an equatorial $j, \Omega = \frac{5}{2}, \frac{5}{2}$ orbit. The two neutrons form an oblate distribution which reduces $M_n$ while leaving $M_p$ unchanged, giving a negative value for the ratio $M_1/M_0$. A more quantitative discussion of this case will be reported in a later publication.

The $M_n/M_p$ ratios in $^{17}$O and $^{18}$O are systematically very large (> 2). Table I includes the $2^+ \to 0^+$ and $4^+ \to 2^+$ transitions in $^{18}$O and the $\frac{1}{2}^+ \to \frac{5}{2}^+$ transition as well as the static quadrupole moment[14] in $^{17}$O. Because the weakly bound neutron or proton is sensitive to the single-particle potential, the radial matrix elements must be corrected for the effect of the Coulomb potential on the binding-energy differences in the two-mirror nuclei. These isospin correction factors, calculated by Brown, Arima, and McGrory[15] have been used for $A = 17$ and 18. These are in agreement with empirical estimates[13] in $A = 17$. These corrections are not very large, even for the weakly bound nuclei, and are even smaller for the other nuclei considered here.

An experiment[8,9] of particular current interest using hadronic probes is the ratio of $M_n/M_p$ in $^{18}$O comparing $(\pi^+, \pi^{+\prime})$ and $(\pi^-, \pi^{-\prime})$. The comparison of oppositely charged pions could be a sensitive method of obtaining $M_n/M_p$ ratios because of the strong isospin dependence of the pion-nucleon force. The ratios $1.75 \pm 0.1$ and $1.27 \pm 0.03$ obtained from these experiments (see Table I) are considerably lower than the value $2.26 \pm 0.16$ from mirror-transition rates; the isospin correction is in the direction of reducing the discrepency but is much too small to account for the differences. We consider this an extreme correction, since consideration of core-polarization form factors and Coulomb corrections to core polarization would be expected to give contributions in the opposite direction. Therefore, assuming the experimental accuracy of the EM lifetimes, the pion results are incorrect. It would be useful to have more tests of the pion method (e.g., in $^{26}$Mg, $^{30}$Si, $^{34}$S, and $^{42}$Ca).

The reaction $^{18}$O($\alpha, \alpha'$) has been performed at 40 MeV; this is the only nucleus for which the distorted-wave Born-approximation calculations are out of phase with experiment, and therefore, the error in the isoscalar transition rate is large.[3] A higher-energy ($\alpha, \alpha'$) experiment is required to obtain significant results in this case. In other cases in which ($\alpha, \alpha'$) has been measured, $^{26}$Mg and $^{42}$Ca, there is agreement with the results using the mirror-transition method. This is encouraging since the ($\alpha, \alpha'$) method has now been tested for $T=0$ and $T=1$ nuclei.[16] The results for the $^{42}$Ca($p,p'$) experiment[7] at 1 GeV are also presented in Table I and are in agreement with the mirror-transition method.

In conclusion, we note that an isospin decomposition of multipole matrix elements is possible from the EM decay of states in mirror nuclei. The information is as accurate as the experimental data except for Coulomb corrections which are somewhat important only for weakly bound nucleons. The sensitivity to $M_n$ is large since this comes directly from the mirror decay. Therefore, the results of the mirror-transition method of determining $M_n$ should serve as a test for nuclear models and hadronic-probe methods.

We gratefully acknowledge the help of P. M. Endt in supplying us with current values of the relevant electromagnetic transition rates. This work was performed in part under the auspices of the U. S. Department of Energy by the Lawrence Livermore Laboratory under Contract No. W-7405-Eng-48, and in part under Contract No. E(11-1)-3039.

[a] Permanent address: Physics Department, Oregon State University, Corvalis, Ore. 97331.

[1] The division of the electromagnetic operator into isoscalar and isovector pieces has been made by many authors; e.g., E. K. Warburton and J. Weneser, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969).

[2] P. M. Endt and C. Van Der Leun, Nucl. Phys. A235, 27 (1974).

[3] A. M. Bernstein, Phys. Lett. 29B, 332, 335 (1969), and in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1969).

[4] H. Rebel, Z. Phys. A277, 35 (1976).

[5] A. M. Bernstein and I. Fischman, to be published.

[6] A. M. Bernstein, V. R. Brown, and V. A. Madsen, Phys. Lett. 71B, 48 (1977).

[7] A. Chaumeaux, V. Layly, and R. Schaeffer, to be published.

[8] S. Iverson *et al.*, Phys. Rev. Lett. 40, 17 (1978), and to be published.

[9] J. Jansen *et al.*, to be published.

[10] P. W. M. Glaudemans and C. Van Der Leun, Phys. Lett. 34B, 41 (1971).

[11] P. M. Endt, private communication.

[12] V. R. Brown and V. A. Madsen, Phys. Rev. C 11, 1298 (1975).

[13] J. L. Durell, C. Harter, and W. R. Phillips, Phys. Lett. 70B, 405 (1977).

[14] The same technique of isotopic-spin decomposition has been used for the static quadrupole moments of $^{17}$F and $^{17}$O as for the transition rates. The resulting neutron quadrupole moment of $^{17}$O is of interest for nuclear models and for the elastic scattering of hadronic probes.

[15] B. A. Brown, A. Arima, and J. B. McGrory, Nucl. Phys. A277, 77 (1977).

[16] Tests of hadronic-probe method for $N>Z$ nuclei is a nontrivial addition to a test in $N=Z$ nuclei since the effective hadron interaction might be modified by the extra neutrons.