By making similar assumptions as in Eqs. (7) and (8), i.e.,

$$\vec{r} \cdot \nabla v(\vec{r}) = F(v(\vec{r})), \qquad (7')$$

one obtains

$$E(z) = \frac{z}{4} N(N-1) F\left(\frac{2}{N(N-1)} \frac{dE}{dz}\right) + z \frac{dE}{dz}. \qquad (9')$$

This very simple equation should be of great use in studying the properties of many-body systems, such as extensivity or saturation.

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Light Scattering from Nonequilibrium Stationary States: The Implication of Broken Time-Reversal Symmetry

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We report a theroetical prediction of a new feature in the spectrum of light scattered from a fluid in a nonequilibrium stationary state with a temperature gradient. The spectrum is not symmetric in the frequency shift and the Brillouin components have different intensities. The phenomenon is linked to the breaking of time-reversal symmetry and to the appearance of a static correlation function between momentum and number densities which is zero in equilibrium, but has a $1/k^2$ dependence in the stationary state. We suggest a light scattering experiment by means of which these predictions can be verified.

We report here some results of a theoretical investigation of nonequilibrium stationary states (NESS) in simple fluids. The theory predicts some new effects and one of the purposes of this communication is to suggest an interesting experiment which can probe the new phenomena.

Our analysis is based on nonlinear response theory. In the context of this theory, we have assumed the existence of a set of "slow" variables, which spans the slow, macroscopic evolution in an N-body system.[1,2] For a simple fluid it is customary to take the densities of the conserved variables (i.e., energy, number, and momentum) to compose this set. In the following we denote such a set by $\underline{A}(X^N(t), \vec{r})$, where X^N is the phase point and $\vec{r}$ is the position in space; we use a shorthand notation and write $\underline{A}(\vec{r}, t)$. The set of conjugate variables is denoted by $\underline{\Phi}(\vec{r})$. An inner product in the space of such sets involves integration over the spatial variables as well as summation on the indices (including Cartesian component indices where appropriate), and will be denoted by $*$, as $\underline{A}(\vec{r}_1, t) * \underline{B}(\vec{r}_1)$.

With this in mind, we have derived the NESS average of an *arbitrary* dynamical variable, say $B(\vec{r}, t)$.[3] Since an isolated system has no NESS we solved the Liouville equation for a system composed of three subsystems, two of which are large and act as reservoirs. The third subsystem is smaller and in it a quasistationary state is established. The result for the NESS average (denoted by $\langle \rangle_{NE}$) is[3]

$$\langle B(\vec{r}) \rangle_{NE} = \langle B(\vec{r}) \rangle_L - \int_0^\infty d\tau \langle \hat{B}(\vec{r}) \underline{\hat{I}}(\vec{r}_1, -\tau) \rangle_L * \nabla \beta \underline{\Phi}(\vec{r}_1). \qquad (1)$$

In this equation and the following, the caret denotes the deviation of a quantity from its equilibrium average. The notation $\langle \rangle_L$ stands for an average on a "local" distribution function

$$f_L(X^N) = \frac{f_{G.C.}(X^N) \exp[\beta \underline{\Phi}(\vec{r}_1) * \underline{A}(\vec{r}_1)]}{\sum_N \int dX^N f_{G.C.}(X^N) \exp[\beta \underline{\Phi}(\vec{r}_1) * \underline{A}(\vec{r}_1)]}, \qquad (2)$$

where $f_{\text{G. C.}}$ is the grand canonical distribution function. The quantities $\underline{I}(r, t)$ are dissipative fluxes defined by

$$\underline{\dot{A}}(\vec{r}, t) = -\nabla \cdot \underline{J}(\vec{r}, t) \tag{3}$$

and

$$\underline{I}(\vec{r}, t) = \underline{J}(\vec{r}, t) - \langle \underline{J}(\vec{r}, t)\underline{A}(\vec{r}_1) \rangle * \langle \underline{A}(\vec{r}_1, t)\underline{A}(\vec{r}_2) \rangle^{-1} * \underline{A}(\vec{r}_2, t), \tag{4}$$

where simple angular brackets denote an equilibrium average. The conjugate variables $\underline{\Phi}$ are chosen so that

$$\langle \underline{A}(\vec{r}) \rangle_{\text{NE}} = \langle \underline{A}(\vec{r}) \rangle_{\text{L}} \tag{5}$$

identically, and for a simple fluid they turn out to be[4]

$$\Phi_N(\vec{r}) = [\beta(\vec{r})/\beta] \mu^+(\vec{r}) - \mu, \quad \Phi_E(\vec{r}) = 1 - \beta(\vec{r})/\beta,$$
$$\vec{\Phi}_p(\vec{r}) = \beta(\vec{r})\vec{v}(\vec{r})/\beta, \tag{6}$$

where $\beta(\vec{r}) = [k_BT(\vec{r})]^{-1}$, $\vec{v}(\vec{r})$ is the fluid velocity, $\mu^+(\vec{r})$ is the local internal chemical potential, and β and μ are the corresponding equilibrium quantities.

Equation (4) projects out the "slow" part of $\underline{J}(\vec{r}, t)$ and therefore $\underline{I}(\vec{r}, t)$ is assumed to vary rapidly. Notice that for the special case $\underline{J}(\vec{r}, t) = \underline{B}(\vec{r}, t)$, Eq. (1) gives the well-known result[4] for the macroscopic hydrodynamic fluxes in a simple fluid. The first term, then, is the Euler (reversible) evolution and the second is the dissipative part.

In this Letter we concentrate on the fluctuations in NESS. Choosing the variable $N(\vec{r}, t+\sigma)N(\vec{r}', t)$, where $N(\vec{r}, t)$ is the number density

$$\sum_{j=1}^{N} \delta(\vec{r} - \vec{r}_j(t)),$$

we find the following for NESS that are close to equilibrium[3,5,6]:

$$\langle N(\vec{r}, \sigma)N(\vec{r}') \rangle_{\text{NE}} = \langle N(\vec{r}, \sigma)N(\vec{r}') \rangle_{\text{L}} - \int_0^\infty \langle N(\vec{r}, \sigma)N(\vec{r}')\underline{I}_T(-\tau) \rangle * \nabla\beta \underline{\Phi}(\vec{r})d\tau. \tag{7}$$

This expression follows from the fact that the correlation function in the NESS can be proved to be stationary and the closeness to equilibrium is used in changing the second average on the right-hand side (RHS) to an equilibrium one. It is also valid only for small gradients and the subscript T stands for the variable integrated over all space. Under the same conditions the first term on the RHS may be computed in an equilibrium ensemble whose thermodynamic properties are the same as the local properties of the actual system.

Consider now the simplest stationary state of a simple fluid—a heat-conducting stationary state. In this case the only dissipative flux present is the heat flux. The most important implication of Eq. (7) is that time-reversal symmetry is now broken. It is easy to verify that the first term on the RHS is symmetric under time reversal whereas the second term is antisymmetric because of the coupling with the flux.

As is well known, the Fourier transform (in space and time) of the density autocorrelation function can be measured by scattering light from the system.[7] Making a Fourier transformation with respect to σ and $\vec{r} - \vec{r}'$ we find

$$S_{\vec{k}\omega}(\vec{r}) = S_{\vec{k}\omega}{}^L(\vec{r}) + V^{-1} \int_0^\infty \langle N_{\vec{k}\omega}N_{-\vec{k}\omega}\vec{I}_{E,T}(-\tau) \rangle \cdot \nabla\beta(\vec{r})d\tau. \tag{8}$$

Every function still contains an $\vec{r}$ dependence because the NESS is not translationally invariant. The first term on the RHS is approximately equal to a structure factor in an equilibrium system whose properties are the same as the local ones in the NESS, and is symmetric in $\vec{k}$ and ω. The second term is antisymmetric in $\vec{k}$ and ω and needs to be evaluated.

Defining the quantity

$$\underline{W}_{NN}(\vec{k}, \sigma | \vec{r}) = V^{-1} \int_0^\infty \langle N_{\vec{k}}(\sigma)N_{-\vec{k}}\vec{I}_{E,T}(-\tau) \rangle \cdot \nabla\beta(\vec{r})d\tau = \tfrac{1}{2}V^{-1} \int_{-\infty}^\infty \langle N_{\vec{k}}(\sigma)N_{-\vec{k}}\vec{I}_{E,T}(-\tau) \rangle \cdot \nabla\beta(\vec{r})d\tau, \tag{9}$$

we see that its evaluation is immediate if we can find expressions for the various elements

$$\vec{\Lambda}^{\alpha\gamma E}(\vec{k},\,\sigma) = \int_{-\infty}^{\infty} d\tau\, \langle A_{\alpha,\,\vec{k}}(\sigma) A_{\gamma,\,-\vec{k}}\vec{I}_{E,T}(-\tau)\rangle, \tag{10}$$

where α and γ stand for N, E, or P in a simple fluid. We do so[6] by deriving the equation of motion:

$$\dot{\vec{\Lambda}}^{\alpha\gamma E}(\vec{k},\,\sigma) = M_{\vec{k}}^{\alpha\epsilon}(\sigma)\,\vec{\Lambda}^{\epsilon\gamma E}(\vec{k},\,\sigma) + i\vec{k}\cdot\vec{\Omega}^{\alpha\gamma E}(\vec{k},\,\sigma), \tag{11}$$

where Eq. (6) has been used,

$$\vec{\Omega}^{\alpha\gamma E}(\vec{k},\,\sigma) \equiv \int_{-\infty}^{\infty} d\tau\, \langle\vec{I}_{\alpha,\,\vec{k}}(\sigma) A_{\gamma,\,-\vec{k}}\vec{I}_{E,T}(-\tau)\rangle, \tag{12}$$

$$M_{\vec{k}}^{\alpha\epsilon}(\sigma) = i\vec{k}\cdot[\langle\vec{J}_{\vec{k}}(\sigma)\underline{A}_{-\vec{k}}\rangle * \langle\underline{A}_{\vec{k}}(\sigma)\underline{A}_{-\vec{k}}\rangle^{-1}]^{\alpha\epsilon}, \tag{13}$$

and repeated indices are to be summed over. In matrix notation, the solution of Eq. (11) for large σ is

$$\underline{\Lambda}^{E}(\vec{k},\,\sigma) = \exp(\underline{M}_{\vec{k}}\,\sigma)\underline{\Lambda}^{E}(\vec{k},\,\sigma=0) + \int_{0}^{\sigma} d\tau\,\{\exp[\underline{M}_{\vec{k}}(\sigma-\tau)]\,i\vec{k}\cdot\underline{\Omega}^{E}(\vec{k},\tau)\}. \tag{14}$$

Use has been made of the fact that $\underline{M}(\sigma)$ becomes independent of σ for σ larger than microscopic times.[8] It is possible to show[6] that for small $\vec{k}$ and large σ (hydrodynamic modes) the second term on the RHS contributes very little to the density-density component of $\underline{\Lambda}^{E}$, and hence will be neglected.

Using similar ideas to those outlined above, one can obtain an expression for $\underline{\Lambda}^{E}(\vec{k},\,\sigma=0)$. This reads[6] (in components)

$$\vec{\Lambda}^{\alpha\gamma E}(\vec{k},\,\sigma=0) = (1 - \epsilon_{\alpha}\epsilon_{\gamma})\int_{0}^{\infty} d\tau\,[\exp(\underline{M}_{\vec{k}}\tau)\,\vec{\Gamma}^{E}(\vec{k},\,\tau=0)\exp(\underline{M}_{\vec{k}}^{\dagger}\tau)]^{\alpha\gamma}, \tag{15}$$

which again, is valid only for small k. Here ϵ_{α} is the signature of the α component under time reversal, and the dagger denotes Hermitian conjugation. Further,

$$\vec{\Gamma}^{\alpha\gamma E}(\vec{k},\,t) \equiv \langle A_{\alpha,\vec{k}} A_{\gamma,-\vec{k}}\vec{I}_{E,T}(-t)\rangle. \tag{16}$$

$\underline{\Gamma}^{E}(\vec{k},\,t=0)$ is easy to calculate in the small-k limit using standard thermodynamic fluctuation theory.

Collecting the above results we see that $\underline{W}_{NN}(\vec{k},\,\sigma|\vec{r})$ can be expressed as

$$\underline{W}_{NN}(\vec{k},\,\sigma|\vec{r}) = V^{-1}[\exp(\underline{M}_{\vec{k}}\,\sigma)]^{NP_x}[\int_{0}^{\infty} d\tau\exp(\underline{M}_{\vec{k}}\tau)\underline{\Gamma}^{E}(\vec{k},\,0)\exp(\underline{M}_{\vec{k}}^{\dagger}\tau)]^{P_xN}\cdot\nabla\beta(\vec{r}). \tag{17}$$

In obtaining this result we used the fact that the matrix $\underline{M}_{\vec{k}}$ is composed of two diagonal blocks.[7-9] The only momentum component that couples to N and E is the longitudinal one. Choosing k to point in the X direction, we obtain Eq. (17). Performing the algebra involved in Eq. (17) and a Fourier transformation in σ, and adding the result to $S_{\vec{k}\omega}^{L}$ in Eq. (8), we find the following for NESS that are not too far from equilibrium:

$$S_{\vec{k}\omega}(\vec{r}) = \left[\frac{2k_BT\rho}{mc_0^2}\left(\frac{\overline{C}_p}{\overline{C}_V} - 1\right)\frac{\Gamma_T k^2}{\omega^2 + (\Gamma_T k^2)^2}\right] + \left\{\frac{2k_BT\rho\Gamma_s k^2}{mc_0^2}\left[\frac{1-\epsilon(\vec{r})}{(\omega-kc_0)^2+(\Gamma_s k^2)^2} + \frac{1-\epsilon(\vec{r})}{(\omega+kc_0)^2+(\Gamma_s k^2)^2}\right]\right\}, \tag{18}$$

where

$$\epsilon(\vec{r}) = \frac{c_0}{2\Gamma_s k}\frac{\vec{k}\cdot\nabla\ln T(\vec{r})}{k^2}, \tag{19}$$

and k_B, T, ρ, $\overline{C}_p$, $\overline{C}_V$, m, c_0, Γ_T, and Γ_s are Boltzmann's constant, the temperature, the density, the heat capacities per particle at constant pressure and volume, the particle mass, the sound speed, and the heat and sound attenuation constants, respectively. All quantities appearing in Eqs. (18) and (19) are those of a homogeneous system whose thermodynamic properties correspond to the actual ones at the point $\vec{r}$. As was mentioned before, the light scattering spectrum is proportional to $S_{\vec{k}\omega}$, and is shown in Fig. 1, for argon at a density of 1 g/cm^3 and a temperature of 235 K which is subjected to a 0.5-K/cm temperature gradient. The wave vector $k = 852$ cm^{-1}.

The spectrum consists of three Lorentzian curves: the central heat (Rayleigh) peak and the two sound (Brillouin) peaks. Although the three peaks have the same widths and positions as are found in equilibrium, notice that the amplitudes of the two sound peaks are no longer equal. The size of the asymmetry is governed by $\epsilon(\vec{r})$ which in Fig. 1 is 0.12. From Eq. (19) one sees that the coupling of the density fluctuations to the sound mode propagating with the heat flux is enhanced at the expense of the one going in the opposite direction. The spectrum can be decomposed into its symmetric and antisymmetric parts (in ω). From Eq. (19) we see that the symmetric
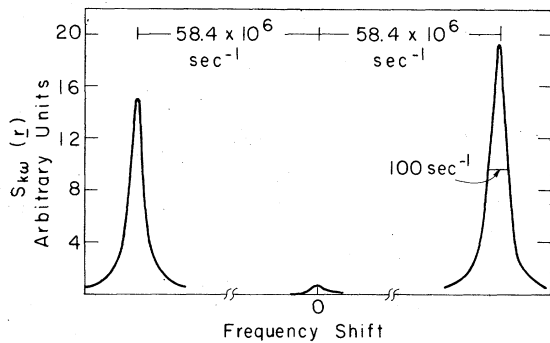
FIG. 1. The predicted spectrum of light scattered from argon subject to a temperature gradient of 0.5 K/cm. The density is 1 g/cm^3 and the temperature at the scattering point is 235 K. Here $k = 852$ cm^{-1}, which for an incident beam with $k_{in} = 10^5$ cm^{-1} corresponds to a scattering angle of 0.4°. Notice the asymmetry in the Brillouin peaks which is attributed to the implication of broken time-reversal symmetry.

part [obtained by putting $\epsilon(\vec{r}) = 0$] is the same as the equilibrium spectrum, whereas the antisymmetric part [the term involving $\epsilon(\vec{r})$] is also antisymmetric in $\vec{k}$, and contains all the new effects.

The origin of this effect is interesting. As sketched above, and as can be seen clearly from the detailed analysis,[2,3] the asymmetric term is a direct consequence of the fact that the NESS static correlation function $\langle P_{\vec{k}}^{-l} N_{-\vec{k}} \rangle_{NE}$ is not zero; here, $P_{\vec{k}}^{l}$ and $N_{\vec{k}}$ are the Fourier components of the longitudinal momentum density and number density, respectively. In equilibrium, the static correlation function is zero because of time-reversal symmetry. In NESS, not only is it not zero, but it has the form

$$\langle P_{\vec{k}}^{-l} N_{-\vec{k}} \rangle_{NE} = \frac{k_B T}{2\Gamma_s k} \frac{\vec{k} \cdot \nabla \ln T(\vec{r})}{k^2} . \quad (20)$$

For small k's this quantity becomes very important, growing like k^{-2}. This is characteristic of a behavior found in broken-symmetry phenomena.[9] The existence of dissipative steady fluxes destroys time-reversal symmetry and the familiar k^{-2} dependence arises. Notice, however, that contrary to the usual broken-symmetry phenomena in equilibrium systems, time reversal is a discrete symmetry which means that no new peaks (Goldstone modes) are expected in the light-scattering spectrum. Our results[6] show that all static correlation functions between hydrodynamic variables that are zero in equilibrium as a result of time-reversal symmetry now exhibit a k^{-2} behavior. The implication of this is that the ef-

fect shown in Fig. 1 grows when k (or the scattering angle) is reduced. It is important to comment, however, that the k^{-2} behavior cannot be taken as a divergence. The theory, at present, is limited to k's that are larger than the macroscopic length scales [$1/\nabla T(r)$, say] and thus the limit $k \rightarrow 0$ is not accessible with this theory.

The k^{-2} behavior is equivalent to $1/r$ decay in physical space. This is an indication of a long-range order that sets in once a NESS is established. We stress that this long-range order is found far from any instability and is just a consequence of the symmetry breaking associated with the existence of the fluxes.

The predictions of our theory could be tested in the following simple light-scattering experiment. The scatterer is a simple fluid with a linear temperature profile. One can then send a laser beam perpendicular to the temperature gradient and observe the scattered light in the beam–temperature-gradient plane. As was mentioned before, the spectrum measured at scattering angles θ and $-\theta$ (in that plane) will not be the same since the new terms in Eq. (1) are antisymmetric under this transformation (which is equivalent to $\vec{k} \rightarrow -\vec{k}$). By "subtracting" the two spectra one can obtain (twice) the new term which is purely antisymmetric in ω. The scattering angle associated with the spectrum depicted in Fig. 1 is 0.4°. Notice that the asymmetry is proportional to $1/k^2$ and thus depends strongly on the scattering angle. Although this angle is fairly small, the shifts associated with the Brillouin peaks in Fig. 1 place such an experiment well within the regime of modern light-scattering techniques.

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Simple Prediction of Quantum Chromodynamics for Angular Distribution of Dileptons in Hadron Collisions

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The quantum-chromodynamic predictions are computed for production of large–transverse-momentum lepton-antilepton pairs from $q\bar{q}$ scattering (which is relevant to $\pi^- p$ or $\bar{p}p$ collisions). A simple formula not involving the parton distributions is derived for two of the three coefficients describing the angular distribution.

It has been increasingly realized[1,2] recently that the perturbation expansion of quantum chromodynamics (QCD)[3,4] is applicable to many hadronic processes, such as production of high-mass lepton-antilepton pairs. Perturbation theory in the coupling g is used for inclusive processes with initial parton states and the cross sections are convoluted with parton distribution functions to give hadronic cross sections. Since the parton distributions depend on the long-distance behavior of QCD, which is asymptotically free,[3] they are not computable by simple perturbative methods.

Moreover the infrared-sensitive part of the parton cross sections can be absorbed[1,2] into a redefinition (i.e., a renormalization) of the parton distributions. Thus the final result of the perturbative calculations comes from momentum scales set only by the large energies of the problem. Consequently asymptotic freedom shows that a small-coupling expansion is valid.

The simplest proof of such statements is for deep-inelastic lepton scattering, when the operator product expansion can be used.[5] For other processes such as dilepton production one can analyze Feynman diagrams—as summarized below or in Ref. 2.

In this note I compute the angular distribution of large–transverse-momentum dileptons produced when the dominant parton collisions are of quark and antiquark, as is expected in $\pi^- p$ or $\bar{p}p$ scattering. There the quark and antiquark can be valence, so that the original Drell-Yan process,[6] $q\bar{q} \to \text{``}\gamma\text{''} \to \mu^+ \mu^-$, certainly dominates at low transverse momentum $q_\perp$ of the dilepton. Large $q_\perp$ is produced with an order g^2 cross section by the diagrams of Fig. 1. From these a simple result,

Eq. (6) below, is obtained in the Collins-Soper[7] frame. However, large $q_\perp$ can also be produced in order g^2 by gluon-quark collisions. But since a large–transverse-momentum parton must be produced in addition to the virtual photon, the initial-state partons must be at fairly large x. For example, with a dilepton invariant mass of $M = 4$ GeV and with $q_\perp = 3$ GeV, the minimum parton center-of-mass energy is $q_\perp + (M^2 + q_\perp{}^2)^{1/2} = 8$ GeV. When the hadrons have $s = 400$ GeV^2, this gives a typical parton x of $8/\sqrt{400} = 0.4$. Since there are probably more valence quarks than gluons at large x, the $q\bar{q}$ process should dominate. Note that in $\pi^+ p$, in contrast to $\pi^- p$ collisions, the dominant valence process is $d\bar{d}$ scattering, rather than $u\bar{u}$, so that there is an extra factor of $\frac{1}{4}$ due to the electric charge of the d quark, and there the gluon-quark process may well be important. Previous work[8] has concentrated on pp collisions, where gluon-quark scattering is important, and has not used the Collins-Soper frame where the simple result Eq. (6) is derivable.

Consider production of a dilepton of momentum q^μ from hadrons of momenta $p_1{}^\mu$ and $p_2{}^\mu$. I write $q^\mu = x_1 p_1{}^\mu + x_2 p_2{}^\mu + q_\perp{}^\mu$, with $q_\perp \cdot p_1 = q_\perp \cdot p_2 = 0$, and I consider the limit $s \equiv (p_1 + p_2)^2 \to \infty$ with x_1 and x_2 fixed and $q_\perp$ arbitrary. By dimensional analysis, this is equivalent to keeping s fixed and let-

FIG. 1. Diagrams for production of large-$q_\perp$ dileptons. The curly lines are gluons.