Inelastic Collision of Spherical Ion-Acoustic Solitons

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The collision of spherical ion-acoustic solitons having different centers of symmetry is studied. Unlike one-dimensional collisions, the solitons are significantly changed by the collision. A new two-dimensional nonlinear object is found to be created. Remnants of the solitons that survive are found to be shifted in phase and reduced in amplitude.

Solitons solutions to *one*-dimensional nonlinear partial differential equations have been of considerable interest in recent years,[1] the Korteweg-de Vries (KdV) equation is probably the best known example. Many characteristics of solitons have been established theoretically, in computer experiments, and in laboratory experiments.[2] The most novel aspect of solitons is the collision property; two solitons preserve their identities and suffer at most a phase shift as the result of a collision[3] even though the underlying equations are nonlinear. This property has been used as a defining characteristic of one-dimensional solitons.[4]

Two special cases of nonplanar solitonlike objects have also been investigated. It has been shown that spherically and cylindrically symmetrical nonlinear density perturbations can be described by modified KdV equations.[5] Although they describe two- and three-dimensional phenomena, these equations are still one dimensional since they depend only on the radius r. Numerical solutions and laboratory experiments with plasmas have found that cylindrical solitons and spherical solitons exhibit some of the same properties as planar solitons, e.g., the relation (amplitude)$\times$(width)^2 = constant[5,6] and arbitrary compressive density perturbations are found to evolve into solitons.[7] However, one can ask whether the basic defining property of solitons, the collision property, holds for such objects and also whether more generally it holds for any collision which takes place in more than one dimension.

In a recent experiment, Nishida, Nagasawa, and Kawamata[8] found that cylindrically symmetrical imploding solitons can undergo inelastic collisions when they collide with themselves at the center of symmetry. Newell and Redekopp[9] examined theoretically the problem of the collision between two planar solitons which takes place at an angle (i.e., in two dimensions) and found that a new soliton was produced. Evidence for the production of a new solitonlike object in the collisions of two cylindrical shallow water wave solitons also exists.[10] In this Letter, we describe an experiment which shows the the collision of two spherical outgoing ion-acoustic solitons, having different centers of symmetry, gives rise to a new nonlinear *two*-dimensional object. The spherical solitons are also found to be "shifted in phase" and to have lost amplitude after the collision.

The experiment was performed in a large (volume $=0.9$ m^3) multidipole plasma device which has been described elsewhere.[11] Typical experimental parameters were neutral-argon pressure $\approx 3\times10^{-5}$ Torr, base pressure $\approx 2\times10^{-7}$ Torr, plasma density $n_e\approx 2\times10^9$ cm^{-3}, electron temperature $T_e\approx 1.7$ eV, and ion temperature $T_i\approx 0.2$ eV. Spherical solitons were excited by applying a large-amplitude pulse (+1900 V, 1 μsec width) to two steel spheres (diameter $=1$ cm, center-to-center separation $=2.0$ cm). The pulse generator was connected to the spheres through a 12-μF capacitor. Density perturbations were detected with a small (diameter ≈ 1 mm) spherical Langmuir probe which was biased to detect electron saturation current. A boxcar averager was used to plot "interferometer traces"—density perturbation versus position at fixed times after application of the launching pulse.

When the pulse was applied to either of the spheres, a pair of outward-going spherical solitons were observed. Figure 1(a) is a plot of the position of the leading soliton 6 μsec after application of the launching pulse. Data for each sphere were obtained separately. In Fig. 1(b), the density perturbation is graphed versus polar angle at each sphere again at 6 μsec. It is appar-

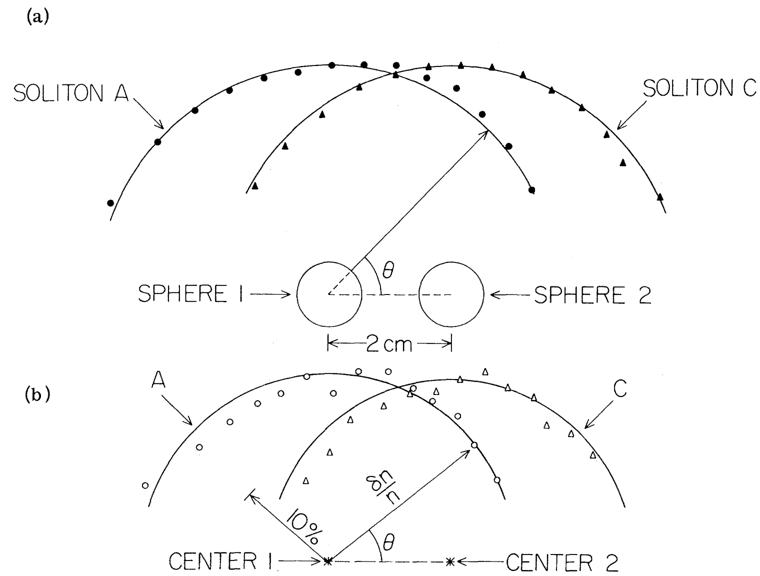

FIG. 1. Characteristics of individually excited spherical solitons. (a) Position of the leading soliton when the launching pulse is applied to each sphere separately. The data correspond to $t = 6$ μsec. (b) Density perturbation as a function of the polar angle at each sphere, at time $t = 6$ μsec.

ent that the velocity and amplitude do not depend on direction. The spherical solitons *separately* obey the relation (amplitude)$\times$(width)$^2 \approx$ constant, and their relative density perturbation $\delta n/n$ decays as $r^{-4/3}$ as expected theoretically. Their velocity is found to obey the relation $v = c_s(1 + \alpha \times \delta n/n)$, where $\alpha = 0.36 \pm 0.1$ for the data shown here and c_s is the ion sound velocity. Note that this behavior is not consistent with the pulses being pseudowaves[12] (i.e., particle bursts rather

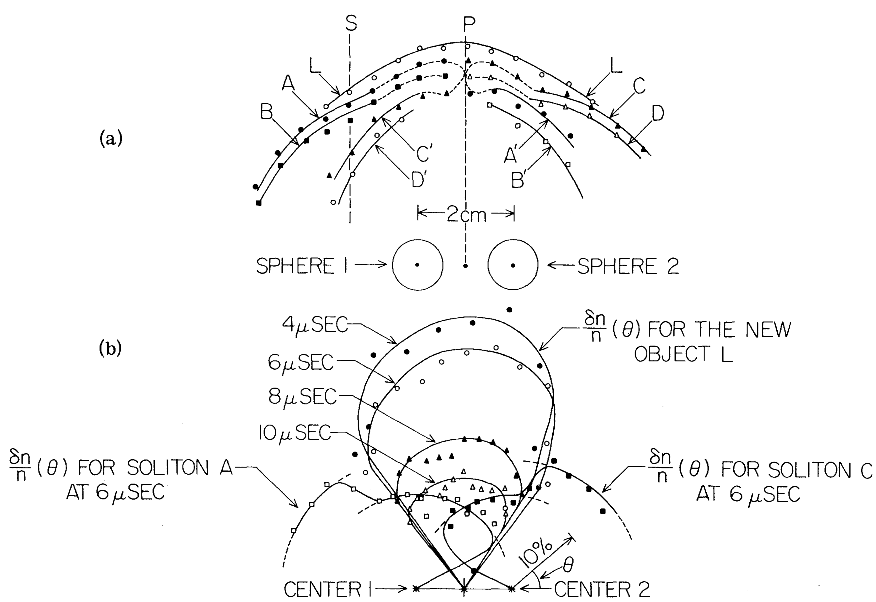

FIG. 2. Characteristics of spherical soliton interaction. (a) Positions of the solitons at $t = 6$ μsec, when the launching pulse is simultaneously applied to both spheres. The soliton-soliton collision has already occurred. The primes indicate the remnants of solitons which have survived the collisions. The *new pulse I* spatially leads the colliding pulses. (b) The amplitude of the *new pulse* vs the polar angle at the midpoint between the sphere centers. Also shown are the amplitudes of the colliding pulses at 6 μsec.

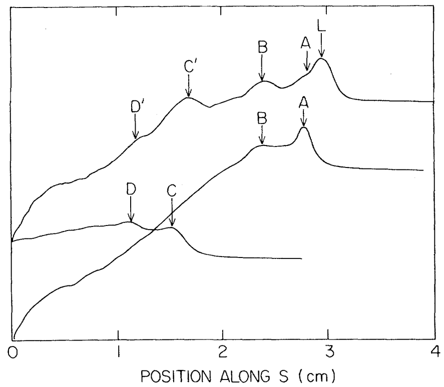

FIG. 3. "Interferometer traces" measured at 6 μsec, along the line S of Fig. 2(a). Traces are shown for pulses applied to both spheres (upper trace) and individually to each sphere (two lower traces). No large density perturbation was detected ahead of these signals.

than nonlinear waves). Because of the spherical geometry, pseudowaves should damp as r^{-2} (much faster than the observed damping) and their velocity should be independent of their local amplitude. We further excluded the possibility of pseudowaves by measuring ion distribution functions with the boxcar averager at times in the neighborhood of the pulse arrival time and much later. Significant ion bursts were not present.

We study the collision of spherical solitons by *simultaneously* applying the same voltage pulse to both spheres. Collisions of the large amplitude spherical density perturbations occur on rings in the perpendicular bisector plane (plane P) of the two spheres. At early times after the pulse is applied, solitons evolve but have not collided and the situation is not significantly different from the superposition of the single-sphere results.

Figure 2(a) shows the position of the solitons at 6 μsec, a time when collisions have already occurred. At coordinates far from plane P, we can identify spherical solitons labeled A, B, C, and D with amplitudes and positions comparable to those of solitons launched from single spheres (see Fig. 1). Closer to plane P, we observe spherical density perturbations labeled A', B', C', and D' (centered on the two spheres) which can be identified as the remnants of solitons which have survived the collisions. Comparison of the radii of the spherical objects before and after collision (A with A', etc.) shows that there is a "phase shift" during collision with the soliton remnants shifting *ahead* by about 0.2 cm. "Interferometer traces" measured at 6 μsec along line S at the edge of the interaction region are shown in Fig. 3. Traces for pulses applied separately to sphere 1, sphere 2, and simultaneously to both spheres are given. The phase shifts of C' and D' with respect to C and D are apparent while the positions of A and B with and without collisions are essentially unchanged.

The most dramatic new result is found close to the collision plane P and is already apparent along line S (see Fig. 3). A *new* object, labeled L, similar to the type predicted by Newell and Redekopp,[9] is seen in Fig. 2(b) to be created during the collision. Near the collision plane this new object has a maximum density perturbation the order of the sum of the maximum perturbations of A and C. It exists only in a narrow wedge between the two spheres, and appears to propagate outward parallel to plane P with a velocity greater than that of the individual spherical solitons from which it was created. At 6 μsec this object is already about 1 cm (or 30%) further away from the line connecting the centers of the spheres. In Fig. 2(b) we have given the amplitude of the new object L versus polar angle as well as the corresponding graphs for A and C. Note that the amplitude of the part of the soliton which has undergone collision is reduced by about 30%. From these data it appears that the new object is created at the expense of the old ones. In Fig. 2(b), we also show how the amplitude perturbation of the new object varies in time. Note that collisions continually take place along plane P but that the location of such collisions [Fig. 2(a)], viewed as superpositions of the spherical waves, lag far behind L.

We have presented data taken only in one plane. Since this experiment has cylindrical symmetry about the line connecting the two spheres, it is likely that the new nonlinear object is really shaped like a torus with a noncircular minor cross section. Such an object may be the first observation of a true two-dimensional soliton which depends on both r and θ.

In summary, we have observed the collision of spherical ion-acoustic solitons in more than one dimension and found that the one-dimensional collision property does *not* hold. Spherical objects with reduced amplitude and shifted phase do survive the collision but the most significant result is the creation of a new, large-amplitude two-dimensional nonlinear object.

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Observation of an Ion-Beam-Driven Instability in a Magnetized Plasma

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We have observed, by CO_2-laser-light scattering, large-amplitude electron-density fluctuations with $k\lambda_D \sim 0.1$ and a narrow angular spread in $\vec{k}$ about the direction normal to a transient shocklike distance. This shock propagates radially inwards in a cylindrical plasma [$n_e = (1-6)\times 10^{13}$ cm^{-3}, $T_e \lesssim 1$ eV] having an axial magnetic bias field of 170 G. The observed fluctuations are seen in a narrow region in front of the shock where a stream of reflected ions passes through the background plasma, and are thought to be driven by this stream.

A variety of microinstabilities driven by the currents and gradients produced when a large-amplitude magnetic pulse is applied to a magnetized plasma have been predicted and observed.[1-7] We report here experimental observations of a previously unrecognized microinstability, apparently driven by streaming of unmagnetized ions at right angles to a magnetic field (and along a steep negative gradient of that field) in a deuterium plasma.[8] These ions are reflected from an imploding potential barrier associated with a large-amplitude magnetic disturbance applied to the surface of a cylindrical plasma in a fast θ-pinch device. The instability is observed as a large (10^4–10^5) enhancement of scattered-light intensity over thermal level near $k\lambda_D = 0.1$, seen by means of small-angle scattering of CO_2-laser light. These fluctuations have wave vector $\vec{k}$ lying in a fan-shaped volume oriented along the (radial) direction of reflected ion streaming, with angular half-width about 20° in the plane perpendicular to the magnetic field, and about 5° out of the plane perpendicular to $\vec{B}$. They occur for ion drift speeds that are several times the local Alfvén speed.

The θ-pinch device has been previously described.[9] Birefly, it has a coil 50 cm in diameter and 100 cm long, surrounding a Pyrex vacuum vessel of 340-cm length and 46-cm i.d. Deuterium plasmas of electron density $(1\text{–}6)\times 10^{13}$ cm^{-3} and quasisteady axial magnetic "bias" field of 170 G are prepared by means of a ringing discharge into the coil followed by a slowly changing "bias" discharge. A shocklike implosion is driven by a magnetic pulse having amplitude 2.5 kG and rise time 300 nsec, applied by switching a high-

