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$$ds^2=(V-\tfrac{1}{2}c_0{}^2x^2)du^2+2du\,(d\varphi-c_0x\,dy)-(dx^2+dy^2),$$

$$V=V(x,y),\quad V_{,xx}+V_{,yy}=0$$

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Broken-Symmetric Theory of Gravity

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A theory of gravity incorporating the concept of spontaneous symmetry breaking is proposed. It is suggested that the same symmetry-breaking mechanism is responsible for breaking a unified gauge theory into strong, weak, and electromagnetic interactions.

Einstein's theory of gravity[1] and Fermi's theory of weak interaction[2] have one feature in common: In contrast to electrodynamics and the modern theory of strong interaction they contain coupling constants with dimension of $1/(\text{mass})^2$. These coupling constants are notably small, with[3] Fermi's coupling constant $G_\text{F}\sim(300m_N)^{-2}$ and Newton's coupling constant $G_\text{N}\sim(10^{19}m_N)^{-2}$. It has long been suggested that the smallness of G_F is due to the massiveness of the intermediate boson W, vis., $G_\text{F}\sim e^2/M_W{}^2$. The successful unification[4] of electromagnetic and weak interactions has made precise this idea. Central to the unification scheme is the concept of spontaneous symmetry breaking, which causes some scalar field to have a vacuum expectation value v, thus generating the mass of the intermediate boson so that

$$G_\text{F}\sim e^2/M_W{}^2\sim 1/v^2. \tag{1}$$

The concept of spontaneous symmetry breakdown[5] has proved to be extraordinarily fruitful in many areas of physics and I consider it worthwhile to try to incorporate it into gravitation. Clearly, the physics of gravitation is very different from the physics of weak interaction since gravity is long ranged and the mediating particle, the graviton, is massless. Nevertheless, I would like to have a relation analogous to Eq. (1) and to attribute the smallness of Newton's constant G_N to the massiveness of some particle.

Motivated by these considerations, I suggest here that Einstein's action for gravity be modified to read

$$S=\int d^4x\sqrt{g}\,[\tfrac{1}{2}\epsilon\varphi^2R+\tfrac{1}{2}g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi-V(\varphi)+\mathcal{L}_w]. \tag{2}$$

Here φ represents a scalar field and ϵ denotes a dimensionless coupling constant which we would take to be of order $\lesssim 1$. $\mathcal{L}_w$ is the Lagrangian for the rest of the world. (We must distinguish two physically different cases depending on whether or not $\mathcal{L}_w$ includes φ, that is to say, whether or not φ interacts with matter fields directly. For the moment we will assume that $\mathcal{L}_w$ does not include φ. See below.) Suppose that $V(\varphi)$ is such that it is minimized when $\varphi=v$. Setting $\varphi=v$ we see that (2) reduces to Einstein's action with the identification

$$G_\text{N}=\frac{1}{16\pi}\left(\frac{1}{\tfrac{1}{2}\epsilon v^2}\right). \tag{3}$$

This should be compared to (1). Thus we need an enormously large vacuum expectation value v, of the order of the Planck mass $m_{\rm Pl} \equiv 10^{19} m_N$. In our discussion we must[6] assume $V(v)=0$. The precise form of $V(\varphi)$, however, need not be specified. Occasionally, in order to be explicit, we will use the form $V_{\rm expl} = \frac{1}{8}\lambda(\varphi^2 - v^2)^2$.

Making the substitution $\varphi = v + \zeta$ we see that this theory requires the existence of a scalar particle ζ with mass naively given by $[V''(\varphi = v)]^{1/2}$. {Actually, because of the peculiar coupling of φ to gravity the mass is lowered somewhat to $[V''(\varphi = v)/(1+6\epsilon)]^{1/2}$. We will postpone the discussion of this point until later.}

If $V_{\rm expl}$ is used ζ is very massive, with a mass $\lambda^{1/2} v \sim (\lambda/8\pi\epsilon)^{1/2} m_{\rm Pl} \sim 10^{19}$ GeV if λ is of order $\lesssim 1$. In general, however, the mass of ζ is not specified by the theory and there is no reason that ζ could not be relatively light. The interaction of ζ with gravity is unlike the interaction of any other particle with gravity.[7] In particular, ζ is unstable and decays into two gravitons rapidly with a width $\Gamma \sim \epsilon^2 m_\zeta{}^3/v^2$. If $V_{\rm expl}$ is used, $\Gamma \sim \epsilon^2 \lambda m_\zeta$. In this paper, I adopt the philosophy that dimensionless coupling constants such as ϵ and λ are all of order $\lesssim 1$. Thus, if $m_\zeta \ll m_{\rm Pl}$, the ζ particle could be relatively narrow.

A particularly attractive speculation is that the spontaneous symmetry breakdown generating Newton's gravitational constant is also responsible for generating the weak, electromagnetic, and strong interactions from a unified theory. It has been suggested that the weak, electromagnetic, and strong interactions merge into one unified Yang-Mills gauge theory at some large mass scale. An especially appealing scheme is the SU(5) gauge theory of Georgi and Glashow.[8] Knowing how coupling constants depend on the mass scale and knowing the values of the weak, electromagnetic, and strong interactions at low energies one can determine the mass scale at which unification occurs. The result of such an analysis is that unification occurs at[9] $\sim 10^{19}$ GeV. The emergence of a mass scale close to the Planck mass $m_{\rm Pl}$ appears to be a numerical coincidence within the framework of this analysis. I would like to suggest, however, that this is not coincidental but that the scalar field φ in Eq. (2) should be replaced by a Higgs field[10] φ transforming like $\underline{24}$ under SU(5). Expressions like φ^2 and $(\partial_\mu\varphi)^2$ in Eq. (2) should be replaced by $\vec{\varphi}^2$ and $(\partial_\mu\vec{\varphi})^2$. In the unified SU(5) theory, the relevant mass scale is 10^{19} GeV and there are presumably vector bosons of this mass and the vacuum expectation value of $|\varphi|$ is also presumably of the order $\sim 10^{19}$ GeV. Thus, I suggest that one unified mechanism is responsible for the mass scale of gravity as well as for the breaking of SU(5) into SU(3)$\otimes$SU(2)$\otimes$U(1). The present suggestion, of course, does not unify all four interactions, but it does provide an intriguing link between gravity and the other three interactions and deserves to be investigated further.

It is straightforward to derive the equations of motion from the action in Eq. (2). One need only recall that in

$$\delta(g^{1/2} g^{\mu\nu} R_{\mu\nu}) = g^{\mu\nu} R_{\mu\nu}\,\delta g^{1/2} + g^{1/2} R_{\mu\nu}\,\delta g^{\mu\nu} + g^{1/2} g^{\mu\nu}\,\delta R_{\mu\nu}, \tag{4}$$

the last term is a total divergence and thus can be dropped in the standard theory of gravity. The first two terms in Eq. (4) generate the Einstein tensor $(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R)$ in Einstein's equation of motion. Here, however, the third term in Eq. (4) must be kept. After integrating by parts and manipulating various identities, one finds the modified equation of motion

$$\tfrac{1}{2}\epsilon\varphi^2 (R^{\mu\nu} - \tfrac{1}{2} g^{\mu\nu} R) = -\tfrac{1}{2}[T_w^{\mu\nu} + T_\varphi^{\mu\nu} + (\epsilon\varphi^2)^{;\mu;\nu} - g^{\mu\nu}(\epsilon\varphi^2)^{;\lambda}{}_{;\lambda}]. \tag{5}$$

Here $T_w^{\mu\nu}$ is the energy-momentum tensor of the world constructed from $\mathcal{L}_w$ in Eq. (2) and

$$T_\varphi^{\mu\nu} = \partial^\mu\varphi\,\partial^\nu\varphi - g^{\mu\nu}[\tfrac{1}{2} g^{\lambda\rho}\partial_\lambda\varphi\,\partial_\rho\varphi - V(\varphi)]. \tag{6}$$

The equation of motion of the field φ reads

$$\varphi^{\mu}{}_{;\mu} + \frac{\delta V}{\delta\varphi} - \epsilon R\varphi = 0 \tag{7}$$

provided $\mathcal{L}_w$ does not depend on φ. If $\mathcal{L}_w$ depends on φ then the right-hand side of this equation would read $\delta\mathcal{L}_w/\delta\varphi - \partial_\mu \delta\mathcal{L}_w/\delta\,\partial_\mu\varphi$. In the present broken-symmetric phase of the world $\varphi(x) = v$ with $V(v)=0$ and $\frac{1}{2}\epsilon v^2 = (16\pi G_{\rm N})^{-1}$. Then Eq. (5) reduces to the standard Einstein's equation as expected. Thus, no present-day experiment can distinguish between this theory and Einstein's theory. [In the present epoch, the term $\epsilon R\varphi$ in Eq. (7) is completely negligible.[11]]

One of the demands we would certainly like to make on this theory is that the exchange of energy between matter and the gravitational field be described exactly as in Einstein's theory. This amounts to one version of the equivalence principle. I now show that this is in fact true, namely,

$$T_{w\ ;\nu}^{\mu\nu}=0 \tag{8}$$

holds, provided that $\mathcal{L}_w$ does not include φ. In Einstein's theory this equation follows from the contracted Bianchi identity satisfied by the Einstein tensor $(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R)$. In the present theory this idenity gives instead the conservation law

$$(a^{-1}[T_w^{\mu\nu}+t^{\mu\nu}])_{;\nu}=0, \tag{9}$$

where the quantity inside the round brackets can be thought of as the "effective" energy-momentum tensor. (We have introduced the notation $a\equiv\frac{1}{2}\epsilon\varphi^2$ and $t^{\mu\nu}\equiv T_\varphi^{\mu\nu}+2a^{;\mu;\nu}-2g^{\mu\nu}a^{;\lambda}{}_{;\lambda}$). I wish to show that Eq. (9) when supplemented by the equation of motion of φ actually implies Eq. (8). Rearranging Eq. (9) somewhat, I can write

$$-\tfrac{1}{2}T_{w\ ;\nu}^{\mu\nu}=a_{;\nu}(R^{\mu\nu}-\tfrac{1}{2}g^{\mu\nu}R)+\tfrac{1}{2}t^{\mu\nu}{}_{;\nu}. \tag{10}$$

Using the identity

$$a^{;\sigma}R_{\sigma\kappa}=a^{;\lambda}{}_{;\lambda;\kappa}-a^{;\lambda}{}_{;\kappa;\lambda} \tag{11}$$

we find that Eq. (10) simplifies to read

$$T_{w\ ;\nu}^{\mu\nu}=a^{;\mu}R-T_{\varphi\ ;\nu}^{\mu\nu}. \tag{12}$$

Finally, we use the equation of motion of φ [Eq. (7)] to find

$$T_{\varphi\ ;\nu}^{\mu\nu}=a^{;\mu}R \tag{13}$$

and hence the assertion is proved. Thus, the metric of space-time determines the motion of mass points.

If, on the other hand, $\mathcal{L}_w$ contains φ as is suggested by my speculation that φ is also responsible for breaking down a unified theory of strong, weak, and electromagnetic interactions, then the equation of motion of φ is no longer given by Eq. (7). Instead of $T_{w\ ;\nu}^{\mu\nu}=0$ we obtain

$$T_{w\ ;\nu}^{\mu\nu}=(\partial_\mu\varphi)\left(\partial_\mu\frac{\delta\mathcal{L}_w}{\delta\partial_\mu\varphi}-\frac{\delta\mathcal{L}_w}{\delta\varphi}\right). \tag{14}$$

However, in a region of space-time in which φ is not changing, such as here and now, we do have $T_{w\ ;\nu}^{\mu\nu}=0$.

There have been proposed theories of gravity involving a scalar field, notable among which is the theory of Brans and Dicke.[12] In contrast to these earlier theories,[13] the theory proposed here incorporates the concept of spontaneous symmetry breaking. This is crucial. For instance, the Brans-Dicke theory is motivated instead by Mach's principle. Thus, the scalar field φ has no self-interaction and has the trace of the energy-momentum tensor of matter as its source. In other words, the potential term $V(\varphi)$ in Eq. (2) is not included. As a result, the Brans-Dicke theory is inconsistent with observation unless a certain parameter is very large.[14] In contrast, in the theory discussed here the field φ is anchored by the symmetry-breaking potential to have a fixed value, provided that the scalar curvature R is not enormous in the space-time region under consideration.

We now return to the propagation of the particle ζ through space-time, which is unlike that of any other physical particle. Considering small vibrations about flat space $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ and $\varphi=v+\zeta$, we expand the equations of motion [Eqs. (5) and (7)] to first order:

$$\partial^2(h_{\mu\nu}-\tfrac{1}{2}\eta_{\mu\nu}h)=\frac{4}{v}(\eta_{\mu\nu}\partial^2\zeta-\partial_\mu\partial_\nu\zeta), \tag{5a}$$

$$[\partial^2+V''(\zeta)]\zeta=\tfrac{1}{2}\epsilon v\partial^2 h \tag{7a}$$

(we have used the harmonic gauge $\partial_\mu h^\mu{}_\nu=\frac{1}{2}\partial_\nu h^\mu{}_\mu$). Note that the source of gravity waves [the right-hand side of Eq. (5a)] is first order in ζ. This is in contrast with the usual situation in which the source of gravity waves is second order in small vibrations of matter fields. Tracing Eq. (5a) and substituting in Eq. (7a) we see that

$$\left(\partial^2+\frac{V''(v)}{1+6\epsilon}\right)\zeta=0. \tag{7b}$$

This mass shift can be thought of as due to a wave-function renormalization. Thus, a ζ particle propagating in flat space drags along a gravitational excitation[15] with amplitude

$$h_{\mu\nu}(k)=-\frac{2}{v}\left(\eta_{\mu\nu}+\frac{2k_\mu k_\nu}{k^2}\right)\zeta(k). \tag{15}$$

While the theory cannot be distinguished experimentally here and now from Einstein's theory, it invites us to contemplate the possibility of gravity being different at another time and another place. In a Robertson-Walker universe the scalar curvature is given by

$$R=6[k/\mathcal{R}^2+(1-q)H^2], \tag{16}$$

where[16] H is Hubble's constant, q is the deceleration parameter, $\mathcal{R}$ is the Robertson-Walker scale factor, which is approximately the "size" of the universe, and $k=+1$, 0, or -1 according to whether the universe is closed, flat, or open. As we follow the universe backwards in time, $\mathcal{R}$ increases and the term $\epsilon\mathcal{R}\varphi$ in Eq. (7) becomes increasingly important, shifting the vacuum expectation value v of φ. Thus, the gravitational "constant" G changes with time, according to[11]

$$\frac{\delta G}{G}=-2\,\frac{\delta v}{v}=\frac{2\epsilon R}{V''(v)}\sim\left(\frac{H}{m_\zeta}\right)^2. \tag{17}$$

This variation is completely negligible until the "age" of the universe H^{-1} becomes comparable to the Compton time of ζ. Another effect contributing to $\delta G/G$ is the rising temperature of the universe as one goes back in time. Crudely, the effect of finite temperature[17] T is to add to the potential $V(\varphi)$ terms like[18] $\sim T^2\varphi^2$. This term leads to a shift

$$\delta G/G\sim T^2/V''(v)\sim(T/m_\zeta)^2. \tag{18}$$

Again, the variation in G is completely negligible until the temperature becomes comparable to m_ζ. Incidentally, it has recently been suggested[19] that other events of great importance for the subsequent evolution of the universe took place at temperatures $\lesssim m_{\rm Pl}$.

Theories with a varying gravitational constant have long been proposed.[20,21] The variation $\delta G/G$ in these theories, largely motivated by Dirac's large-number hypothesis, is typically too large to be consistent with observation. No such difficulty afflicts the present theory, because the expectation value of φ is essentially fixed by $V(\varphi)$.

I note that near a Schwarzschild black hole the scalar curvature R vanishes and so the present theory and Einstein's theory lead to identical physical consequences.

The present theory and Einstein's theory can differ substantially only with ultrahigh values of the scalar curvature and/or the temperature, in particular near the initial (or final) singularity. Unfortunately, this is also the regime in which quantum effects presumably become important and one may justifiably be somewhat reluctuant to treat the theory classically. An especially interesting question is whether or not the initial (or final) singularity can be avoided, if the evolution of the universe follows Eqs. (5) and (7).

The starting motivation for this work was to write down a theory of gravity in which the only dimensional constant is associated with a mass term. This would be the case if we use $V_{\rm expl}$ in Eq. (2). Naively, one might hope that the resultant theory is renormalizable. This is, of course, not[22] true if one does perturbation theory about $\varphi=v$. However, it might be possible, perhaps by resumming or by expanding about another point, to show that the theory is renormalizable. Unfortunately, I am here only expressing a hope. Much further investigations will be necessary.

This work was supported in part by the U. S. Energy Research and Development Administration Contract No. AT(E11-1) 3071 Theoretical.

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Horizontal Interaction and Weak Mixing Angles

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The formula $\tan^2\theta_C \simeq m_d/m_s$ is derived from a continuous symmetry. Cabibbo universality is guaranteed in a natural way. All weak mixing angles are determined in terms of quark masses. The b quark is predicted to decay mainly to the u quark, and the lifetime of the associated mesons is $\sim 10^{-10}$–10^{-11} sec. We argue that this additional symmetry is (essentially) the only one which can be added to the standard $SU_c(3)\otimes SU(2)\otimes U(1)$ model without generating anomalies.

The standard Weinberg-Salam-Glashow-Iliopoulos-Maiani $SU(2)\otimes U(1)$ model[1] has scored remarkable successes in correlating weak-interaction data.[2] But, like all other models proposed so far, it fails to offer any explanation for the values of The Cabibbo-like mixing angles[3] and of the quark masses. Another unsatisfactory feature is that with the proliferation of quarks (five known "experimentally," at least six according to theoretical prejudice) one is forced to add more left-handed doublets and right-handed singlets to the model. The famous question of "Who ordered the muon?" has now been escalated to "Why does Nature repeat herself?" Furthermore, strict universality as defined by Cabibbo[4] is no longer an elagantly automatic feature of the theory with more than two left-handed doublets. In this paper, we offer no fundamental answers to the questions raised above but we show that, by linking these questions together, one may determine all the mixing angles in terms of quark masses. In particular, we guarantee Cabibbo universality and obtain the relation[5]

$$\tan^2\theta_C \simeq m_d/m_s. \tag{1}$$

This relation is known to be well satisfied with m_d and m_s as determined by soft-meson analysis.[6]

In the standard model the gauge symmetry fixes in an elegant fashion the interaction of gauge bosons with left-handed fermions within a given doublet but does not relate different doublets to each other (Fig. 1). Were it not for the weak mixing angles linking the different left-handed doublets, weak-interaction theory would break up into disjoint pieces each with its own conserved quantum number. A number of authors[5] have proposed to remedy this situation by imposing discrete symmetries interchanging or permuting the various doublets and singlets and have obtained interesting relations. Unfortunately the number of possible discrete symmetries is very

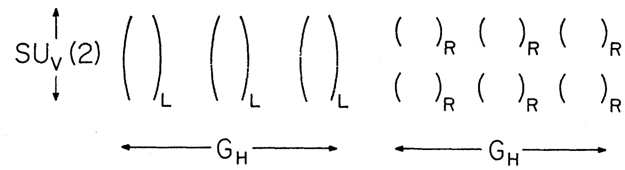

FIG. 1. Multiplet structure of the standard $SU(2)\otimes U(1)$ model with six quarks. We propose to gauge the horizontal group G_H.