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Current-Algebra Calculation of the Decay $\tau \to 3\pi\nu$

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A fit to the integrated rate and 3π mass distribution of the decay mode $\tau^+ \to \pi^+\pi^+\pi^-\bar{\nu}_\tau$ has been performed using hard-pion current algebra, including the approximate imposition of unitarity. We find that the existence of an A_1 with low mass ($m_A = 1.1$ GeV) and narrow width $[\Gamma_A^{\text{tot}} = 150$ MeV, $\Gamma(A_1 \to \rho\pi) = 100$ MeV$]$ is consistent with the data if there is an appreciable non-$\rho\pi$ contribution to the matrix element $\langle 3\pi | A_\mu | 0\rangle$. Such a contact term is predicted to exist by current algebra.

The recent observation[1,2] of the decay $\tau^+ \to \pi^+\pi^+\pi^-\bar{\nu}_\tau$ and measurement of the resulting 3π mass distribution provides an ideal laboratory for study of the axial-vector matrix element $\langle 3\pi | A_\mu | 0\rangle$ for timelike (momentum)^2. Because the theoretical calculation[3] of this matrix element is based on detailed application of current algebra and partial conservation of axial-vector current away from the soft-pion limit, and depends sensitively on the parameters of the A_1 meson, one may ostensibly confront these calculations with experimental data and hope to learn more about hard-pion current algebra (HPCA) and the A_1 meson. There are caveats to this goal: The experimental situation is unclear because of poor statistics, and also, there are ambiguities in the theoretical calculation of the matrix element. Both of these problems will be clarified in the following text, where we sketch our calculation, leaving details to a forthcoming publication; the principal results are summarized in the concluding section.

The matrix element for the decay $\tau^+ \to \pi^+\pi^+\pi^-\bar{\nu}_\tau$ is given by

$$\mathfrak{M} = (G_F \cos\theta_c/\sqrt{2}) L_\mu H^\mu, \tag{1}$$

where $L_\mu = \bar{u}_\nu \gamma_\mu (1-\gamma_5) u_\tau$ and $H_\mu = \langle \pi^+\pi^+\pi^- | (A_\mu^1 + iA_\mu^2) | 0\rangle$. The differential decay rate $d\Gamma/dM_{3\pi}$ may then be written

$$\frac{d\Gamma}{d\sqrt{s}} = 2\sqrt{s}\left(\frac{G_F\cos\theta_c}{\sqrt{2}}\right)^2 \frac{(m_\tau^2 - s)^2 \pi^2}{m_\tau^3} \times \left\{ \frac{1}{2!} \int \prod_i \left[\frac{d^3k_i}{(2\pi)^3 2k_{i0}}\right] \delta^4(p - \sum k_i) \left[\frac{1}{3}\left(2 + \frac{m_\tau^2}{s}\right) H_\mu^* \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{s}\right) H^\nu + \frac{m_\tau^2 (H\cdot p)^2}{s^2}\right]\right\}, \tag{2}$$

where $s = p^2 = M_{3\pi}^2$. The first term in (2) gives the decay rate into a 1^+ final state, and the second into a 0^-.

$\rho^0\pi^+$ contribution.—In this first stage of the calculation, we shall assume that the final state is entirely $\rho^0\pi^+$. The helicity amplitude defined in a Cartesian isospin basis, i.e.,

$$F_\mu^{(\lambda)} \epsilon^{Aab} = \langle \rho^a(k,\lambda)\pi^b(q) | A_\mu^A(0) | 0\rangle, \tag{3}$$

has a general expression

$$F_\mu^{(\lambda)} = F_0\epsilon_\mu{}^\rho(\lambda) + F_1 p\cdot\epsilon^\rho(\lambda)(k-q)_\mu + F_2 p\cdot\epsilon^\rho(\lambda)p_\mu, \quad (4)$$

where $p = k+q$. An expression analogous to (2) for $\tau^+ \to \rho^0\pi^+\bar\nu_\tau$ may then be written in terms of the F_i, and it is these amplitudes which are directly calculable from various models of HPCA.[4-7] The predictions of the various models are essentially the same for the amplitudes F_1 and F_2. However, in the dispersive approach[6,7]

$$F_0 = B + R/D_A, \quad (5)$$

while in the Lagrangian[5] and Ward-identity[4] approaches

$$F_0 = [B(m_A{}^2 - s) + R]/D_A. \quad (6)$$

In the above equations, B and R are constants, and $D_A = m_A{}^2 - s - i\Gamma_A m_A$, with m_A the mass and Γ_A the *total* width of the A_1.

The choice of F_0 is crucial, because it gives the overwhelmingly dominant contribution to the decay rate. While the two forms, Eqs. (5) and (6), are identical for $\Gamma_A = 0$, there are substantial differences in the region $s = m_A + \Gamma_A$ for $\Gamma_A \neq 0$, *even for a narrow* A_1 ($\Gamma_A \approx 100$ MeV)! On the basis of unitarity, we have chosen F_0 to be given by Eq. (6). More precisely, the linear elastic *unitarity* relation for the S- and D-wave amplitudes $\mathfrak{F}_L$ corresponding to Eq. (4), i.e.,

$$\mathrm{Im}\mathfrak{F}_L = \sum_{L'} T_{L'L}{}^*\rho\mathfrak{F}_{L'}, \quad (7)$$

is satisfied to better than 95% for the dominant 1^+ S-wave amplitude. We obtain this result using a factorizable ρ-π scattering amplitude $T_{L'L} = (a_L a_{L'}/\rho)/D_A$, and HPCA parameters in the range of interest.[8] The 1^+ D-wave and 0^- P-wave amplitudes violate unitarity, but this fact is of little import since they contribute less than 5% of the decay rate at the $M_{3\pi}$'s under consideration. On the other hand, for the dispersive form, Eq. (5), there are 100% violations of unitarity by the dominant 1^+ S-wave amplitude near the resonance even for a narrow A_1. Furthermore, in the presence of inelastic channels (e.g., $\epsilon\pi$), the form (6) for F_0 is at least consistent with unitarity, whereas (5) gives a substantial imaginary part for $T^\dagger\rho\mathfrak{F}$ in the vicinity of the resonance, in violation of unitarity.

Thus, we obtain F_0, F_1, and F_2 from Refs. 4 or 5. These amplitudes depend on the parameters g_ρ and g_A (defined by $\langle\rho^a|V_\mu{}^b|0\rangle = g_\rho\delta^{ab}\epsilon_\mu{}^\rho$, $\langle A_1{}^a|A_\mu{}^b|0\rangle = g_A\delta^{ab}\epsilon_\mu{}^A$); λ_A is the anomalous moment of the A_1, defined in Ref. 5 (and related to δ of Ref. 4 by $\lambda_A = 1+\delta$); and m_A and Γ_A which have been defined earlier. The first Weinberg sum rule[9] is used to express $g_A{}^2$ in terms of $g_\rho{}^2$, $m_\rho{}^2$, and $F_\pi{}^2$, and $g_\rho{}^2$ is written as $g_\rho{}^2 = z^2(2m_\rho{}^2F_\pi{}^2)$. Experimentally, $z^2 = 1.3 \pm 0.2$. Finally, the ρ width can be calculated in terms of all these parameters, and its known value places a constraint on λ_A, z^2, and m_A. The amplitudes F_0, F_1, and F_2 then depend on the parameters z^2, m_A, and Γ_A, while $\Gamma(A_1\to\rho\pi)$ depends only on z^2 and m_A. As an example, if $m_A = 1.1$ GeV, then $\Gamma(A_1\to\rho\pi)$ is *constrained* to lie between 70 and 130 MeV (for $1.1 \lesssim z^2 \lesssim 1.5$).

Comparison with data.—We proceed as follows: *First*, we compare[10] the integrated branching ratio with respect to leptonic modes,

$$R \equiv \left[\int_{m_\rho+m_\pi}^{m_\tau} d\sqrt{s}\; d\Gamma_{\tau^+\to\rho^0\pi^+\bar\nu_\tau}/d\sqrt{s}\right]/\Gamma_{\tau^+\to l^+\nu_l\bar\nu_\tau}, \quad (8)$$

with the experimental value $R = 0.26 \pm 0.20$ ($M_{3\pi} \gtrsim 0.95$ GeV) obtained from the data of Jaros *et al.*[1] *Next*, for acceptable values of R, the theoretical mass distribution with an arbitrary normalization constant as a fitting parameter is fitted (in a χ^2 sense) to the experimental 3π mass distribution above $\rho\pi$ threshold. We use the data of Jaros *et al.* because it has error bars on the points, and does *not* include ρ mass cuts on the data. The latter aspect will be useful in our later discussion of the role of background.

Figure 1, curve *a*, shows the best fit for $\Gamma(A_1 \to \rho\pi) = \Gamma_A = 100$ MeV (corresponding to $z^2 = 1.3$). The value for R in this case is 0.31, which is acceptable, but the mass distribution is obviously much too low on the high side of resonance. Varying z^2 between the acceptable limits does little to better the fit, which has χ^2 per degree of freedom = 13.0/7. Thus, we conclude that although the integrated decay rate is acceptable, the mass distribution is not.

From a χ^2 point of view, the situation actually improves as m_A increases, but the mass distribution above resonance is still not acceptable. Best fits for m_A = 1.2, 1.3, and 1.4 GeV are shown in Fig. 2.[11] The widths for $A_1\to\rho\pi$ in these cases are 200, 350, and 500 MeV, respectively, corresponding to $z^2 = 1.3$. The values of R are acceptable, ranging from 0.25 to 0.18.

Background contributions.—In spite of the shortcomings of the fit in Fig. 1, curve *a*, we do *not* conclude that the 1.1-GeV A_1 implies a conflict between τ decay data and HPCA. The

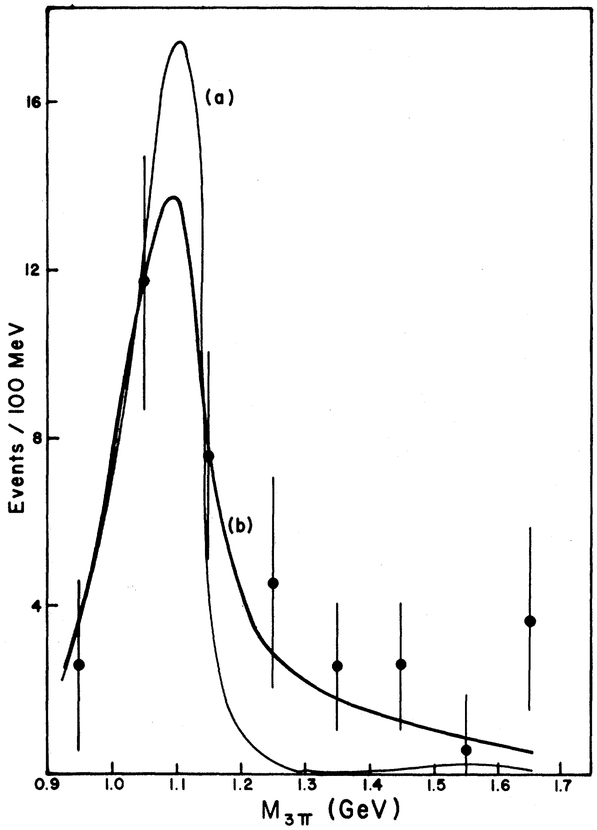

FIG. 1. Curve a, best fit to 3π mass distribution for case $m_A = 1.1$ GeV, $\Gamma_A = \Gamma(A_1 \to \rho\pi) = 100$ MeV, no background; $R = 0.31$. Curve b, best fit for case $m_A = 1.1$ GeV, $\Gamma_A = 150$ MeV, $\Gamma(A_1 \to \rho\pi) = 100$ MeV, with non-$\rho\pi$ background; $R = 0.33$.

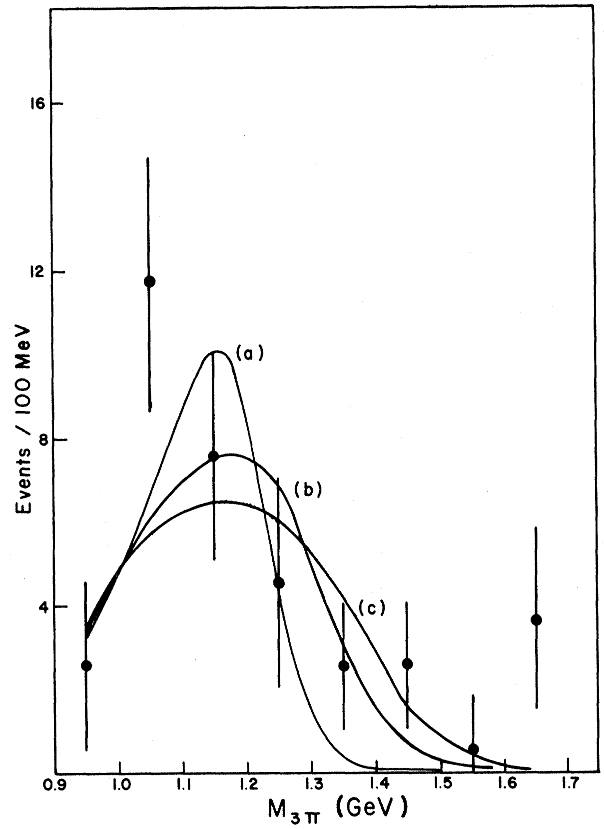

FIG. 2. Best fits for curve a, $m_A = 1.2$ GeV, $\Gamma_A = \Gamma(A_1 \to \rho\pi) = 200$ MeV, $R = 0.26$. Curve b, $m_A = 1.3$ GeV, $\Gamma_A = \Gamma(A_1 \to \rho\pi) = 350$ MeV, $R = 0.22$. Curve c, $m_A = 1.4$ GeV, $\Gamma_A = \Gamma(A_1 \to \rho\pi) = 500$ MeV, $R = 0.18$.

reason is that both the Ward-identity and Lagrangian approaches *require* the existence in the four-point function[12,13] of a contact term

$$\langle \pi^a(k_1)\pi^b(k_2)\pi^c(k_3)|A_\mu{}^A|0\rangle = g_A D_A{}^{-1}\Big(\{[a + b(s - m_A{}^2)]k_{1\mu} + c(k_1\cdot k_2 k_{3\mu} + k_1\cdot k_3 k_{2\mu})\}\delta^{Aa}\delta^{bc} + \text{cyclic}\Big). \quad (9)$$

This leads to terms in the decay rate proportional to three-particle phase space, and thus contribute to the high-mass tail in Fig. 1. The coefficients a, b, c in (9) are unfortunately only partly determined by HPCA, because they depend sensitively on undetermined on- and off-shell $A_1\pi\epsilon$ and $\pi\pi\epsilon$ couplings. Thus, we have approached the problem in the following phenomenological manner: (1) Set $c = 0$. (2) Fixed a and b by requiring (i) an acceptable value of R, (ii) a reasonable fit to the 3π mass distribution, and (iii) a non-$\rho\pi$ contribution to the A_1 width of, for example, $\lesssim 50$ MeV for $m_A = 1.1$ GeV. A fit along these lines is shown in Fig. 1, curve b.[14] The parameters are $m_A = 1.1$ GeV, $\Gamma_A = 150$ MeV, $\Gamma(A_1 \to \rho\pi) = 100$ MeV, $a = 168$ GeV^{-1}, $b = 306$ GeV^{-3}. The values of a and b are in line with the scale ($a \sim b m_A{}^2 \sim m_A{}^2/g_A F_\pi \approx 100$ GeV^{-1}) which is typical of the terms which *are* calculable from the four-point $1PI$ functions in Refs. 12 and 13. The total value of R that we obtain is 0.33, with 0.18 coming from the $\rho\pi$ channel, and 0.15 coming from the background.

Our conclusions are the following:

(1) Because of the small width required by HPCA, a fit to the data is not likely if the A_1 has mass 1.1 GeV, and the final state consists entirely of $\rho\pi$ with no background.

(2) However, a fit *consistent* with the full content of HPCA for the four-point function $\langle 3\pi|A_\mu|0\rangle$ *is* obtainable with a narrow A_1; i.e., $m_A = 1.1$ GeV, $\Gamma_A = 150$ MeV, $\Gamma(A_1 \to \rho\pi) = 100$ MeV. The background term can be thought of as simulating

a broad $\epsilon\pi$ final state, as well as non-A_1 contributions [proportional to the coefficient b in Eq. (9)]. Our background term is a small perturbation (15%) on the $\rho\pi$ term in the region $\sqrt{s} < 1.15$ GeV, but dominates for $\sqrt{s} \gtrsim 1.25$ GeV.

After, a $\rho\pi$ cut, Alexander *et al.*[2] find the region $1.2 < M_{3\pi}/\text{GeV} < 1.5$ to be populated with $\pi^+\pi^+\pi^-$ events at a density of about 1 event/0.1 GeV. This number is claimed to be consistent with the estimated background due to electron misidentification. Our background term contributes about 2 events/0.1 GeV in this region (see Fig. 1, curve b), of which, about *half* contain $\pi^+\pi^-$ pairs which *do*, in fact, populate the ρ mass bands. Hence we make a *prediction* of about 1 event/0.1 GeV in this region which will *not* fall in the ρ mass bands; within statistics, this is consistent with the data.

(3) If indeed there is a narrow 1.1-GeV A_1, one must reconcile this result with the absence of such a resonance in partial-wave analyses (PWA) of the 3π final states produced in diffractive and charge-exchange hadronic interactions. In diffractive production we can only guess that the complicated interplay between Deck background and resonance production prevents the extraction of the resonance parameters in a PWA (with present statistics). The absence of an A_1 signal in forward charge-exchange reactions is more mysterious because of the negligible background. Here, it has been suggested[15] that the amplitude is suppressed in the forward direction because of a zero in the $\pi\rho A_1$ vertex function at $k_\rho = 0$. Finally, a narrow 1.1-GeV 3π peak is seen in backward K^-p and π^-p reactions,[16] and it *is* presumably the A_1.

(4) In a previous analysis of diffractive data[17] one of us (R.A.) obtains a wide (~500 MeV) $J^P = 1^+$ resonance near 1500 MeV. The presence of such a structure does not vitiate the present work, because our procedure of expanding vertex functions to second order in momenta includes the effects of such *gentle*, high-mass singularities in the first term in Eq. (6) for the amplitude F_0 (corresponding to a subtraction in a dispersive approach), whose normalization is fixed by HPCA.

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[1]J. A. Jaros *et al.*, Phys. Rev. Lett. 40, 1120 (1978).

[2]G. Alexander *et al.*, Phys. Lett. 73B, 99 (1978).

[3]A hard-pion calculation has been performed by D. A. Geffen and Warren J. Wilson, to be published. Our calculation differs from this one in some crucial aspects: (i) the imposition of constraints placed on $\Gamma(A_1 \to \rho\pi)$ by the full content of the hard-pion current algebra, including the first Weinberg sum rule [S. Weinberg, Phys. Rev. Lett. 18, 507 (1967)], which results from the c-number nature of various Schwinger terms; (ii) the imposition of approximate unitarity; and (iii) most importantly, the inclusion of the effects of a non-$\rho\pi$ background, whose existence is predicted by current algebra. The latter will make possible a fit with narrow A_1.

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[5]R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. 174, 1999, 2008 (1968).

[6]S. G. Brown and G. B. West, Phys. Rev. Lett. 19, 812 (1967), and Phys. Rev. 168, 1605 (1968).

[7]D. A. Geffen, Phys. Rev. Lett. 19, 770 (1967).

[8]Unitarity also requires $\Gamma_A \propto \rho \propto k$ in the $\rho\pi$ threshold region, but we ignore this complication, and take $\Gamma_A = \text{const}$.

[9]Weinberg, Ref. 3.

[10]Using $m_\tau = 1.782$ GeV [J. Kirkby, SLAC Report No. SLAC-PUB-2127, 1978 (unpublished)].

[11]For large m_A, $F_1, F_2 \to 0$ and $F_0 \to g_\rho/F_\pi$, which is the soft-pion result.

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[14]The contact amplitude [in which we include pieces $\propto (s' - m_\rho^2)/(s' - m_\rho^2 + i m_\rho \Gamma_\rho)$, $s' = (p_{\pi^+} + p_{\pi^-})^2$] and the piece of the $\langle 3\pi | A_\mu | 0\rangle$ matrix element containing a ρ pole interfere to a negligible degree on summing, squaring, and integrating over phase space. The unitarity corrections will not be large because the ρ-pole and contact terms dominate in different regions of s.

[15]G. L. Kane, private communication; G. Preparata, private communication.

[16]Ph. Gavillet *et al.*, Phys. Lett. 69B, 119 (1977); A. Ferrer *et al.*, Phys. Lett. 74B, 287 (1978).

[17]R. S. Longacre and R. Aaron, Phys. Rev. Lett. 38, 1509 (1977).