Second-Harmonic Emission at Resonance Absorption

G. Auer
Institute for Theoretical Physics, Innsbruck University, Innsbruck, Austria

and

K. Sauer and K. Baumgärtel
Central Institute for Electron Physics, Academy of Sciences of the German Democratic Republic, Berlin, German Democratic Republic

(Received 22 February 1979)

This paper reports on numerical studies of the generation of second-harmonic radiation by a p-polarized electromagnetic wave incident on an inhomogeneous overdense plasma. Maximum second-harmonic emission coincides with maximum absorption of the fundamental wave. The importance for second-harmonic emission of electromagnetic structure resonances due to density plateaus near the critical layer is demonstrated.

The generation of the second harmonic (SH) has been observed in laser[1-3] and microwave[4,5] plasma irradiation when a p-polarized electromagnetic wave is incident obliquely on an imhomogeneous overdense plasma. Analytical studies[6,7] have predicted some of the main characteristics of SH emission: The SH increases with the square of the amplitude of the fundamental wave, it is emitted specularly in the same direction as the reflected wave, and it assumes a maximal value for a certain angle of incidence which is determined by the density scale length of the plasma.

In this work, the reflection of a p-polarized wave and the associated SH generation in an inhomogeneous cold plasma are investigated by considering a plasma composed of a large number of homogeneous layers[8,9] with a stepwise change in plasma density from one layer to the next. Only a rigid plasma is studied here; i.e., no plasma motion is allowed. Calculations including the reaction of the plasma to the ponderomotive forces will be presented in a forthcoming paper.

When the influence of the second harmonic back on the fundamental wave is neglected, the electric field oscillating at twice the frequency ω_0 of the incident wave is governed in each homogeneous layer by the usual linear wave equation with a given nonlinear source term,

$$\nabla^2\vec{E}^{(2)} - \nabla(\nabla\cdot\vec{E}^{(2)}) + \left(\frac{2\omega_0}{c}\right)^2 \epsilon(2\omega_0)\vec{E}^{(2)} = -\frac{4\pi}{c^2}\frac{\partial \vec{j}_{NL}{}^{(2)}}{\partial t}. \quad (1)$$

The dielectric of the cold plasma, including an effective collision frequency, is

$$\epsilon(\omega) = 1 - (\omega_p{}^2/\omega^2)(1+i\nu/\omega)^{-1}. \quad (2)$$

The nonlinear current density of frequency $2\omega_0$ due to the fundamental wave is given by

$$\vec{j}_{NL}{}^{(2)} = -en^{(1)}\vec{v}^{(1)} - en_0\vec{v}_{NL}{}^{(2)} = -en^{(1)}\vec{v}^{(1)} - i(n_0e^3/4m^2\omega_0{}^3)\nabla(\vec{E}^{(1)}\cdot\vec{E}^{(1)}). \quad (3)$$

Because in a cold plasma only transverse waves can propagate, the density perturbation $n^{(1)}$ and hence the first part of the nonlinear current vanish within each layer. At the boundaries, however, a surface charge is induced due to the jump of the longitudinal component of the electric field $E_L{}^{(1)}$. (By longitudinal component we mean that parallel to the density gradient.) This leads to a surface current and consequently to a jump of the transversal magnetic field,

$$B_{T(i-1)}{}^{(2)} - B_{T(i)}{}^{(2)} = -\frac{4\pi}{c}\frac{ieE_T{}^{(1)}}{m\omega_0}(E_{L(i)}{}^{(1)} - E_{L(i-1)}{}^{(1)}). \quad (4)$$

The second part of the nonlinear current is a gradient, and thus the field due to this term is irrotational and is given by

$$\vec{E}_S{}^{(2)} = -\frac{\omega_p{}^2}{\omega^2}\frac{e}{m\omega_0\epsilon(2\omega_0)}\tfrac{1}{8}\nabla(\vec{E}^{(1)}\cdot\vec{E}^{(1)}). \quad (5)$$

When the fundamental wave quantities are known, the SH in each layer is given by the sum of the source field (5) and the solution of the homogeneous wave equation. Application of the boundary conditions for the transverse electric and magnetic fields yields a set of coupled equations for the amplitudes of the SH in the various layers. This set has a pentadiagonal structure and can be solved numerically by standard methods.

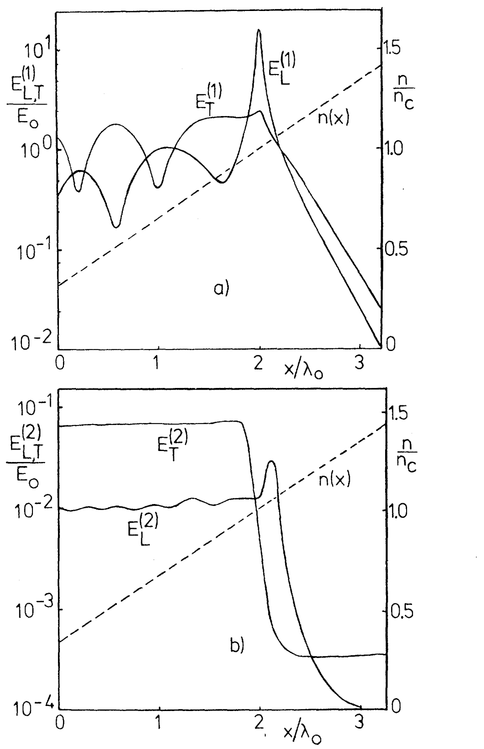

FIG. 1. Longidutinal and transverse electric fields for a linear density profile ($L = 3\lambda_0$, $n_{max} = 2n_c$, $eE_0/m\omega c = 0.005$, $\nu/\omega_0 = 0.005$, $\theta = 15°$). (a) Fundamental wave; (b) SH.

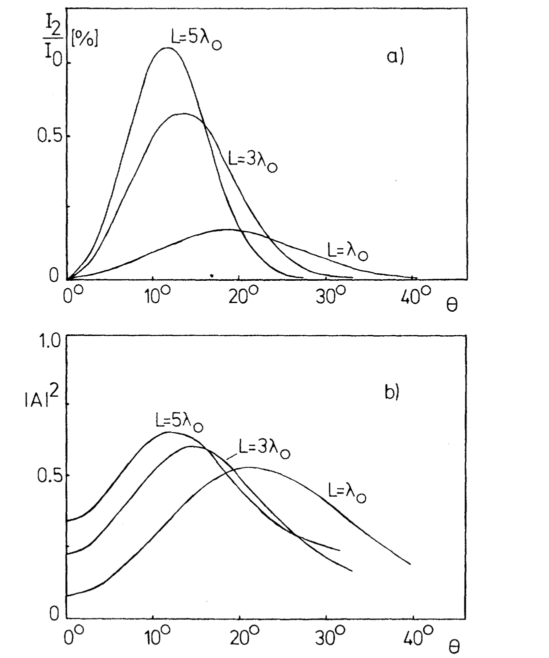

FIG. 2. (a) Relative intensity of backscattered SH and (b) absorption of the fundamental wave for a linear density profile as function of the angle of incidence and for different density scale lengths ($n_{max} = 2n_c$, $eE_0/m\omega c$ 0.005, $\nu/\omega_0 = 0.005$).

In the following some results of our calculations will be presented. In Fig. 1 the longitudinal and transverse field components are plotted for both the fundamental wave and the SH in the case of a linear density profile. The fundamental wave shows the well-known behavior, a standing-wave pattern up to the critical density, a sharp resonance of the longitudinal component at the critical density (note the logarithmic scale!), and then a quick decrease. The SH is generated around the critical layer and is emitted into the direction of lower density only, which is in agreement with analytical results.[7]

In Fig. 2(a) the intensity of the backscattered SH is shown as a function of the angle of incidence of the fundamental wave. With growing density scale length L [defined by $(d\ln n/dx)^{-1}$ at the critical density n_c] the maximum of the SH emission increases and shifts towards smaller angles; the angular region for SH emission, however, becomes smaller. When comparing with Fig. 2(b), where the corresponding absorption of the fundamental is shown, we see that the angle for maximum SH emission is somewhat smaller

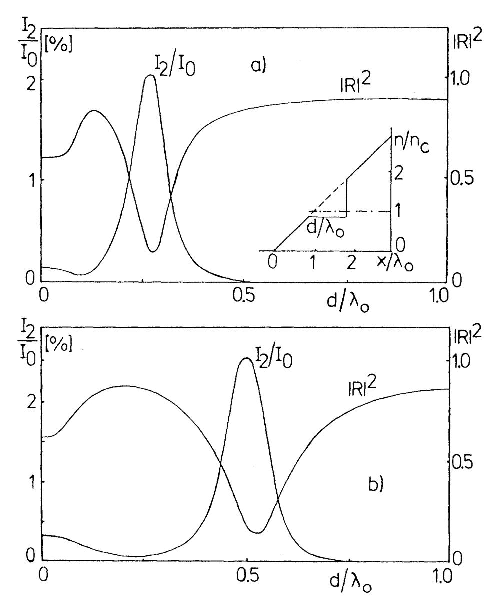

FIG. 3. Relative intensity of the backscattered SH and the reflection coefficient for a density profile with a plateau as functions of the plateau length ($n_{max} = 3n_c$, $eE_0/m\omega c = 0.005$, $\nu/\omega_0 = 0.005$, $\theta = 30°$). (a) $n_{plateau} = 0.9n_c$; (b) $n_{plateau} = 0.85n_c$.

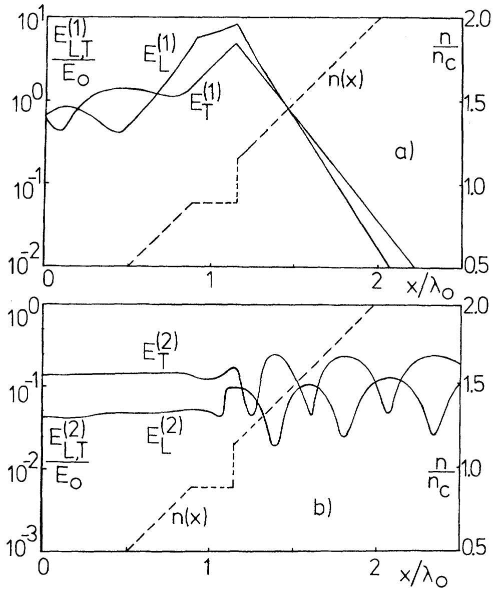

FIG. 4. Longitudinal and transverse electric field for the situation of Fig. 3(a) and a plateau length of $0.25\lambda_0$. (a) Fundamental wave; (b) SH.

than that for maximum absorption. We further see that for a scale length of $3\lambda_0$ (λ_0 vacuum wavelength) and a ratio of electron quiver velocity to velocity of light ($eE_0/m\omega c$) of 0.005, the intensity of the emitted SH is maximally half a percent of the incident intensity, which is also the order of magnitude found in experiments.[2]

It is well established now that for strong incident fields a plateau is formed due to the ponderomotive forces near the critical density.[10-12] We investigated such a modified profile also, and found that this plateau formation and the structure resonances resulting therefrom can very strongly influence both the reflection of the fundamental and the SH emission. In Figs. 3(a) and 3(b) the change of these two quantities with the length of the plateau is shown when all other parameters are kept constant. In Figs. 4(a) and 4(b) the fields for a profile with plateau are plotted. A characteristic enhancement of both the longitudinal and the transverse fundamentals is observed which can be understood in terms of the excitation of surface waves[13] which are supported by the modified density profile. The significant feature in the SH behavior is that the SH does not decrease beyond the critical density in contrast to the simple linear profile considered earlier. A closer investigation shows that the SH is still generated near the critical density (only here the source terms are nonnegligible), but it is emitted now in both directions. Thus, in this case, the properties of the plasma up to the critical point for the SH, that is up to $4n_c$, are crucial for the resulting pattern of the SH, since now the SH emitted in the forward direction can be reflected and thus interfere with itself. The emission of SH radiation in the forward direction was also observed by Batanov and Silin[5] who investigated a Gaussian profile with a peak density of $3n_c$.

This work was supported in part by the Fonds zur Förderung der wissenschaftlichen Forschung (Austria) under Grant No. 2781/S.

[1]K. Eidman and R. Sigel, Phys. Rev. Lett. 34, 799 (1975).

[2]J. L. Bobin, in *Plasma Physics: Nonlinear Theory and Experiments*, edited by H. Wilhelmsson (Plenum, New York, 1977).

[3]E. A. McLean, J. A. Stamper, B. H. Ripin, H. R. Griem, J. M. McMahon, and S. E. Bodner, Appl. Phys. Lett. 31, 825 (1978).

[4]A. Y. Wong, in *Laser Interaction and Related Phenomena*, edited by H. Schwarz and H. Hora (Plenum, New York, 1977), Vol. 4.

[5]G. M. Batanov and V. A. Silin, Tr. Fiz. Inst. (Moskva) 73, 87 (1974).

[6]N. S. Erokhin, V. E. Zakharov, and S. S. Moiseev, Zh. Eksp. Teor. Fiz. 56, 179 (1969) [Sov. Phys. JETP 29, 101 (1969)].

[7]N. S. Erokhin, S. S. Moiseev, and V. V. Mukhin, Nucl. Fusion 14, 333 (1974).

[8]M. V. Mueller, Phys. Rev. Lett. 30, 582 (1973).

[9]K. Baumgärtel and K. Sauer, to be published.

[10]K. C. Estabrook, E. Valeo, and W. L. Kruer, Phys. Fluids 18, 1151 (1975).

[11]D. W. Forslund, J. M. Kindel, K. Lee, and E. L. Lindman, Phys. Rev. Lett. 36, 35 (1976).

[12]N. E. Andreev and V. P. Silin, Fiz. Plazmy 4, 908 (1978).

[13]J. M. Kindel, K. Lee, and E. L. Lindman, Phys. Rev. Lett. 34, 134 (1975).