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Simple Prediction of Quantum Chromodynamics for Angular Distribution of Dileptons in Hadron Collisions

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The quantum-chromodynamic predictions are computed for production of large-transverse-momentum lepton-antilepton pairs from $q\bar{q}$ scattering (which is relevant to $\pi^- p$ or $\bar{p}p$ collisions). A simple formula not involving the parton distributions is derived for two of the three coefficients describing the angular distribution.

It has been increasingly realized[1,2] recently that the perturbation expansion of quantum chromodynamics (QCD)[3,4] is applicable to many hadronic processes, such as production of high-mass lepton-antilepton pairs. Perturbation theory in the coupling g is used for inclusive processes with initial parton states and the cross sections are convoluted with parton distribution functions to give hadronic cross sections. Since the parton distributions depend on the long-distance behavior of QCD, which is asymptotically free,[3] they are not computable by simple perturbative methods.

Moreover the infrared-sensitive part of the parton cross sections can be absorbed[1,2] into a redefinition (i.e., a renormalization) of the parton distributions. Thus the final result of the perturbative calculations comes from momentum scales set only by the large energies of the problem. Consequently asymptotic freedom shows that a small-coupling expansion is valid.

The simplest proof of such statements is for deep-inelastic lepton scattering, when the operator product expansion can be used.[5] For other processes such as dilepton production one can analyze Feynman diagrams—as summarized below or in Ref. 2.

In this note I compute the angular distribution of large-transverse-momentum dileptons produced when the dominant parton collisions are of quark and antiquark, as is expected in $\pi^- p$ or $\bar{p}p$ scattering. There the quark and antiquark can be valence, so that the original Drell-Yan process,[6] $q\bar{q} \to$ "γ" $\to \mu^+\mu^-$, certainly dominates at low transverse momentum $q_\perp$ of the dilepton. Large $q_\perp$ is produced with an order g^2 cross section by the diagrams of Fig. 1. From these a simple result, Eq. (6) below, is obtained in the Collins-Soper[7] frame. However, large $q_\perp$ can also be produced in order g^2 by gluon-quark collisions. But since a large-transverse-momentum parton must be produced in addition to the virtual photon, the initial-state partons must be at fairly large x. For example, with a dilepton invariant mass of $M = 4$ GeV and with $q_\perp = 3$ GeV, the minimum parton center-of-mass energy is $q_\perp + (M^2 + q_\perp^2)^{1/2} = 8$ GeV. When the hadrons have $s = 400$ GeV^2, this gives a typical parton x of $8/\sqrt{400} = 0.4$. Since there are probably more valence quarks than gluons at large x, the $q\bar{q}$ process should dominate. Note that in $\pi^+ p$, in contrast to $\pi^- p$ collisions, the dominant valence process is $d\bar{d}$ scattering, rather than $u\bar{u}$, so that there is an extra factor of $\frac{1}{4}$ due to the electric charge of the d quark, and there the gluon-quark process may well be important. Previous work[8] has concentrated on pp collisions, where gluon-quark scattering is important, and has not used the Collins-Soper frame where the simple result Eq. (6) is derivable.

Consider production of a dilepton of momentum q^μ from hadrons of momenta p_1^μ and p_2^μ. I write $q^\mu = x_1 p_1^\mu + x_2 p_2^\mu + q_\perp^\mu$, with $q_\perp \cdot p_1 = q_\perp \cdot p_2 = 0$, and I consider the limit $s \equiv (p_1 + p_2)^2 \to \infty$ with x_1 and x_2 fixed and $q_\perp$ arbitrary. By dimensional analysis, this is equivalent to keeping s fixed and let-

FIG. 1. Diagrams for production of large-$q_\perp$ dileptons. The curly lines are gluons.

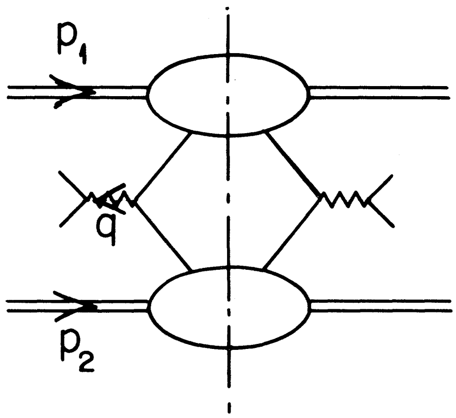

FIG. 2. Structure of dominant diagrams for dilepton production according to Ref. 10. The lines cut by the dashed line are on shell.

ting all masses go to zero. A renormalization-group transformation[5] is used to keep the subtraction mass μ constant; this implies use of the usual effective coupling.[5] The Feynman diagrams would then be ultraviolet dominated were it not that there are infrared divergences in the zero-mass limit. So it must be shown that these can be absorbed by a redefinition of the parton distributions, and that this is the same redefinition as is needed in lepton scattering.

The program for the proof is as follows:

(1) If parton-hadron amplitudes fall off rapidly enough as the partons go off shell, then the methods of Landshoff, Polkinghorne, and Short[9] would show[10] that the dominant diagrams are those of Fig. 2. Diagrams with exchanges between top and bottom do not contribute[11] in the Drell-Yan limit.

(2) In a renormalizable theory, parton-hadron amplitudes do not fall off quite rapidly enough.[12] But this effect is ultraviolet dominated, so that it is governed by subgraphs connected to the rest of a diagram for the process by the minimal number of lines.[13] Hence diagrams of the form of Fig. 3 dominate. The upper and lower pairs of parton lines are close to their mass shell.

(3) By the usual analysis, infrared divergences correspond to possible on-shell processes, so that Fig. 3 is an ultraviolet-dominated amplitude convoluted with two on-shell parton amplitudes, which are the same as in deep-inelastic scattering.

(4) Feynman graphs for dimuon production from parton scattering have the form of the middle three bubbles of Fig. 3. Their infrared divergences can be absorbed into a renormalization of the parton distributions.

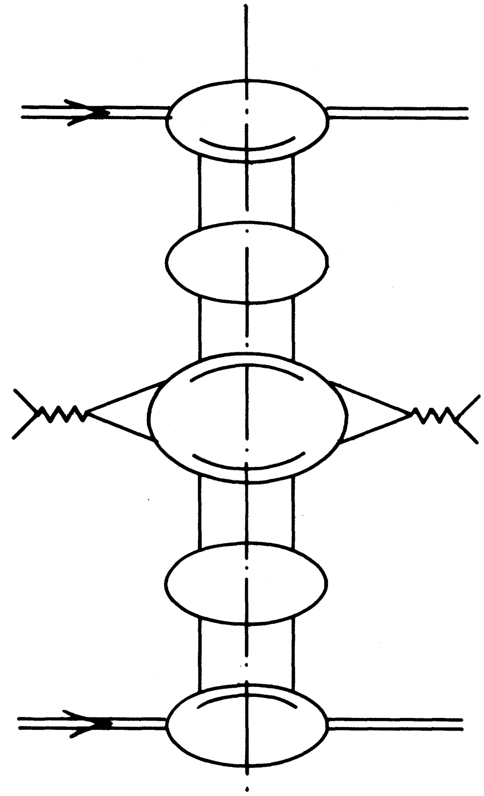

FIG. 3. Dominant diagrams for dilepton production. Two-particle irreducibility of a bubble is denoted by a double line.

(5) In a gauge theory, diagrams other than Fig. 3 can contribute,[14] and the infrared divergences due to gauge degrees of freedom are not of the form stated in step (3). But the axial gauge, where all gluons have transverse polarization, avoids these problems.[2]

A proof within perturbation theory along these lines has been announced by Ellis *et al.*[2] It should be possible to follow the methods of Landshoff and Polkinghorne to make the proof nonperturbative as far as the infrared-sensitive part is concerned.

Since I only want the large-$q_\perp$ cross section (say at $q_\perp > 1$ GeV), in the lowest relevant order in g, no renormalization of the parton distributions has been done in the following calculations.

Lam and Tung[15] define

$$\begin{aligned} W_{\mu\nu} &= s\int d^4z\, e^{iq\cdot z}\langle p_1, p_2|J_\mu(z)J_\nu(0)|p_1, p_2\rangle \\ &= (-g_{\mu\nu}+q_\mu q_\nu/q^2)W_1+\tilde{P}_\mu\tilde{P}_\nu W_2 \\ &\quad -\tfrac{1}{2}(\tilde{P}_\mu\tilde{p}_\nu+\tilde{p}_\mu\tilde{P}_\nu)W_3+\tilde{p}_\mu\tilde{p}_\nu W_4, \end{aligned} \tag{1}$$

where J_μ is the electromagnetic current, $\tilde{P}_\mu \equiv [p_{1\mu}+p_{2\mu}-q_\mu q\cdot(p_1+p_2)/q^2]/\sqrt{s}$, $\tilde{p}_\mu \equiv [p_{1\mu}-p_{2\mu}-q_\mu q\cdot(p_1-p_2)/q^2]/\sqrt{s}$, and $g_{\mu\nu}=\mathrm{diag}(1,-1,-1,-1)$. Then the cross section for dilepton production (when masses are ignorable) is

$$\frac{d\sigma}{d^4q}=\frac{\alpha^2}{12\pi^3 s^2 M^2}(-W_\mu{}^\mu). \tag{2}$$

The angular distribution of one of the leptons in the dilepton center of mass is[7,15]

$$\frac{dN}{d\Omega}\equiv\frac{d\sigma}{d^4q\,d\Omega}\left(\frac{d\sigma}{d\Omega}\right)^{-1}=\frac{3}{8\pi}\,\frac{(1+\cos^2\theta)W_T+(1-\cos^2\theta)W_L+W_\Delta\sin2\theta\cos\varphi+W_{\Delta\Delta}\sin^2\theta\cos2\varphi}{2W_T+W_L}, \tag{3}$$

where W_T, W_L, W_Δ, $W_{\Delta\Delta}$ are linear combinations of the W_i's defined in Ref. 15 and depend on the frame in which the polar angles θ and φ are measured.

From the diagrams of Fig. 1, I compute

$$W_i=\tfrac{1}{18}\pi\int_{x_1+q_\perp^2/s(1-x_2)}d\xi_1\int_{x_2+q_\perp^2/s(1-x_1)}d\xi_2\,\delta[(\xi_1-x_1)(\xi_2-x_2)-q_\perp^2/s]\times\sum_a f_{a/1}(\xi_1)f_{\bar{a}/2}(\xi_2)\omega_i(q^2,q_\perp^2,s,\xi_1,\xi_2), \tag{4}$$

where

$$\omega_1=\frac{16e_a^2g^2}{\xi_1\xi_2q_\perp^2/s}\left[\frac{q^4}{s^2}-\frac{2q_\perp^2}{s}\xi_1\xi_2+\xi_1^2\xi_2^2\right], \tag{5a}$$

$$\omega_2=\omega_4=-16e_a^2g^2q^2(\xi_1^2+\xi_2^2)/(\xi_1\xi_2q_\perp^2), \tag{5b}$$

$$\omega_3=32e_a^2g^2q^2(\xi_1^2-\xi_2^2)/(\xi_1\xi_2q_\perp^2). \tag{5c}$$

The sum in Eq. (4) is over quark flavors a, and I have taken color into account. A quark of flavor a has electric charge e_a and there is probability density $f_{a/i}(\xi)$ for it to occur in hadron i with momentum ξp_i. The distribution of quarks in a nucleon is known from lepton scattering, while π^-p data at low $q_\perp$ imply the $\bar{u}$-quark distribution in the π^- via the simple Drell-Yan process.[6]

Notice that each ω_i behaves as $q_\perp^{-2}$ when $q_\perp\to0$. This singularity, which implies that the cross section integrated over $q_\perp$ is logarithmically divergent, is to be removed by the "infrared renormalization" of the parton distributions; our formulas as they stand are not valid at low $q_\perp$.

Using Appendix B of Ref. 15 to compute W_T, W_L, W_Δ, and $W_{\Delta\Delta}$ I find that the angular distribution given by Fig. 1 is

$$\frac{dN}{d\Omega}=\frac{3}{16\pi}\left[\frac{q^2+\frac{3}{2}q_\perp^2}{q^2+q_\perp^2}+\frac{q^2-\frac{1}{2}q_\perp^2}{q^2+q_\perp^2}\cos^2\theta+\frac{2W_\Delta}{2W_T+W_L}\sin2\theta\cos\varphi+\frac{\frac{1}{2}q_\perp^2}{q^2+q_\perp^2}\sin^2\theta\cos2\varphi\right], \tag{6}$$

in the Collins-Soper[7] frame: In the dilepton center of mass θ is the angle between one lepton and the bisector of $\vec{p}_1$ and $-\vec{p}_2$, while the azimuthal angle φ is relative to the plane of $\vec{p}_1$ and $\vec{p}_2$. If the leptons have momenta k_1^μ and k_2^μ and if $k=k_1-k_2$, then $\cos\theta=2(q\cdot p_1k\cdot p_2-k\cdot p_1q\cdot p_2)/[sq(q_\perp^2+q^2)^{1/2}]$ and $\sin\theta\cos\varphi=2(q\cdot p_2k\cdot p_1+k\cdot p_2q\cdot p_1)/[sq_\perp(q_\perp^2+q^2)^{1/2}]$, where lepton and hadron masses have been ignored. There appears to be no simple form for the coefficient of $\sin2\theta\cos\varphi$. Note that Eq. (6) reduces to the simple Drell-Yan form $dN/d\Omega=(3/16\pi)(1+\cos^2\theta)$ when $q_\perp=0$.

Integration over φ or $\cos\theta$ gives the following:

$$\frac{dN}{d\cos\theta}=\frac{3}{4}\left[\frac{2q^2+3q_\perp^2+(2q^2-q_\perp^2)\cos^2\theta}{4(q^2+q_\perp^2)}\right], \tag{7}$$

$$\frac{dN}{d\varphi}=\frac{1}{2\pi}\left[1+\frac{q_\perp^2\cos2\varphi}{4(q^2+q_\perp^2)}\right]. \tag{8}$$

Equation (6) is clearly of great importance in testing QCD. The deviation from $1+\cos^2\theta$ is large when $q_\perp$ is large, and the parton distributions are not involved. The only assumption other than that perturbative QCD correctly describes dilepton production is that $q\bar{q}$ annihilation dominates. This can be tested: Gluon-quark scattering increases the cross section at large $q_\perp$ above that given by Eqs. (2), (4), and (5). But note that intrinsic transverse momentum for the partons tends to increase $q_\perp$ by a few hundred MeV.

Equation (6) implies Eq. (4.1) of Ref. 7 when $q_\perp^2 \ll q^2$, but is more general. No integration over $q_\perp$ is needed and a value for A_2 is obtained. Unlike the result in Ref. 15 for A_2, no assumption about the parton distributions is needed. The present proof is on more solid ground than in Refs. 7 and 15. The reason that the new result implies the old one may be that in time-ordered perturbation theory in the center-of-mass frame the energy denominator is of order $q_\perp$ for annihilation but of order q for bremsstrahlung.[16]

A similar remark should apply to quark-gluon scattering, of which a calculation will be made.

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How to Include Parton Transverse Momentum in Quantum Chromodynamics

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I show that parton transverse-momentum effects can be included in factorized quantum-chromodynamics inclusive cross sections, but that factorization is maintained only if the transverse-momentum variable which is held fixed is independent of the scale of the parton momentum. I also emphasized that other nonperturbative effects can be just as important as the transverse momentum.

Despite several recent proofs[1,2] and explanations,[1] the factorization of inclusive cross sections in quantum chromodynamics (QCD) is still sometimes misunderstood. A particularly sticky point concerns the question of how to introduce a phenomenological transverse-momentum distribution of partons within hadrons. This question is of some practical importance because fits to

