[5]This technique was extended and applied to φ^4 field theory in four dimensions with encouraging results: N. N. Khuri, Phys. Lett. 82B, 83 (1979). I follow the Borel-like summation technique introduced by Khuri.

[6]It is well known that QCD Green's functions are not Borel summable [see G. 't Hooft, in Proceedings of the School of Physics "Ettore Majorana," "Subnuclear Physics," Erice, Sicily, 1977 (to be published)]. This is because of at least two problems: (1) The Borel function is known to have singularities along the real positive axis; (2) QCD Green's functions have accumulation of singularities at the origin of the complex coupling constant plane. The first problem is avoided by the technique introduced in Ref. 5. The second problem comes from the q^2 dependence of the Green's function and it does not apply to the β function.

[7]The detail of this analysis is presented by A. I. Sanda, Rockefeller University Report No. COO-2232B-178 (to be published).

[8]'t Hooft, Ref. 6.

[9]$\alpha_2(Q^2)$ is a good approximation to $\alpha(Q^2)$ because of the the smallness of J_3.

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Spin and Spin-Isospin Distribution in Some Medium-Heavy Nuclei

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Pairs of odd-even nuclei with the wave functions of the valence nucleons different simply by an interchange of neutrons and protons (pseudo mirror pairs) have been identified and mirror symmetry in the subspace of valence nucleons conjectured. Isoscalar and isovector magnetic moments have been studied, for a given shell and for an increasing number of odd nucleons. Impressive regularities become evident.

The peculiar convenience of the isospin formalism for discussing nuclear magnetic moments and β decay has been noticed long ago.[1] If all the $2T+1$ magnetic moments of an isospin multiplet are known, they should be completely determined by the two constants μ_s and μ_v in the relation

$$\mu(T, T_3) = \mu_s(T) + \mu_v(T)T_3,$$

where $\mu_s(T)$ is the isoscalar magnetic moment and $\mu_v(T)$ is the isovector magnetic moment. The sum of the moments of two states of an isospin multiplet with T_3 and $-T_3$ *(mirror pairs)* is thus equal to $2\mu_s$ and the difference is equal to $2\mu_v T_3$; then the magnetic moments of *two* members of an isospin multiplet completely determine the others.

If we ignore mesonic effects and relativistic corrections, we have

$$\langle \textstyle\sum_i \sigma_{iz} \rangle = \frac{[\mu(T, T_3) + \mu(T, -T_3)] - J}{g_p + g_n - 1}, \tag{1}$$

$$\langle \textstyle\sum_i \sigma_{iz}\tau_{i3} \rangle = \frac{[\mu(T, T_3) - \mu(T, -T_3)] - \langle \sum_i j_{iz}\tau_{i3}\rangle}{g_p - g_n - 1}. \tag{2}$$

In the following we will use the symbols $\langle\sigma\rangle$ and $\langle\sigma\tau\rangle$ for the left-hand sides of formulas (1) and (2).

In general these expressions are independent and there are *no further relations among them.* Only for definite models, which imply certain prescriptions for the nuclear wave function, can we write down relations involving these quantities. Such relations offer tests for the consistency of the nuclear wave function assumed. When analyzed within an isospin multiplet, the accumulation of data on nuclear magnetic moments of several nuclei resulted in the discovery of several outstanding regularities along the shells for $\langle\sigma\rangle$ and $\langle\sigma\tau\rangle$. This greatly helped to clarify properties of definite models and wave functions.[2,3] The main results of the analysis have been traced back in several review articles and text books.[4] Because of the experimental difficulties of determining magnetic moments of states belonging to triads or even higher isospin multiplets, most of the established regularities concern the magnetic moments of light mirror nuclei (isodoublets). In spite of the lack of data on magnetic moments within isospin multiplets higher than doublets, in this Letter we try to obtain phenomenological information on $\langle\sum_i\sigma_i\rangle$ and $\langle\sum_i\sigma_i\tau_i\rangle$ along a given shell, for several odd-even medium-heavy nuclei. To this end we have conjectured that isospin symmetry could be respected within the isospace of the valence nucleons so that combining the magnetic moments of nuclei which are mirror in the subspace of the valence nucleons we could obtain information on isoscalar and isovector magnetic moments separately. More specifically the nuclear function of some odd-even nuclei $|A,J,T,T_3\rangle$ has been supposed to factorize into a core part $(J=0)\,|\overline{A},J=0,\overline{T}_3\rangle$ and an odd-nucleon-group part $|(A-\overline{A}),J,t,t_3\rangle$ $(t_3=T_3-\overline{T}_3,\ t=|t_3|)$ carrying the angular momentum of the nucleus (valence nucleons). Guided by the j-j shell-model scheme, we have selected pairs of nuclei with the odd-group part of the wave function differing simply by an interchange of neutrons and protons. (Their cores may differ considerably both in $\overline{A}$ and $\overline{T}_3$.) These pairs of nuclei are obviously mirror in the subspace of the valence-group nucleons $\langle T_3-\overline{T}_3=-(T_3'-\overline{T}_3')$, where the apex refers to the other member of the pair]; furthermore, since the core of these pairs has $J=0$, the magnetic moment of each of these nuclei is described in terms of the (valence) odd-group part of the wave function:

$$\vec{\mu}=\vec{\mu}_{\text{valence}}+\vec{\mu}_{\text{core}},$$

and

$$\langle A,J,T,T_3|\vec{\mu}|A,J,T,T_3\rangle =\langle A-\overline{A},J,t,t_3|\vec{\mu}_{\text{valence}}|A-\overline{A},J,t,t_3\rangle.$$

In the following we shall refer to such pairs of nuclei as "pseudo mirror pairs."

Pseudo mirror pairs may be considered mirror members of an isospin multiplet in the isospin subspace $\vec{t}$ of the valence nucleons. In this space we can attempt an isospin analysis following formulas (1) and (2). (For mirror pairs and tentatively for pseudo mirror pairs the *sum* of the magnetic moments is twice the isoscalar part, whereas the *difference* is twice the isovector one.)

In the case of mirror pairs, formula (2) has been further approximated as

$$|\langle\textstyle\sum_i j_{iz}\tau_{i3}\rangle|=J. \tag{3}$$

Within a j-j single-particle basis, first-order corrections are included in (3). (A further improvement of this approximation, within the seniority scheme, is discussed in Talmi.[4])

In this work we have considered the following pseudo mirror pairs (in parentheses we report the experimental magnetic moment[5]): In the $J^\pi=\frac{1}{2}^-$ shell,

^{89}Y (− 0.14), ^{75}Ge (0.51);

^{91}Y (− 0.16), ^{77}S (0.53).

The first pair have a proton and neutron, respectively, in the $2p$ shell. The remaining nucleons fill completely various shells, so that they are included in the $(J=0)$ core. The second pair is obtained from the previous one by adding a pair of neutrons and protons, respectively. (Because of the Pauli principle, a pair of odd-type nucleons cannot be added to the $2p_{1/2}$ shell.)

In the $J^\pi=\frac{3}{2}^-$ shell,

^{65}Cu (2.39), ^{61}Ni (− 0.75);

^{67}Ga (1.85), ^{63}Zn (− 0.28).

The first pair have a proton and neutron, respectively, in the $2p_{3/2}$ shell, ^{67}Ga has three protons in the $2p_{3/2}$ shell, whereas ^{63}Zn has three neutrons in the same shell. Note that these nuclei all have four neutrons on the $1f_{5/2}$ shell and this shell is just in between the two partners of the p shell.

In the $J^\pi=\frac{3}{2}^+$ shell,

^{199}Au (0.27), ^{139}Ce (0.95);

^{169}Tm (0.54), ^{125}Te* (0.74, 443 keV).

The first pair have a proton and a neutron *hole*, respectively, in the $2d_{3/2}$ shell; the second pair has three neutron and proton *holes* on the same shell.

In the $J^\pi=\frac{5}{2}^+$ shell,

^{141}Pr (4.14), ^{91}Zr (− 1.30);

^{147}Pm* (3.54, 91 keV), ^{109}Cd (− 0.83);

^{151}Eu (3.46), ^{111}Cd* (− 0.79, 247 keV).

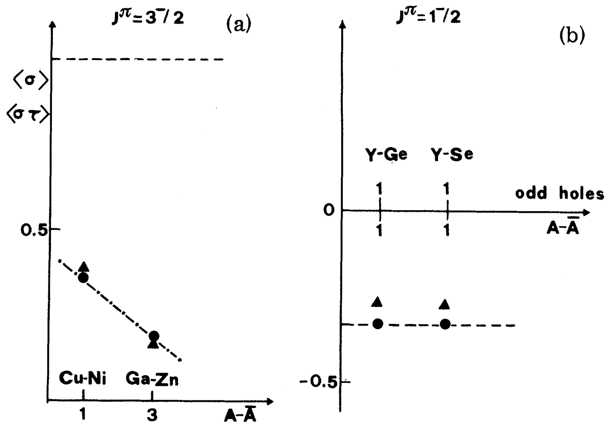

FIG. 1. Values of $\langle\sigma\rangle$ (solid circles) and $\langle\sigma\tau\rangle$ (solid triangles) vs $A-\bar{A}$ as deduced from magnetic moment data using formulas (1)–(3) ($A-\bar{A}$ is the number of the valence nucleons of the considered pair). The broken horizontal line indicates the Schmidt values of $\langle\tau\rangle$ and $\langle\sigma\tau\rangle$ for the shell. The dot-dashed line is the best straight fitting our data.

These three pseudo mirror pairs are characterized by having one, three, five protons and neutrons, respectively, on the $2d_{5/2}$ shell.

In the $J^\pi = \frac{5}{2}^-$ shell,

^{85}Rb (1.35), ^{67}Zn (0.88);

^{73}As* (1.62, 67 keV), ^{65}Zn (0.77).

The first pair have one proton hole and a neutron hole, respectively; the second pair have five proton holes and neutron holes, respectively (^{65}Zn might have only three holes).

In the $J^\pi = \frac{7}{2}^-$ shell,

^{37}K* (5.2, 1.38 keV), ^{37}Ar (− 1.33);

^{41}Sc (5.43), ^{41}Ca (− 1.59);

^{51}V (5.15), ^{43}Ca (− 1.31);

^{59}Co (4.62), ^{51}Cr (− 0.94).

The first two pairs have a proton and a neutron in the $f_{7/2}$ shell; the third pair have three protons and neutrons, respectively, on the shell; the last pair have seven protons and neutrons, respectively.

In Figs. 1–3 we show the values of $\langle\sigma\rangle$ and $\langle\sigma\tau\rangle$ as extracted from formulas (1)–(3) for a given shell and for an increasing number of odd nucleons. In spite of approximation involved in formulas (1)–(3) many regularities become evident. The results are very impressive and follow strictly the behavior of the data deduced in analyzing the mirror nuclei.[2] (The reader should refer back to this article to appreciate fully the present results.)

(a) The systematics suggest that $\langle\sigma\rangle$ and $\langle\sigma\tau\rangle$ vary essentially in the same way along the shell. This implies that corrections for the deviation of magnetic moments from Schmidt values affect both the isoscalar and isovector parts of the odd-group nucleons.

(b) $\langle\sigma\rangle$ and $\langle\sigma\tau\rangle$ decrease quite regularly when pairs of odd-group nucleons are added in the same optical shell (a mean decrease for each added pair is about $0.2\mu_N$.) Note that in the case

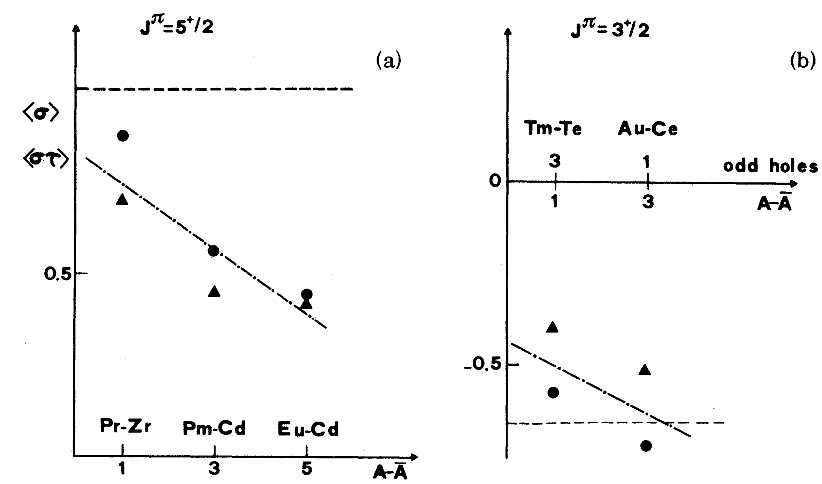

FIG. 2. Same as Fig. 1.

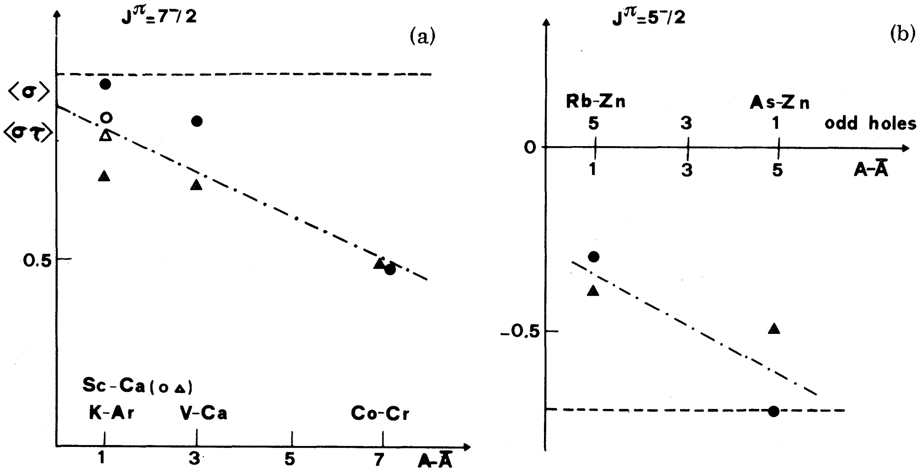

FIG. 3. Same as Fig. 1. Open symbols refer to the Sc-Ca pair.

of $J=\frac{1}{2}^-$ the pair added belongs to the even group: We do not have any decrease in this case. [Compare with Fig. 1(c) of Ref. 2.]

(c) Keeping in mind that in the extreme j-j scheme $\langle\sigma\rangle$ and $\langle\sigma\tau\rangle$ values are 1 for $J^\pi=\frac{1}{2}^+,\frac{3}{2}^-,\frac{5}{2}^+,\frac{7}{2}^-$, etc., and $-0.33, -0.66, -0.71$ for $J^\pi=\frac{1}{2}^-,\frac{3}{2}^+,\frac{5}{2}^-$, we can follow in an instructive manner the effects of the configuration mixing along the shells. The data reflect the expectation that the j-j coupling is an adequate approximation at the beginning of the $l+\frac{1}{2}$ orbit and at the end of the $l-\frac{1}{2}$ orbit. In fact, at the beginning of the $\frac{5}{2}^+$ and $\frac{7}{2}^-$ shells we have values for $\langle\sigma\rangle$ and $\langle\sigma\tau\rangle$ near unity; the role of the mixed levels ($\frac{3}{2}^+$ and $\frac{5}{2}^-$, respectively) becomes more and more important with increasing $A-\bar{A}$[see Figs. 2(a) and 3(a)]. Similarly, at the end of the shells $\frac{1}{2}^-$, $\frac{3}{2}^+$, and $\frac{5}{2}^-$ ($l-\frac{1}{2}$ type) the Schmidt predictions emerge quite well [see Figs. 1(b), 2(b), and 3(b)]. For discussing these shells it is convenient to use the *hole* picture. Schmidt predictions emerge when *one hole* is present in the shell. With an increased number of *holes* in the shells, configuration mixing is allowed with the spin-orbit partner shells ($l+\frac{1}{2}$ type) and $\langle\sigma\rangle$ and $\langle\sigma\tau\rangle$ increase [read Figs. 1(b), 2(b), and 3(b) from right to left] towards the values of the mixed levels. For the $\frac{1}{2}^-$ shell there is no spin-orbit partner available, so that (first-order) configuration mixing is zero. The situation is very similar to that discussed in Ref. 2 for $1p_{1/2}$ shell and $1d_{3/2}$ shell of mirror pairs. Remember that for $J=l+\frac{1}{2}$ odd-Z ($-N$) nuclei, if there is more than one proton (neutron) in the shell, the admixed configuration can be performed by breaking a pair of odd $J=0$ nucleons in the $J=l-\frac{1}{2}$ state. Similarly for $J=l-\frac{1}{2}$ nuclei, if there is more than one hole in this state, the admixed configuration can be performed by breaking a pair of odd $J=0$ holes and transferring one hole from the state $l-\frac{1}{2}$ to the state $l+\frac{1}{2}$.

A special comment is needed for the case $J^\pi=\frac{3}{2}^-$ ($l+\frac{1}{2}$ type). Comparing the Cu-Ni pair with the Ga-Zn pair we can see the quenching effect on $\langle\sigma\rangle$ and $\langle\sigma\tau\rangle$ due to the configuration mixing of the $p_{3/2}, p_{1/2}$ orbits by adding a couple of *odd* nucleons on the p shell. However, the absolute value of $\langle\sigma\rangle$ and $\langle\sigma\tau\rangle$ is influenced by other effects: mixing between $1p_{3/2}$ and $f_{5/2}$ levels and possibly a contribution from a $J=2$ core part (as a matter of fact all these nuclei have a large quadrupole moment), so that it seems natural to consider that the zeroth-order configurations for these nuclei are essentially mixed. This should explain why $\langle\sigma\rangle$ and $\langle\sigma\tau\rangle$ are much more depressed than the Schmidt values at the beginning of the $2p$ shell.

It is a remarkable fact that all these contributions affect the isoscalar and the isovector parts in a similar way.

A quantitative explanation of our systematics can be easily worked out from the pioneer first-order configuration mixing calculations of the nuclear magnetic moments.[6,7] Using a δ-function spin-dependent interaction and making certain simplifying assumptions (disregard the mass-number dependence of the radial integrals and energy denominators and take into account $l\pm\frac{1}{2}$ mixing of odd nucleons only) the correction $\Delta\sigma$ on $\langle\sigma\rangle$ and $\langle\sigma\tau\rangle$ induced by the first-order mixing turns out to lie between $|\Delta\delta|\simeq 0.18$ ($J=\frac{7}{2}^-$) and $|\Delta\delta|\simeq 0.25$ ($J=\frac{3}{2}^-$) for each pair of odd nucleons or holes broken ($\Delta\delta = 0$ for $J^\pi = \frac{1}{2}^-$), so that our data should lie on a straight line. (The results deduced from our tables are consistent with these predictions.) Finally, let us note that these pseudo mirror pairs all satisfy the relations

$$[\mu(t, t_3) - J]/\mu(t, -t_3) = -1.20,$$

within a few percent. This is an enlargement of the so-called De Shalit relation $(g_s - g_l)_p/(g_s - g_l)_n = -1.20$ deduced in the frame of the extreme single-particle model.

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Total Scattering Cross Section for Na on He Measured by Stimulated Photon Echoes

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A technique which employs heretofore unappreciated aspects of the optical stimulated echo was developed and used to measure the total scattering cross section for Na($3S_{1/2}$)-He and Na($3P_{1/2}$)-He collisions. The repulsive-core–dominated Na($3S_{1/2}$)-He collision is found to be relatively soft. The cross section for degradation of the $3S_{1/2}$-$3P_{1/2}$ superposition, determined from photon-echo–decay measurements, is anomalous in that it is smaller than the total elastic-scattering cross section with Na in either state separately.

The study of atomic scattering, traditionally carried out by atomic-beam measurements,[1] has recently been supplemented by laser techniques.[2] In this Letter we report the first application of a coherent-transient laser technique to the study of velocity-changing collisions (VCC) in an atomic system.[3] Our technique utilizes unappreciated novel aspects of the three-pulse stimulated photon echo[4] (SP echo) to provide relatively direct measurements of VCC. We predict and demonstrate experimentally (e.g., Fig. 1) that phase memory information stored in *either* the ground

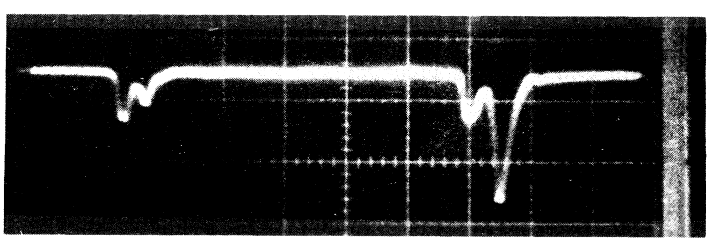

FIG. 1. Oscilloscope trace showing scattered light from three excitation pulses (third row of the table in Fig. 3 with $\vec{K}'\cdot\hat{z}/|\vec{K}'| = -1$) and the subsequent SP echo (fourth pulse) pronounced on the $3S$-$3P_{1/2}$ transition of Na when the second-to-third pulse separation is $\cong 17$ times the 16-nsec lifetime of the $3P_{1/2}$ state. At the time of the third pulse a negligible $\cong 10^{-8}$ of the intially excited population remains in the $3P_{1/2}$ state. The echo persists because of the information stored in the ground state alone. (Horizontal: 50 nsec/div.)

