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## Exclusive $\rho$ Production by Virtual Photons

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The cross section for production of  $\rho^0$  mesons has been measured in the reaction  $e + p \rightarrow e$  $+p+\pi^++\pi^-$ . The cross section is presented as a function of W, the c.m. energy of the virtualphoton-proton system, and  $-Q^2$ , the square of the virtual-photon mass. The vector-dominance model is not able to describe the dependence of the cross section on the parameters  $Q^2$  and W. The slope parameter B describing the scattering of the proton exhibits a significant variation with  $Q^2$ .

The production of the  $\rho^0$  meson in high-energy electroproduction has been understood in terms of a model in which the hadronic constituents of the photon are produced by diffractive dissociation at the target hadron. The generalized vectordominance model<sup>1</sup> is used to explain the variation of the  $\rho^0$  production cross section as a function of  $Q^2$ , the square of the momentum transfer to the electron, and W, the center-of-mass energy of the virtual-photon-proton system. The angular distribution of the produced  $\rho^0$  is expected to reflect the structure of the target hadron. We have measured the production cross section and this angular distribution as functions of  $Q^2$  and W; we present the data here.

The Cornell University electron synchrotron was used to produce an 11.5-GeV electron beam focused on a 7.5-cm liquid-hydrogen target. The secondary particles were detected by a multiwire proportional counter system in the gap of a large-aperture magnet. The apparatus in which these data were taken was described elsewhere.<sup>2</sup> The sample was drawn from a set of events in which four tracks were found with a zero overall charge. One of the negative tracks was identified as an electron by matching the measured momentum and the energy deposited in a lead-scintillator shower counter. The event was then fitted to the hypothesis

$$e + p - e + p + \pi^{+} + \pi^{-}$$
. (1)

The principal ambiguity in these events was the assignment of the proton and the  $\pi^+$  between the positive tracks. Scintillation counters measured the time of flight of the reconstructed hadrons from the hydrogen target with a geometric aperture of about 0.6. With use of this information, the fraction of this event sample that remained ambiguous was less than 1%. Contamination of this event sample by other distinct hypotheses was negligible.

The majority of the events that satisfied event hypothesis (1) were from the following two reactions:

$$e + p \rightarrow e + \Delta^{++} + \pi^{-}, \qquad (2)$$

$$e + p - e + p + \rho^0. \tag{3}$$

We kept for analysis those events for which Wwas greater than 2.3 GeV; this cut reduced the contribution from Reaction (2) relative to that from Reaction (3). The events that we analyze here were distributed between this limit and a W of 3.75 GeV.  $Q^2$  varied between 0.7 and 3.0 GeV<sup>2</sup>.

The dependence of the  $\rho$  production cross section on  $Q^2$ , W, and t, the square of the fourmomentum transfer to the proton, were extracted by the following procedure. The data occurring in a particular kinematic bin were divided individually by the virtual-photon flux factor,

$$\Gamma = \frac{\alpha}{4\pi^2 Q^2} \frac{E'}{E} \frac{W^2 - M^2}{2M} \frac{2}{1 - \epsilon} , \qquad (4)$$

where E is the incident electron energy, E' is the final electron energy, and  $\epsilon$  is the polariza-

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tion of the virtual photon. The weighted sum was then corrected for the acceptance of the apparatus in the bin. For example, in the study of the tcross-section dependence, the acceptance was calculated independently in bins which divided the available range in  $Q^2$  twice, W twice,  $t - t_{\min}$ five times, and  $m_x$  (the invariant mass of the di-pion system) four times. The Monte Carlo integration required to determine the bin acceptance then assumed generating functions in all eight kinematic variables describing each event, but the binning procedure reduced the dependence of the result on the generating function chosen.

This procedure yielded the virtual-photon cross section, conventionally defined as  $\sigma_r + \epsilon \sigma_L$ , into the final state  $(p\pi^+\pi^-)$  for each bin. In order to extract the  $\rho^0$  cross section, a fit was carried out against the variables  $m_x^2$  and  $m_y^2$ , where  $m_y$  is the invariant mass of the  $\pi^+ p$  system. Specifically,

$$F(m_x^2, m_y^2) = a_1 F_{\rho}(m_x^2) (1 + a_4 \cos^2 \theta_{\pi}) + a_2 F_{\Delta}(m_y^2) + a_3 F_{DS}.$$
(5)

 $a_1$ ,  $a_2$ ,  $a_3$ , and  $a_4$  were determined from the fit;  $F_{\rho}$  and  $F_{\Delta}$  are Breit-Wigner resonance functions modified for the orbital angular momentum in the final state. The form was taken from Jackson,<sup>3</sup> except that the  $\rho^0$  was skewed by a factor<sup>4</sup>

$$\left[ (m_0^2 - t + Q^2) / (m_x^2 - t + Q^2) \right]^2.$$
(6)

The phase-space function  $F_{\rm ps}$  was taken to be a constant. In the  $\rho^0$  fit the mass  $m_{\rho}$  was fixed at 0.770 GeV/ $c^2$  and the width  $\Gamma_{\rho} = 0.15$  GeV/ $c^2$ .

The  $\Delta^{++}$  was assumed to decay uniformly in its c.m. system.<sup>5</sup> It is important to note the fact that the finite (~ 60%/track) geometrical acceptance of this apparatus has little effect on the cross sections we present here. The particles that are lost are those with large dip angles and although the spin state of the  $\rho^0$  influences the angular distribution of the  $\pi^+\pi^-$  system, the effect of the spin state on the average value of the acceptance is small. Particularly, the cross sections we present here depend only weakly on the value of  $a_4$ , the polar anisotropy of the  $\rho^0$ . A detailed account of this procedure is presented by Ahrens.<sup>6</sup>

We have made an estimate of the radiative corrections to this reaction; they are insensitive to  $Q^2$  and W and amount to less than 20% in any bin. We have made an overall correction to the cross sections based on this estimate.



FIG. 1. The virtual-photon-proton cross section for  $\gamma_v + p \rightarrow \rho_0 + p$  as a function of  $Q^2$ . The data for this experiment are divided into two bins in W as shown. The photoproduction point is at  $E_\gamma = 4.7$  GeV: W = 3.1 GeV.  $\bigcirc$ ,  $\langle W \rangle = 2.5$ ;  $\blacktriangle$ ,  $\langle W \rangle = 3.2$ ;  $\times$ , Ref. 7,  $\langle W \rangle \sim 2.4$ ;  $\bullet$ , Ref. 8, W = 3.1;  $\nabla$ , Ref. 9,  $\langle W \rangle \sim 4.5$ .

Figure 1 shows the dependence of the  $\rho^{0}$  cross section as a function of  $Q^{2}$ , with the data divided into two W regions. In each bin, the fraction of  $e\rho\pi^{+}\pi^{-}$  cross section assigned by the fit to the  $\Delta^{++}$  channel is always less than 10% of the total; the fraction due to phase space is less than 30% and decreases with W. Also in this figure are the photoproduction values for an appropriate  $W^{8}$ and some electroproduction results.<sup>7,9</sup> Other results in this kinematic region are consistent with these data.

We have used the paper of Fraas and Schildknecht<sup>1</sup> to make a comparison between the data presented here and the generalized vector-dominance model. We have chosen to use the crosssection predictions rather than more detailed distributions because the cross-section data are particularly insensitive to systematic effects. The curves in Fig. 1 are derived from Eq. (38) of Ref. 1 but allow a ratio of transverse to longitudinal cross section ( $\xi$ ) different from unity.<sup>10</sup> The function plotted is

$$\frac{P_{\rm in}^*(Q^2=0)}{P_{\rm in}^*(Q^2)} \left(\frac{m_{\rho}^2}{m_{\rho}^2+Q^2}\right)^2 \left\{1 + \epsilon \xi^2 \frac{Q^2}{m_{\rho}^2}\right\} \sigma_{(\gamma \rho \to \rho^0 \rho)}(Q^2=0, W) \exp[B(Q^2)t_{\rm min}(Q^2) - B(0)t_{\rm min}(0)]$$

with  $P_{\text{in}}^*$  the photon momentum in the hadron c.m. system. At the lower value of W = 2.5, and with  $\xi = 0.3$ , the curve approximates the  $Q^2$  dependence of the data but at the higher value of W (3.2) there is no correspondence between data and theoretical curve. This discrepancy cannot be resolved by any reasonable value of  $\xi$ .

The difficulties indicated here become more apparent in Fig. 2. Figure 2 shows the variation of the cross section as a function of W for the two  $Q^2$  bins. Two points from the experiment of Joos et al.<sup>7</sup> are included as well as values from the muon experiment of Francis et al.<sup>11</sup> The relatively flat behavior of the photoproduction cross section is also shown. The formulation of Fraas and Schildknecht leads to a prediction that the cross section varies slowly with W in keeping with the diffractive character of the model. In photoproduction, shown by the curve in Fig. 2, this is approximately true. However, the data we present show anomalous behavior from this point of view in that at low  $Q^2$  the W dependence is very marked in the range of this experiment.



In the high- $Q^2$  bin the *W* dependence becomes less marked. This behavior, namely, that the rate of *W* dependence is not monotonic with increasing  $Q^2$ , seems difficult to explain in any reasonable variation of the vector-dominance approach.

The *t* dependence of the cross section is studied with the data divided into two  $Q^2$  and two *W* regions. In each region the differential cross section  $d\sigma/dt$  is fitted by the form  $A \exp[B(t - t_{\min})]$ . We have plotted the parameter *B* in Fig. 3 together with data from comparable experiments. The values of *B* vary somewhat with changes in the fitting procedure, particularly in the form of the skewing function described above. These variations are comparable to the statistical errors shown and generally cause the value of *B* to be smaller than the values shown in Fig. 3. Other experimental work<sup>14</sup> is consistent with the dis-



FIG. 2. The virtual-photon-proton cross section for  $\gamma_v + p \rightarrow \rho_0 + p$  as a function of W. The mean  $Q^2$  for each bin is shown both for this experiment and for Ref. 11.

FIG. 3. The values of the slope parameter B as a function of  $Q^2$ .  $\bigcirc$ , this experiment,  $\langle W \rangle = 2.5 \text{ GeV}; \bullet$ , this experiment,  $\langle W \rangle = 3.2 \text{ GeV}; \times$ , Ref. 7,  $\langle W \rangle = 2.4$  GeV;  $\blacksquare$ , Ref. 9,  $\langle W \rangle = 4.5 \text{ GeV}; \blacktriangle$ , Ref. 12,  $\langle W \rangle = 2.4$ , GeV;  $\blacktriangledown$ , Ref. 13, W = 3.1 GeV.

tribution shown here although the situation previously could be described as murky.

We conclude that there is a variation of the parameter B with  $Q^2$  and any reformulation of the theory should take this into account.

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## **Reactive Content of the First-Order Optical Potential**

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It is shown that the total reaction cross section appropriate to the optical potential for scattering of a projectile by a target of uncorrelated particles (nucleons) can be written to a good approximation as a sum of exclusive cross sections for n-nucleon knockout. The association made by some authors of the reactive content of this optical potential with single-nucleon knockout is shown to apply to the inclusive-reaction cross section only.

The subject of this Letter is the connection between the optical potential for elastic scattering of a particle from a complex target, such as a nucleus, and the cross sections for inelastic reactions induced by the incident particle. In general, one may relate the total reactive cross section  $\sigma_r$ , defined as the difference between the total and elastic cross sections  $\sigma_r = \sigma_T - \sigma_{\rm el}$ , to the optical potential  $\mathfrak{A}$ , by<sup>1</sup>

$$\sigma_{r} = C \langle \varphi_{0}^{(+)} | \mathfrak{u}^{\dagger} - \mathfrak{u} | \varphi_{0}^{(+)} \rangle$$

$$= -2iC \operatorname{Im} \langle \varphi_{0}^{(+)} | \mathfrak{u} | \varphi_{0}^{(+)} \rangle, \qquad (1)$$

where  $C = -i(2\pi)^3 u^{-1}$ , with *u* the projectile velocity, and  $\varphi_0^{(+)}$  is the elastic wave which satisfies the optical scattering equation with the potential  $\mathfrak{A}$ . Equation (1) is an expression of the unitary relation between the flux lost from the elastic channel and the imaginary part of  $\mathfrak{A}$ .

The question of the reactive content may be posed: Assuming that some approximate theory is given for  $\mathfrak{U}$ , what is implied about the nonelastic reactions; for example, which reaction channels are included in the theory and how are the partial cross sections to these channels to be calculated? This question has been discussed recently by a number of authors<sup>2-6</sup> in the context of multiple-scattering theory, mostly concerning the first-order approximation

 $\mathfrak{U} \simeq \langle \sum_{i} t_{i} \rangle \tag{2}$ 

in terms of the scattering of the projectile from individual nucleons, where the expectation value is over the target ground state. The first correction to (2) is due to correlations among the nucleons. The form (2) defines a class of theories (appropriate to uncorrelated targets) which differ in what is used for  $t_i$ . It is generally agreed that inelastic scattering in which one target nucleon is certainly implied by (2). This has been studied in detail in a three-body model, by Tandy, Redish, and Bolle.<sup>2</sup> Application of their results to