than that perturbative QCD correctly describes dilepton production is that $q\bar{q}$ annihilation dominates. This can be tested: Gluon-quark scattering increases the cross section at large $q_\perp$ above that given by Eqs. (2), (4), and (5). But note that intrinsic transverse momentum for the partons tends to increase $q_\perp$ by a few hundred MeV.

Equation (6) implies Eq. (4.1) of Ref. 7 when $q_\perp^2 \ll q^2$, but is more general. No integration over $q_\perp$ is needed and a value for A_2 is obtained. Unlike the result in Ref. 15 for A_2, no assumption about the parton distributions is needed. The present proof is on more solid ground than in Refs. 7 and 15. The reason that the new result implies the old one may be that in time-ordered perturbation theory in the center-of-mass frame the energy denominator is of order $q_\perp$ for annihilation but of order q for bremsstrahlung.[16]

A similar remark should apply to quark-gluon scattering, of which a calculation will be made.

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How to Include Parton Transverse Momentum in Quantum Chromodynamics

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I show that parton transverse-momentum effects can be included in factorized quantum-chromodynamics inclusive cross sections, but that factorization is maintained only if the transverse-momentum variable which is held fixed is independent of the scale of the parton momentum. I also emphasized that other nonperturbative effects can be just as important as the transverse momentum.

Despite several recent proofs[1,2] and explanations,[1] the factorization of inclusive cross sections in quantum chromodynamics (QCD) is still sometimes misunderstood. A particularly sticky point concerns the question of how to introduce a phenomenological transverse-momentum distribution of partons within hadrons. This question is of some practical importance because fits to data in hadron-hadron scattering into high-transverse-momentum final states at presently accessible energies depend sensitively on the assumed form of these nonperturbative effects.[3]

In fact, there are two distinct questions: (1) Is it possible to include transverse-momentum distributions in a way which is consistent with the factorization formalism and, if so, how? (2) Is it physically sensible to do so? In this note, I will answer the first question by giving a self-consistent algorithm for calculating transverse-momentum effects. But I will also emphasize that the answer to the second question is nontrivial and may depend on the process to be studied.

The result of the factorization analysis for an incoming parton can be written as follows:

$$d\sigma_j(p) = \sum_k \int d\xi\, d\tilde{\sigma}_k(\xi p, M)\, \Gamma_{kj}(\xi, M), \tag{1}$$

where $d\sigma_j(p)$ is the perturbative QCD differential cross section for some inclusive process involving an incoming j-type parton (j is a quark, antiquark, or gluon) with momentum p^μ (for simplicity, I ignore quark masses and take $p^2=0$). $d\sigma_j(p)$ is divergent as p^2 goes to zero, but the import of Eq. (1) is that it can be factored into the convolution over a single variable, ξ, of a finite effective cross-section $d\tilde{\sigma}$ with a (matrix) function Γ which contains all the mass singularities.[1,2] Both $d\tilde{\sigma}$ and Γ depend on the renormalization mass M which is necessary not only to renormalize the QCD coupling constant but also to implement the factorization of the logarithmic divergences. Since M is completely arbitrary (the moments of) $d\tilde{\sigma}$ and Γ satisfy renormalization-group equations.

To make contact with hadron scattering, some assumption must be made about how the partons studied in perturbation theory are related to hadrons. The most general assumption consistent with the impulse approximation is that the inclusive cross section for hadrons of momentum P (neglecting effects of the hadron mass as well as quark-mass effects[4,5]) is

$$d\sigma_H(P) = \sum_j \int d^4p\, d\sigma_j(p)\, \mathfrak{F}_j(p, P), \tag{2}$$

where $\mathfrak{F}_j(p,P)$ describes the probability of finding a j-type parton of momentum p in the hadron.[6] If the transverse momentum of the partons can be neglected, then

$$\mathfrak{F}_j(p,P) = \int dy\, f_j(y)\, \delta^{(4)}(p - yP). \tag{3}$$

Then the d^4p integration in Eq. (2) is trivial, and I can write

$$\begin{aligned} d\sigma_H(P) &= \sum_j \int dy\, d\sigma_j(yP) f_j(y) \\ &= \sum_j \int d\xi\, d\tilde{\sigma}_j(\xi P, M) \tilde{f}_j(\xi, M), \end{aligned} \tag{4}$$

where

$$\tilde{f}_j(\xi, M) = \sum_j \int \frac{dy}{y} \Gamma_{jk}\left(\frac{\xi}{y}, M\right) f_k(y). \tag{5}$$

$\tilde{f}_j$ are the "renormalized" distribution functions which are actually measured in deep-inelastic scattering experiments. The moments of $\tilde{f}_j(\xi, M)$ obey the standard renormalization-group equations.

An analogous analysis is possible even if the transverse momentum is not zero. To define a measure of the transverse momentum, introduce another lightlike vector m^μ (for example, in $P + P \to \mu^+ + \mu^- +$ anything, m^μ should be chosen to be the momentum of the other incoming hadron; in electroproduction, $m^\mu = q^\mu - xP^\mu$ where q^μ is the virtual-photon momentum and $x = -q^2/2Pq$). Then define

$$p^\mu = y(P^\mu + \alpha m^\mu + \rho_T{}^\mu), \tag{6}$$

where ρ_T is a spacelike vector such that $\rho_T P = \rho_T m = 0$.

For $p^2 = 0$, if we write

$$\mathfrak{F}_j(p,P) = 2y^{-1}\delta(p^2) F_j(y, \vec{\rho}_T), \tag{7}$$

then Eq. (2) becomes

$$\begin{aligned} d\sigma_H(P) &= \sum_j \int dy\, d^2\rho_T\, d\sigma_j\left(y\left(P + \frac{\vec{\rho}_T{}^2}{2mP}\right)\right) F_j(y, \rho_T) \\ &= \sum_j \int d\xi\, d^2\rho_T\, d\tilde{\sigma}_j\left(\xi\left(P + \frac{\vec{\rho}_T{}^2}{2mP} m + \rho_T\right), M\right) \tilde{F}_j(\xi, \rho_I, M), \end{aligned} \tag{8}$$

where

$$\tilde{F}_j(\xi, \rho_T, M) = \sum_k \int \frac{dy}{y} \Gamma_{jk}\left(\frac{\xi}{y}, M\right) F_k(y, \rho_T). \tag{9}$$

Hopefully, the role of the ρ_T variable is clear to the reader. Because ρ_T is an "angular" measure of transverse momentum (in the sense that it does not depend on the scale of the parton momentum p^μ), it can be held fixed during the rescaling of the parton momentum which is essential to factorization. Thus the ξ moments of $\tilde{F}_j(\xi,\rho_T,M)$ at fixed ρ_T satisfy the same renormalization-group equations as the moments of $\tilde{f}_j(\xi,M)$. In particular, it is consistent with the factorization analysis (though not obviously the best thing to do phenomenologically) to write $\tilde{F}$ as a product

$$\tilde{F}_j(\xi,\rho_T,M)=\tilde{f}_j(\xi,M)g(\rho_T), \qquad (10)$$

or as a sum of such products. This form is very convenient for actual calculations.

On the other hand, the actual transverse momentum of the parton in Eq. (8), $\vec{p}_T=\xi\vec{\rho}_T$, is a very complicated variable from the point of view of the factorization analysis. The moments of the distribution functions F_j at fixed p_T do not factor and the moments of the renormalized distribution function $\tilde{F}_j$ do not satisfy simple renormalization-group equations, all because p_T scales with p^μ and interferes with factorization.

Note that it would be wrong to make an artificial separation of $d\tilde{\sigma}$ into a zeroth-order ("naive parton") piece and higher-order contributions and then treat the two differently in transverse momentum. The parton model only makes sense because it is possible to define the $d\tilde{\sigma}$ which goes with the finite renormalized distribution functions. There is nothing special about the zeroth-order part of $d\tilde{\sigma}$ except that it is sometimes bigger than the higher-order contributions.

I have now answered the first question posed at the beginning of this note. Before going on to the second, I will review the answer once more. The crucial formula is Eq. (8) in which transverse-momentum dependence is introduced in the standard formalism in terms of a variable ρ_T which is independent of rescalings of the parton momentum. Equation (9) shows that the transverse-momentum–dependent distribution functions $\tilde{F}_j(\xi,\rho_T,M)$ have the standard renormalization-group behavior at fixed ρ_T. And Eq. (10) gives a convenient form for actual calculations.

The answer to the second question posed above involves not formalism but physics. The important thing to remember is that the effect of a finite ρ_T distribution is not the only nonperturbative effect left out in the standard factorization analysis. Other effects which can be equally important include soft initial- or final-state interactions and corrections because the bound partons are off shell. These effects are more difficult to model than the ρ_T dependence of Eqs. (8)–(10). And what is worse, the effects of initial- and final-state interactions are obviously process dependent. They will not be the same, for example, in electroproduction and $P+P\rightarrow\mu^++\mu^-$ + anyting.

In electroproduction, the relative magnitude of the final-state–interaction and transverse-momentum effects can be inferred from an analysis of $1/Q_2^2$ terms in scaling violation.[7] This phenomenological analysis suggests that final-state–interaction effects are slightly larger than transverse-momentum effects and go in the opposite direction (to increase the inclusive cross section at large x). Clearly final-state interactions cannot be neglected. It seems reasonable to speculate that one reason why nonperturbative effects appear to be larger in processes involving hadron-hadron scattering into high–transverse-momentum final states than in electroproduction is that the various nonperturbative effects do not tend to cancel in the high-p_T processes as they do in electroproduction. Either the initial-state interactions are unimportant or they reinforce the effect of nontrivial ρ_T distributions.

It is clear from this discussion that to extract the full predictive power of perturbative QCD, one must try to model or fit a variety of nonperturbative effects, not just $p_T\neq 0$, or try to find measurable quantities in which nonperturbative effects are small. Transverse-momentum effects summarize important physics, but they do not tell the whole story.

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Resonance Properties in Quantum Chromodynamics

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Quantum-chromodynamic sum rules are derived which are sensitive to resonance contributions and in fact allow one to compute leptonic decay widths and masses of low-lying resonances. The crucial point is the inclusion of power corrections which are related to the vacuum structure of quantum chromodynamics.

The nonperturbative effects of quantum chromodynamics (QCD) seem to be crucial for understanding quark confinement.[1,2] Here we will study phenomenological implications of the nonperturbative terms in QCD for resonance physics.

We will approach resonances from short distances at which QCD becomes especially simple because of asymptotic freedom.[3] The basic idea is that nonperturbative effects induce power corrections which violate asymptotic freedom if one tries to extend QCD to larger distances. This breaking is correlated with the appearance of resonances which bring structure into the otherwise smooth quark cross sections. Phenomenologically power corrections are introduced via vacuum expectation values such as

$$\langle 0|\bar{\psi}\psi|0\rangle, \quad \langle 0|G_{\mu\nu}{}^a G_{\mu\nu}{}^a|0\rangle, \tag{1}$$

where ψ is the quark field and $G_{\mu\nu}{}^a$ is the gluon-field–strength tensor.

Following this line of reasoning, we obtain constraints on the resonance properties which in fact amount to computing the ρ-meson electronic width and mass, the $\pi\to\mu\nu$ decay constant, and so on. We will concentrate here on the π-ρ-A_1 system just because there are classical papers on the subject[4] and it is instructive to compare new results with the well-known ones. Extensions to other resonances and other details will be published elsewhere (some aspects of the QCD approach to mesons based on power terms have been discussed previously[5] by us).

We start with the T product of two $\Delta I = 1$ vector currents:

$$\Pi_{\mu\nu} = i\int e^{iqx}dx\langle 0|T\{j_\mu(x)j_\nu(0)\}|0\rangle, \quad j_\mu = \tfrac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d), \tag{2}$$

where u and d are quark fields, and q is the photon momentum which is taken to be spacelike and large, $q^2 \equiv -Q^2$, $Q^2 \gg \mu^2$ (μ is some typical hadronic mass). The Wilson operator expansion allows one to represent $\Pi_{\mu\nu}$ as a series in Q^{-2}:

$$\Pi_{\mu\nu} \equiv (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi(Q^2), \quad \Pi = \langle 0|[C_I I + Q^{-4}C_G G_{\mu\nu}{}^a G_{\mu\nu}{}^a + Q^{-6}C_\Gamma(\bar{\psi}\Gamma\psi)(\bar{\psi}\Gamma\psi)]|0\rangle, \tag{3}$$

where I is the unit operator and Γ are matrices acting in the color, flavor, and spinor spaces. Higher powers in Q^{-2} are neglected as well as terms proportional to $m_{u,d}{}^2$, $m_u\bar{u}u$, etc., which are suppressed by the smallness of the quark masses.[6] The expansion coefficients $C_{I,G,\Gamma}$ can be found perturbatively since the effective coupling constant $\alpha_s(Q)$ is small.

The Wilson operator expansion can be proved within standard perturbation theory. Then taking the vacuum-to-vacuum matrix element singles out the unit operator which absorbs all of perturbation theory. Now, we want to include nonperturbative terms and the question arises as to the validity of Eq. (3).