[18]We dropped numerical factors and coupling constants assumed to be order $\lesssim 1$.

[19]M. Yoshimura, Phys. Rev. Lett. 41, 381 (1978); S. Dimopoulos, and L. Susskind, to be published; D. Toussaint, S. Treiman, F. Wilczek, and A. Zee, to be published; S. Weinberg, to be published.

[20]For a review, see for example, Weinberg, Ref. 12, and C. Misner, K. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[21]A recent theory is that of V. M. Canuto, Nuovo Cimento 1, 1 (1978).

[22]For a review of the situation in Einstein's theory see M. J. G. Veltman, in *Methods in Field Theory*, edited by R. Balian and J. Zinn-Justin (North-Holland, Amsterdam, 1976). Recent literature includes G. 't Hooft and M. J. G. Veltman [Ann. Inst. Henri Poincaré 20, 69 (1974)], S. Deser and P. van Nieuwenhuizen [Phys. Rev. D 10, 401, 411 (1974)], and S. Deser, H.-S. Tsao, and P. van Nieuwenhuizen [Phys. Rev. D 10, 3337 (1974)].

Horizontal Interaction and Weak Mixing Angles

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The formula $\tan^2\theta_C \simeq m_d/m_s$ is derived from a continuous symmetry. Cabibbo universality is guaranteed in a natural way. All weak mixing angles are determined in terms of quark masses. The b quark is predicted to decay mainly to the u quark, and the lifetime of the associated mesons is $\sim 10^{-10}$–10^{-11} sec. We argue that this additional symmetry is (essentially) the only one which can be added to the standard $SU_c(3)\otimes SU(2)\otimes U(1)$ model without generating anomalies.

The standard Weinberg-Salam-Glashow-Iliopoulos-Maiani $SU(2)\otimes U(1)$ model[1] has scored remarkable successes in correlating weak-interaction data.[2] But, like all other models proposed so far, it fails to offer any explanation for the values of The Cabibbo-like mixing angles[3] and of the quark masses. Another unsatisfactory feature is that with the proliferation of quarks (five known "experimentally," at least six according to theoretical prejudice) one is forced to add more left-handed doublets and right-handed singlets to the model. The famous question of "Who ordered the muon?" has now been escalated to "Why does Nature repeat herself?" Furthermore, strict universality as defined by Cabibbo[4] is no longer an elagantly automatic feature of the theory with more than two left-handed doublets. In this paper, we offer no fundamental answers to the questions raised above but we show that, by linking these questions together, one may determine all the mixing angles in terms of quark masses. In particular, we guarantee Cabibbo universality and obtain the relation[5]

$$\tan^2\theta_C \simeq m_d/m_s. \tag{1}$$

This relation is known to be well satisfied with m_d and m_s as determined by soft-meson analysis.[6]

In the standard model the gauge symmetry fixes in an elegant fashion the interaction of gauge bosons with left-handed fermions within a given doublet but does not relate different doublets to each other (Fig. 1). Were it not for the weak mixing angles linking the different left-handed doublets, weak-interaction theory would break up into disjoint pieces each with its own conserved quantum number. A number of authors[5] have proposed to remedy this situation by imposing discrete symmetries interchanging or permuting the various doublets and singlets and have obtained interesting relations. Unfortunately the number of possible discrete symmetries is very

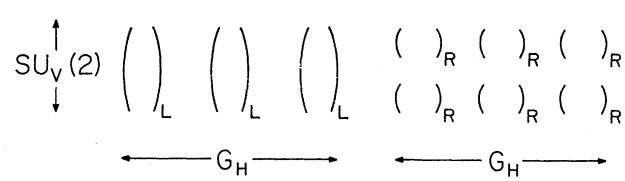

FIG. 1. Multiplet structure of the standard $SU(2)\otimes U(1)$ model with six quarks. We propose to gauge the horizontal group G_H.

large (unlike the case of continuous groups, where there are only a few groups and representations of low dimension) and the choice is essentially arbitrary. As we remarked in an earlier paper[7] an alternative is to introduce a gauge symmetry in the "horizontal direction"[8] (see Fig. 1), transforming among the left-handed doublets and among the right-handed singlets. We will explore this possibility here.

We assume six quarks and six leptons. With three doublets the horizontal gauge group G_H can be SU(2) or SU(3). SU(3) generates anomalies in the lepton sector and can thus be ruled out. We thus propose that the standard model be extended to an $\mathrm{SU}_V \otimes \mathrm{U}(1) \otimes \mathrm{SU}_H(2)$ gauge model[9] with the vertical[10] $\mathrm{SU}_V(2)$ doublets and singlets transforming as triplets under the horizontal $\mathrm{SU}_H(2)$. To generate fermion masses we can introduce Higgs fields transforming as scalar σ, vector η, or tensor φ under $\mathrm{SU}_H(2)$ [and as spinor under $\mathrm{SU}_V(2)$]. Explicitly, the couplings have the form

$$\bar{\psi}_{L\alpha}{}^i \sigma_\alpha U_R{}^i, \quad \bar{\psi}_{L\alpha}{}^i \eta_\alpha{}^k U_R{}^j \epsilon_{ijk}, \text{ and } \bar{\psi}_{L\alpha}{}^i \varphi_\alpha{}^{ij} U_R{}^j,$$

respectively. Here α denotes indices transforming under $\mathrm{SU}_V(2)$ and i, j, k denote indices transforming under $\mathrm{SU}_H(2)$. $U_R{}^i$ denotes the three charge $+\frac{2}{3}$ (up) quarks. For the down-quark (charge $-\frac{1}{3}$) sector we couple with $\tilde{\sigma}, \tilde{\eta}, \tilde{\varphi}$, the isoconjugate transforms under $\mathrm{SU}_V(2)$. φ^{ij} is a traceless symmetric tensor.

It is easy to see that with any number of scalars and vectors an interesting fermion mass matrix is not obtained. The most economical choice is then a tensor φ and a vector η. There will be terms in the Higgs potential coupling φ and η of the form $\eta^\dagger \varphi^\dagger \varphi \eta$, $\eta^\dagger \varphi \varphi \eta^\dagger$, $\eta \varphi^\dagger \varphi \eta^\dagger$. (There are also terms cubic in η and linear in φ. However, one may readily verify that, by suitable choice of parameters, one can arrange to reach the minimum given below.) Being a symmetric tensor, φ can always be diagonalized to have the form $\cos\theta\, \lambda_3 + \sin\theta\, \lambda_8$. These terms will be minimized with vacuum expectation values[11]

$$\langle \varphi_{\alpha=+1}{}^{ij} \rangle \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

and $\quad (2)$

$$\langle \eta_{\alpha=+1}{}^i \rangle \propto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

The mass matrix in the up-quark sector will then read

$$m_{\text{up}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -a & +b \\ 0 & -b & +a \end{pmatrix} \quad (3)$$

leading to a massless u quark, a c quark with mass $= b - a$, and a t quark with mass $= b + a$. The mass matrix in the down-quark sector has the same form (but with different constants a', b'). Thus, at this stage $m_u = m_d = 0$ and all weak mixing angles vanish.

Now we imagine that, by perturbative mechanisms to be discussed below, the vacuum expectation value of η is shifted slightly to

$$\begin{pmatrix} 1 \\ 0 \\ \epsilon \end{pmatrix}$$

with $\epsilon \ll 1$. The up-quark mass matrix then reads

$$m_{\text{up}} = \begin{pmatrix} 0 & +c & 0 \\ -c & -a & +b \\ 0 & -b & +a \end{pmatrix},$$

with $c \ll a, b$, and similarly for the down-quark mass matrix. Diagonalizing the two mass matrices

$$M_{\text{up}} = R_L^{\text{up}\dagger} M_{\text{up}}^{\text{diagonal}} R_R^{\text{up}},$$
$$M_{\text{down}} = R_L^{\text{down}\dagger} M_{\text{down}}^{\text{diagonal}} R_R^{\text{down}},$$

we find that the generalized Cabibbo rotation $R_L{}^{\mathrm{C}} \equiv R_L{}^{\text{up}} R_L{}^{\text{down}\dagger}$ is determined to be *approximately* (in the $\{u, c, t; d, s, b\}$ basis)

$$R_L{}^{\mathrm{C}} = \begin{pmatrix} 1 & -(m_d/m_s)^{1/2} + (m_u/m_c)^{1/2} & -(m_d m_s)^{1/2}/m_b + (m_u m_c)^{1/2}/m_t \\ (m_d/m_s)^{1/2} - (m_u/m_c)^{1/2} & 1 & 0 \\ (m_d m_s)^{1/2} m_b - (m_u m_c)^{1/2}/m_t & 0 & 1 \end{pmatrix}.$$

In particular, we find that the Cabibbo angle is given by

$$\theta_{\mathrm{C}} \simeq (m_d/m_s)^{1/2} - (m_u/m_c)^{1/2} \simeq (m_d/m_s)^{1/2},$$

a result obtained previously.[5,7] What is interesting is that in the present scheme Cabibbo universality is preserved to the extent that $(m_d m_s/m_b{}^2)^{1/2} - (m_u m_c/m_t{}^2)^{1/2}$ is small.

A number of remarks are now in order.

(I) We can couple the same Higgs fields to leptons. Since the neutrinos are massless, lepton-mixing angles are of no interest at low energies. One noteworthy feature of our scheme is that the smallness of m_e is correlated with the smallness of m_u, m_d, θ_C, and other mixing angles.

Indeed, the "form universality[7]" of the mass matrices implies the relations

$$m_e m_\mu / m_\tau^2 = m_d m_s / m_b^2 = m_u m_c / m_t^2. \quad (4)$$

Taking the measured lepton masses we find the first equality in Eq. (4) to be reasonably well satisfied, considering the uncertainties in estimates[12] of quark masses. The second equality in Eq. (4) yields the rough extimate $m_t \sim 15$ GeV.

(II) The horizontal gauge bosons (and various Higgs bosons) have to be several orders of magnitude more massive than the W bosons, or much more weakly coupled, in order to avoid unwanted quark-flavor- and lepton-flavor-changing transitions. This we regard as a rather unsatisfactory feature. Technically, this requires the introduction of additional Higgs fields transforming like singlets under $SU_V(2) \otimes U(1)$ to give the large masses to the horizontal gauge bosons. Since these Higgs fields cannot couple to fermions, the fermion mass matrix and the calculation of angles are not affected.

(III) Given the remark (II), we note that all the successful phenomenological predictions of the standard model, including the prediction for the ratio M_W/M_Z, are preserved.

(IV) The coupling of b to u is suppressed by the factor $(m_d m_s/m_b^2)^{1/2} - (m_u m_c/m_t^2)^{1/2}$ relative to the coupling of d to u. If we assume that Υ is a $\bar{b}b$ bound state and that the t quark is very heavy, then the lifetime of a bottom meson B may be estimated to be roughly

$$\frac{1}{9}\left(\frac{m_b^2}{m_d m_s}\right)\tau(\mu)\left(\frac{m_\mu}{m_B}\right)^5 \sim 10^{-10} \text{ or } 10^{-11} \text{ sec}$$

taking[12] $m_s \sim 150$ MeV and $m_b \sim 4$ GeV. The factor $\frac{1}{9}$ allows for the larger number of available channels. The second mass relation in Eq. (4) actually constrains the t-quark mass in such a way that the coupling of the b to the u quark nearly vanishes in the approximate form of R_L^C. Since there are indications that the b quark is not exceedingly long lived,[13] it may be necessary to loosen our framework. Two possibilities naturally suggest themselves. Firstly, one might double the number of Higgs fields instead of using a Higgs field and its isoconjugate transform. Alternatively, one may take into account an expected shift of the tensor vacuum expectation value into the (11) direction.[14] The first alternative leaves the t-quark mass a free parameter again while maintaining the relation between lepton masses and charge $-\frac{1}{3}$ quark masses. The second alternative allows one to keep both mass relations as a first approximation—however, the b-quark couplings are undetermined in detail, although constrained to be small (the lifetime estimate above provides a rough upper bound).

(V) One attractive possible mechanism to generate the small parameter ϵ in the vacuum expectation value of η is through radiative correction. We note that the vacuum expectation values exhibited in Eq. (2) are invariant under a rotation of 180° in the 2-3 plane. If this discrete invariance is broken by the Higgs fields which transform as singlets under $SU_V(2) \otimes U(1)$ then radiative corrections in general will induce terms in the Higgs potential which will generate ϵ. This mechanism may naturally explain the smallness of m_e, m_u, m_d, θ_C, and other mixing angles.

(VI) A more mundane mechanism will be simply to construct terms in the Higgs potential which will generate[15] ϵ in tree approximation. This appears to be possible with the help of $SU_V(2) \otimes U(1)$ singlet Higgs fields.

(VII) As was pointed out by Kobayashi and Maskawa,[3] it is possible to incorporate CP nonconservation in the fermion mass matrix with six quarks. By supposing that some of the vacuum expectation values are complex one may violate CP invariance in this model but unfortunately with an undetermined phase at the present level of understanding. The exchange of horizontal gauge bosons can also induce a form of superweak CP nonconservation.[16]

(VIII) Our symmetry-breaking scheme involves use of the antisymmetric ϵ_{ijk} symbol, and therefore only works smoothly when the fermion multiplets fall into horizontal triplets.

(IX) In the standard formulation the striking hierarchy of quark and lepton masses is accounted for by vastly different vacuum expectation values and/or vastly different Yukawa couplings. In our model, this is avoided; we need only take the parameters a and b (and the corresponding a' and b') in Eq. (3) to be roughly equal. In particular, this consideration and remark (VIII) suggest that Nature may not repeat herself indefinitely. An end to quark-lepton proliferation is also indicated by a cosmological argument[17] limiting the number of neutrinos to $\leqslant 4$.

(X) It is attractive to suppose that quark and lepton masses, as well as vector-boson masses, arise via dynamical symmetry breaking. However, with only the usual $SU(2)\otimes U(1)$ weak interactions and $SU_c(3)$ color interactions the massless theory has an enormous global horizontal symmetry, whose spontaneous breakdown would produce unwanted Nambu-Goldstone bosons. Our horizontal SO(3) gauge interaction is a simple anomaly-free addition to the standard interactions. The full $SU_c(3)\otimes SU_V(2)\otimes U(1)\otimes SU_H(2)$ gauge interaction leaves no further anomaly-free symmetry[18] of the massless theory, and so it is a candidate for spontaneous mass generation without Nambu-Goldstone bosons. An alternative philosophy might be to suppose that all symmetries of the massless world should be gauged. This symmetry group is enormous, or course; for instance, the quark sector alone has a $U(3N)\otimes U(3N)$ symmetry with N quarks. To gauge this enormous symmetry in its entirely would lead to uncanceled anomalies. An outstanding question in particle physics is why a particular subgroup, namely $SU_c(3)\otimes SU(2)\otimes U(1)$, is singled out. Here we can address ourselves to a much more modest question: Given $SU_c(3)\otimes SU(2)\otimes U(1)$ and the multiplet structure of quarks and leptons, what additional symmetry group can one gauge without anomalies? The possibilities appear to be limited.[19] The first is the one utilized in this paper. The second[20] is to have separate horizontal groups for the quark sector and for the lepton sector.

(XI) It is possible to extend the present discussion to the SU(5) model of Georgi and Glashow[21] unifying strong, weak, and electromagnetic interactions. Assuming that fermion masses are generated by a Higgs field belonging to $\underline{5}$, one obtains the same results as above and in addition various relations between the mixing angles in the lepton sector and the down-quark sector. Presumably, these relations will have to be renormalized down from the unification scale.

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[1]S. Weinberg, Phys. Rev. Lett. 19, 1204 (1967); A. Salam, in *Elementary Particle Theory*, edited by N. Svartholm (Wiley, New York, 1969); S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[2]With the possible exception of the atomic parity experiments; L. L. Lewis *et al.*, Phys. Rev. Lett. 39, 795 (1977); P. E. Baird *et al.*, Phys. Rev. Lett. 39, 798 (1977). For a review, see, for example, J. J. Sakurai, University of California, Los Angeles Report No. UCLA/78/TEP/9 (to bepublished).

[3]M. Kobayashi and K. Maskawa, Prog. Theo. Phys. 49, 652 (1973).

[4]N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960).

[5]An incomplete list of papers which attempted to derive this relation include R. Gatto, G. Sartori, and M. Tonin, Phys. Lett. 28B, 128 (1968); N. Cabibbo and L. Maiani, Phys. Lett. 28B, 131 (1968); H. Pagels, Phys. Rev. D 11, 1213 (1975); S. Weinberg, in *A Festschrift for I. I. Rabi*, edited by Lloyd Motz (New York Academy of Sciences, New York, 1977); F. Wilczek and A. Zee, Phys. Lett. 70B, 418 (1977); A. De Rújula, H. Georgi, and S. L. Glashow, to be published; H. Fritzsch, Phys. Lett. 70B, 436 (1977); D. Sutherland and N. Zoupanos, to be published. In addition, there are a number of papers which obtain some other relations. These include H. Georgi and A. Pais, Phys. Rev. D 10, 539 (1974); R. Mohapatra and G. Senjanović, Phys. Lett. 73B, 176 (1978); S. Pakvasa and H. Sugawara, Phys. Lett. 73B, 61 (1978); H. Fritzsch, Phys. Lett. 73B, 317 (1978); A. Zee, Phys. Rev. D 9, 1772 (1974), and 13, 713 (1976); F. Wilczek and A. Zee, Phys. Rev. D 15, 3701 (1977).

[6]M. L. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); S. L. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968); for a recent review, see Weinberg, Ref. 5.

[7]Wilczek and Zee, Ref. 5. See also A. Zee, in *Particles and Fields—1977*, AIP Conference Proceedings No. 43, edited by P. A. Schreiner, C. H. Thomas, and A. B. Wicklund (American Institute of Physics, New York, 1978), p. 43.

[8]Horizontal gauge interactions have been used previously be S. Barr and A. Zee [Phys. Rev. D 17, 1854 (1978), and to be published] to calculate the electron-muon mass ratio.

[9]Notice that the cancellation of anomalies in the standard model relating the number of leptons and quarks [C. Bouchiat, J. Iliopoulos, and Ph. Meyer, Phys. Lett. 38B, 519 (1972)] is preserved.

[10]To distinguish the two SU(2)'s we refer to the ordinary weak interaction as the "vertical" gauge interaction.

[11]That parameter in the Higgs potential could be suitably chosen so that electric charge is conserved.

[12]Weinberg, Ref. 5.

[13]D. Cutts *et al.*, Phys. Rev. Lett. 41, 363 (1978).

[14]The mechanisms which we have explicitly examined tend to shift $\langle\varphi\rangle$ slightly as well.

[15]In general, it must be broken [H. Georgi and A. Pais, Phys. Rev. D 10, 1246 (1974)].

[16]A similar remark has been made by M. A. B. Bég and H.-S. Tsao [Phys. Rev. Lett. 41, 279 (1978)] in a different context.

[17]V. F. Shvartsman, Pis'ms Zh. Eksp. Teor. Fiz. 9,

315 (1969) [JETP Lett. 9, 184 (1969)]; G. Steigman, D. N. Schramm, and J. E. Gunn, Phys. Lett. 66B, 202 (1977).

[18]It is straightforward to verify that any additional U(1) gauge symmetry leads to anomalies.

[19]There also exists the awkward possibility of allowing two $SU_V(2)$ multiplets to transform as an $SU_H(2)$ doublet and the third $SU_V(2)$ multiplet to transform as an $SU_H(2)$ singlet.

[20]This includes the possibility of having $SU_H(2)$ for leptons and $SU_H(3)$ for quarks at the price of a more complicated Higgs structure.

[21]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1975).

Isospin Decomposition of Nuclear Multipole Matrix Elements from γ Decay Rates of Mirror Transitions: Test of Values Obtained with Hadronic Probes

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The electromagnetic decay rates of mirror states are used to decompose the multipole electromagnetic matrix elements into isoscalar and isovector or equivalently proton and neutron matrix elements. This method is sensitive to the neutron matrix element because it is measured directly, and it is accurate since it depends only on electromagnetic interactions; it therefore can be used to test methods using inelastic hadron scattering. There is agreement with the results for (α,α') on ^{26}Mg and ^{42}Ca but disagreement with the recent $^{18}O(\pi,\pi')$ experiments.

An incisive method to study the dynamical properties of nuclear states is to observe their electromagnetic (EM) transition rates, which is the square of the proton transition matrix element M_p. In the long-wavelength limit one can write the electric multipole operator of order λ as a sum over protons. One can also define a similar sum for neutrons. These operators can be written in one equation as[1]

$$O_{n\,(p)}{}^{\lambda} = \sum_{k=1}^{A}\left[\frac{1\pm\tau_3(k)}{2}\right] r_k{}^{\lambda} Y_{\lambda\mu}(\hat{r}_k). \quad (1)$$

The matrix elements of $O_p{}^\lambda$ and $O_n{}^\lambda$ between nuclear states are defined to be M_p and M_n, respectively. The apparent lack of a physical process to make *precision* measurements of M_n has left a gap in our arsenal of nuclear probes. In this paper we utilize, for the first time, a purely EM emthod to obtain M_n from the EM decay rates of mirror transitions [see Eq. (5) below]. This method has both sensitivity and precision and, to the accuracy of the data, solves this outstanding problem for the case of light nuclei. At present there are approximately 25 measured $E2$ mirror transitions for which this method is applicable.[2] In this paper a subset of the known $E2$ data is presented and compared to the predictions of the schematic model of core polarization. In addition, these results can be used to test other methods of obtaining M_n.

An alternative method to measure M_n has been to use the inelastic scattering of hadrons (which interact with both neutrons and protons); this can then be combined with EM data (or another hadronic-probe experiment) to obtain both M_n and M_p. This procedure usually depends on several features. First, $O_p{}^\lambda$ and $O_n{}^\lambda$ have $r^{\lambda+2}$ in the radial matrix elements which weights the surface region, an effect similar to the surface weighting that occurs for strongly absorbed hadronic projectiles.[3] In addition, the effective interaction between the probe and the bound nucleons must be known, and be of the form of a sum of single-body operators. The validity of this latter condition is far from obvious.

There have been several papers which utilize the results of inelastic hadron scattering experiments such as α particles,[3-6] protons,[7] and, more recently, pions,[8,9] to obtain M_n and M_p. Unfortunately, there have been very few tests of the ba-

