lor series expansion does not appear to converge rapidly. For nonbackbending CHFB spectra, the expansion seems well behaved at all spins.

In conclusion, a simple expression is derived which uses the angular momentum fluctuations in CHFB wave functions to obtain high-spin spectra which are approximately angular momentum projected. Although the fluctuations in J_x are most important, those in J_y and J_z should not be omitted. The projected spectra are less compressed than the CHFB spectra and are closer to the experimental excitation energies.

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Direct Evidence for Multiple Sequential Fission in the Interaction of ^{208}Pb and ^{238}U with Uranium

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In coincidence experiments using mica and glass track detectors four- and five-pronged events have been observed in 1785-MeV $^{238}U + U_{nat}$ and 1535-MeV $^{208}Pb + U_{nat}$. The events are shown to be due to multiple sequential fission. Angular distributions are presented.

Experiments on collisions between fissile nuclei revealed that the dominant reaction channels are those leading to more than two heavy reaction products.[1-3] The reaction mechanism expected to account for most of these multibody events is the sequential process: In a first step two primary fragments are formed after a quasielastic or deep-inelastic interaction and one or both subsequently fission as isolated nuclei.[4] Alternative reaction mechanisms could be instantaneous fission induced by Coulomb or nuclear forces[5] or multibody breakup after fusion.[6] Only complete correlation measurements are likely to provide direct information on the actual reaction mechanism and, having established this, on further details of the dynamical processes during the collison.[7] In particular, in the case of a sequential process, it is possible to infer from the observed angular distribution of the final products valuable information on the transfer of orbital angular momentum into intrinsic angular momenta of the fragments of the first reaction step.[8]

In this Letter we report on a detailed study of fragment-fragment correlations in the interactions 1785-MeV $^{238}U + U_{nat}$ (reaction I) and 1535-MeV $^{208}Pb + U_{nat}$ (reaction II) leading to four heavy reaction products, confining ourselves to establishing the sequential nature of the processes. Further details of the processes will be presented in a forthcoming publication.[9] Also we report on the first observation seen so far of five heavy fragments in heavy-ion collisions. Complete correlation experiments have been performed with the 2π-geometry technique using mica and glass track detectors. All the heavy fragments have been identified with respect to their masses as well as to their energies by a three-dimensional measurement of tracks and by employing an empirical velocity-range relation.

The experiments were performed at the UNILAC in Darmstadt. A low-intensity dose of heavy ions (10^6 particles cm^{-2}) irradiates mica or glass which is covered with approximately 1-mg-cm^{-2} UF_4 or U, respectively. As described in Ref. 1 fragment tracks are made visible by etching. In this technique all reaction products with mass $\geqslant 30$ amu and scattered into the forward hemisphere $\vartheta_{lab} \leqslant 85°$ are registered. Spherical coordinates of the tracks of coincident reaction products are measured event-by-event and constitute the data to be analyzed.

The lower limits for the four- and five-body channels in reaction I are $\sigma_4 \geqslant 920 \pm 110$ mb and $\sigma_5 \geqslant 12 \pm 8$ mb, respectively. These cross sections refer to direct and indirect events (e.g., see the definition in Ref. 1). So far 398 events with four tracks, but only 1 event with five track tracks, have been observed. In reaction II we found $\sigma_4 \geqslant 200 \pm 60$ mb and so far thirteen events with four tracks but no event with five tracks. Together with the observed three-pronged events[9] the four- and five-body reaction channels completely exhaust the total reaction cross section $\sigma_R = 1775 \pm 290$ mb in reaction I, but account for only approximately 78% of $\sigma_R = 1650 \pm 300$ mb in reaction II. Figure 1 shows an example of a four- and five-pronged event. The probability of an accidental overlap of two independent nuclear events is less than 10^{-4}. Also a nonnuclear origin of tracks such as crystal defects can be excluded by optical inspection.

At present a randomly selected sample of 193

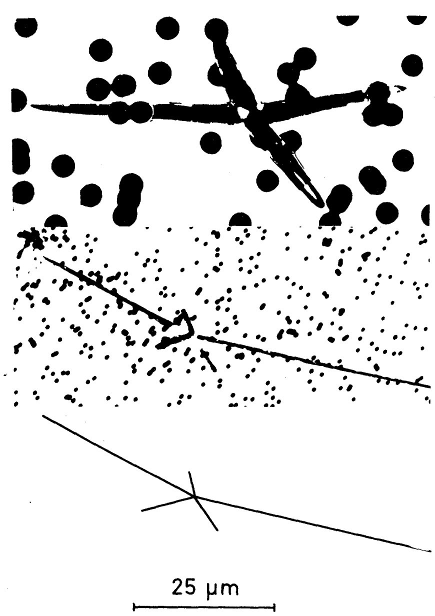

FIG. 1. Microphotograph showing a four- and a five-pronged event in 1785-MeV U+U in glass and mica track detectors, respectively, as seen in the plane perpendicular to the beam. The dots are due to the incident projectiles.

four-pronged events of reaction I and 13 four-pronged events of reaction II have been analyzed in detail. On the basis of the measured track lengths l_i and track directions $\vec{e}_i$, the correlated fragment masses m_i and velocities $\vec{v}_i$ from an individual interaction are calculated under the assumption of conservation of momentum and mass. For this purpose, we solved event by event the coupled equations

$$\sum_i m_i v_i(m_i, l_i)\vec{e}_i = \vec{p}_{in}, \quad \sum_i m_i = m_p + m_t, \qquad (1)$$

where $\vec{p}_{in}$ denotes the incident (linear) momentum and m_p and m_t are the masses of the projectile and the target, respectively. $v(m, l)$ is an empirical velocity-range relation with analytical structure

$$v(m, l) = \sum_{\mu=0}^{2} \sum_{\nu=0}^{4} c_{\mu\nu} m^{\mu} l^{\nu}. \qquad (2)$$

The coefficients $c_{\mu\nu}$ have been found by independent calibration experiments. Obviously, there is no simple relation between track lengths and fragment masses.

The ability of the present method to reproduce known mass and energy distributions has been checked in the case of thermal-neutron–induced fission of U_{nat} using the 4π-geometry technique.[9] The distributions are in agreement with radiochemical or time-of-flight data, allowing for a resolution of 10% for masses $A \sim 100$ amu and energies $E \sim 180$ MeV. Furthermore, the second equation of (1) is redundant for two- and three-pronged events. The conservation of total mass can thus also be used as a test of the accuracy of the present technique to convert track data into masses and energies. In this way the standard deviations in the computed individual masses of reaction I have been determined to be $\Delta m = 13$ amu and $\Delta m = 20$ amu for two- and three-pronged events, respectively. Guided by these results and the number of degrees of freedom, we expect $\Delta m = 24$ amu in the case of four-pronged events.

For some events the error in the computed total masses was significantly larger and far beyond 2 standard deviations. These events are generally characterized by one very short track which was observed at angles $\vartheta_{lab} \geqslant 70°$. Such events are rejected in the subsequent detailed analysis. From the initial sample of 193 four-pronged events in reaction I a subset of 137 events remained.

Evidence for the sequential nature of the investigated multibody events was obtained from the study of kinematical correlations between the heavy reaction products. The relative velocities $v_{ij} = |\vec{v}_i - \vec{v}_j|$ of two fragments in reaction I are shown plotted in Fig. 2 against the relative velocities v_{kl} of the other two complementary fragments. One cluster is evident at $v_{ij} = v_{kl} \simeq (0.26$ fm)/(10^{-23} s). Here, the four fragments (full dots) are grouped together in two pairs where each pair yields very similar relative velocities as expected from ordinary fission. No four-pronged event was found which could not be grouped into two such pairs. The two alternative combinations (open dots) of grouping the four fragments into two pairs results in very different and unphysical values of v_{ij}. The groups of full and open dots are well separated. These facts lend strong support to the sequential nature of the investigated events. Similar results have been obtained for reaction II.

From the pairs of correlated fission fragments according to the clustering seen in Fig. 2 and from the individually computed masses the reaction products prior to fission are known. Thus, the reaction plane is easily reconstructed on an event-by-event basis by the beam and the two prefission fragments. Some results concerning the mass and angular distributions of the reaction products of the first and second steps of reaction I may be summarized as follows.

The distribution of the scattering angles of the prefission reaction products integrated over mass and energy peaks at $\vartheta_{lab} = 45°$ with a full width at half maximum (FWHM) of 20°. The distribution of the prefission masses integrated

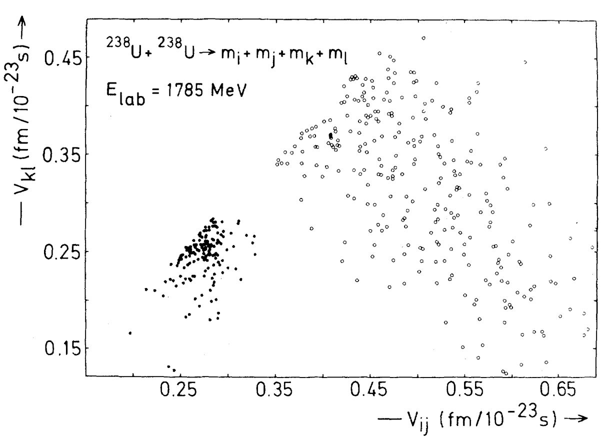

FIG. 2. Relative velocities $v_{ij} = |\vec{v}_i - \vec{v}_j|$ of two fragments for 1785-MeV U+U→$m_i m_j m_k m_l$ plotted against the relative velocities of the other two complementary fragments. All three combinatorial possibilities $v_{ij} \leq v_{kl}$ ($i, j, k, l = 1, 2, 3, 4$) are shown.

over energies and angles is Gaussian around $A = 238$. The extent to which the rather broad distribution having an FWHM of 55 amu is broadened by the resolution has not yet been analyzed.

In the intrinsic system of the fissioning primary reaction products the angular distribution of the final fragments integrated over mass and energy is constant in the reaction plane after appropriate corrections have been made for detection inefficiencies near scattering angles $\vartheta_{lab} = 0°$ and $\vartheta_{lab} = 90°$ [Fig. 3(a)]. In particular, no enhancement[4] in the direction of the recoiling prefission fragment has been found within the present statistics. Rather, two minima in the distribution at $\varphi_{in-plane} \simeq \pm 45°$ with respect to the recoil direction (lab) of the primary fragment can be accounted for by the missing cross section due to events which have been registered but eliminated from the quantitative analysis.[9]

The fragment angular distribution integrated over mass and energy out of the reaction plane is highly anisotropic. As shown in Fig. 3(b) this distribution is focused in the reaction plane. The missing events manifest themselves in a dip at out-of-plane angles $\psi = 90°$. Allowing for this inefficiency (~ 30%) the distribution of the intrinsic spin J of the primary fragments can be estimated to be $W(J) \propto (2J+1)\exp[-(J-J_0)^2/2\sigma_J^2]$ with a peak value $J_0 = 30\hbar$ and a width σ_J less than $10\hbar$. This assumes $0 \leq J \leq 79\hbar$, a maximal polarization ($M = J$) of the primary fragments along a z direction perpendicular to the reaction plane [e.g., formula (12) of Ref. 8], and a Gaussian population of K states with a width $K_0(J,E)$ according to the rotating charged-liquid-drop model.[10] In the calculation we furthermore assumed fixed excitation energies $E = 40$ MeV and equal masses $A = 238$ for the prefission fragments and approximated the level density parameter by $a = A/10$.

Further evidence of the sequential nature of the four-body channel is provided by the apparent lack of nontrivial correlations between the out-of-plane angles for both fission-fragment pairs as shown in Fig. 3(c). The distribution is consistent with the product of both angular distributions, indicating the independent decay of both primary fragments.

For calculating five masses in the case of a five-body final channel, the four equations (1) are not sufficient. As an additional constraint we fix the sum of the masses of two arbitrary fragments $m_{ij} = m_i + m_j$. Varying m_{ij} and the ten possible ways of grouping the fragments into pairs

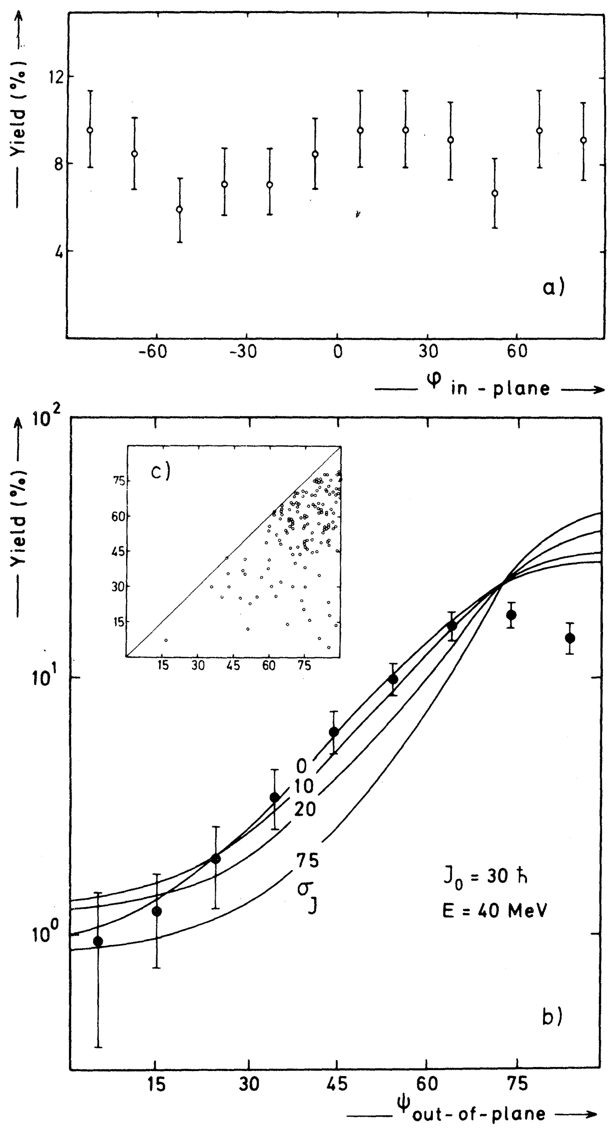

FIG. 3. Fission-fragment angular distributions for (1785 MeV) $U+U \rightarrow m_1m_2m_3m_4$ measured (a) in and (b) out of the reaction plane. $\psi=0$ and $\varphi=0$, respectively, define the normal to the reaction plane and the recoil direction (lab) of the prefission fragments. The family of solid curves is explained in the text. The distribution of the out-of-plane angles [$\psi(34)$, the ordinate] of one fission-fragment pair versus the other [$\psi(12)$, the abscissa] is shown in (c).

$(ij)(klm)$ three solutions could be found for the five-pronged event in reaction I. All of these three solutions support the view of a multiple sequential decay scheme where the event is the result of three distinct reaction steps. The decay chain best in accordance with empirical fission Q values is $U+U \rightarrow 218^* + 258 \rightarrow (75+143) + (104 + 154^*)$, $154^* \rightarrow 120 + 34$. The Q values deduced for the intermediate fragments $A = 218$ and $A = 154$ are $Q_f = 169$ MeV and $Q_f = 84$ MeV, respectively, which agree reasonably well with those expected from ordinary fission.[11] The other two combinations differ from the scheme described above in that the tracks are grouped into different sequential decay sequences with slightly different intermediate masses. But the resulting final masses are very similar.

The analysis indicates an asymmetric mass division even for the light systems. However, the masses obtained are also compatible with a broad symmetric distribution, broadened additionally by our resolution. Varying the track data within limits expected from uncertaincies in measuring lengths and angles, the deduced individual masses varied within ± 15 mass units. The multiple sequential picture, however, is not affected by this change in the input quantities.

In conclusion, we have found direct experimental evidence which strongly supports the sequential picture for the bulk of the data, excluding therefore fast fission processes such as, e.g., Coulomb fission. There is no need to assume a different reaction mechanism for the eliminated events either, since these events show up as missing cross sections in the angular distributions. Obviously, other reaction mechanisms cannot be excluded on the basis of a cross section of a few millibarns or less.

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Effects of Spin-Orbit Deformation in Inelastic Scattering at 0.8 GeV

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New differential cross section and analyzing-power data for 800-MeV $p_{\mathrm{pol}}+{}^{12}\mathrm{C}, {}^{116,124}\mathrm{Sn}$ inelastic scattering to the first 2^+ states are presented. A distorted-wave Born-approximation analysis which utilizes collective form factors and includes deformation of the spin-orbit potential is shown to provide a reasonable description of the analyzing-power data.

Distorted-wave Born-approximation (DWBA)[1] analyses of some of the available medium-energy (~ 1 GeV) proton-nucleus inelastic differential cross-section data were found generally to give good overall agreement for both shapes and magnitudes of the cross sections, using *spin-independent* collective form factors and deformation lengths consistent with averages of results of many low-energy determinations.[2-4] A recent investigation[5] of the sensitivity of the calculations to a spin-orbit contribution to the macroscopic collective form factor suggested considerable sensitivity of the predicted analyzing powers to deformation of the spin-orbit potential, but unfortunately, no inelastic analyzing-power data were available for comparison with the predic-