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# Nonperturbative Determination of $\alpha(q^2)$ and Its Experimental Implications

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The lowest-order perturbative expression for $\alpha(q^2)$, the quantum-chromodynamic running coupling constant, is inadequate for describing the moments of nucleon structure functions at present energies. I propose to determine $\alpha(q^2)$ with use of a new Borel-like summation technique described by Khuri. The $\alpha(q^2)$ which emerges from this procedure is free of the Landau ghost and it is consistent with experiments in both spacelike and timelike regions.

Recently, scaling violation in nucleon structure function has been studied in detail.[1] The first indications are that the observed scaling violation is in quantitative agreement with theoretical predictions based on quantum-chromodynamic (QCD) perturbation theory. This is an exciting result. It is, therefore, important to ask the following question: How valid is the perturbation theory in the range of momentum transfer (1 GeV$^2 \lesssim Q^2 \lesssim 10$ GeV$^2$) most relevant in this study?

Consider the $\beta$ function including the two-loop effects[2]:

$$\beta_2(g) = -\beta_0 g^3/16\pi^2 - \beta_1 g^5/(16\pi^2)^2, \tag{1}$$

where $\beta_0 = 11 - \frac{2}{3}N_f$, $\beta_1 = 102 - \frac{38}{3}N_f$, and $N_f$ is the number of quark flavors. The effective coupling corresponding to $\beta_2$ is[3]

$$\alpha_2(Q^2) = [\beta_0/4\pi)\ln(Q^2/\Lambda^2) + \eta f(Q^2)]^{-1},$$

where (2)

$$f(Q^2) = \ln[1 + \beta_0{}^2/\beta_1 \ln(Q^2/\Lambda^2) + f(Q^2)].$$

$Q^2 = -q^2$ in the spacelike region, $\eta = \beta_1/4\pi\beta_0$, and $\Lambda$ is an adjustable scale parameter. In Fig. 1, we compare $\alpha_1(Q^2) = [\beta_0/4\pi \ln(Q^2/\Lambda^2)]^{-1}$ (curve $A$), and $\alpha_2(Q^2)$ (curve $B$). Clearly, more convergent $\alpha(Q^2)$ is necessary in order to compare the theory and experiments.

As a possible solution[4,5] to this problem, I propose to use a Borel-like summation technique[6,7] in computing the $\beta$ function. Consider a perturbative expression for the $\beta$ function

$$g\beta(g) = \sum_{n=2}^{\infty} a_n (g^2)^n. \tag{3}$$

It is widely suspected that the radius of convergence of this power series is zero. In order to

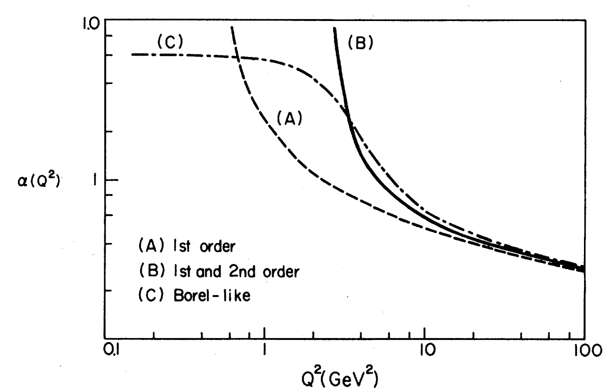


FIG. 1. The running coupling constants based on one leading-order term (curve $A$) and two leading-order terms (curve $B$), in the perturbation expansion for the $\beta$ function are compared. Curve $C$ is obtained from the Borel-like summation. All three curves are normalized at so that $\alpha(100\text{ GeV}^2) = 0.27$ corresponding to $\Lambda = 0.75$ GeV (see Ref. 1) for curve $A$.

define (3), consider the Borel function

$$B(z)=\sum_{n=2}^{\infty}\frac{a_n}{(n-1)!}z^{n-1}+B_{\rm dyn}, \quad (4)$$

where $B_{\rm dyn}$ is a dynamical term absent in the perturbation theory. It is known[8] that $B(z)$ has singularities along both positive and negative real axis from $z=32\pi^2/\beta_0$ to $\pm\infty$. I assume, following additional properties for $B(z)$, that (i) $B(z)$ has a nonzero radius of convergence; (ii) there are no singularities on the $z$ plane except on the real axis; (iii) the integral (5) given below converges for some $g^2$; (iv) the dynamical term $B_{\rm dyn}$ can be ignored for the present application. With the above assumptions, one can define[5]

$$g\beta(g)=\tfrac{1}{2}[g\beta_+(g)+g\beta_-(g)],$$
$$g\beta_\pm(g)=\int_{C_\pm}B(z)\exp(-z/g^2)\,dz, \quad (5)$$

where $C_\pm$ represents a path defined by $z=|z|e^{i\varphi}$, $0<\pm\varphi<\pi/2$, $0<z<\infty$. Equation (5) is the only choice which preserves the reality[5] of $\beta(z)$.

Now I compute $g\beta(g)$, making full use of known and assumed analytic properties of $B(z)$. The result is[5,7]

$$g\beta(g)=\sum_{n=2}^{\infty}\frac{b_n}{(n-1)!}\mathcal{G}_{n-1},$$
$$\mathcal{G}_n=\int_0^\infty \mathrm{Re}(\omega^n)\exp(-z/g^2)\,dz, \quad (6)$$
$$\omega=[a-i(z^2-a^2)^{1/2}]/z,\quad a=32\pi^2/\beta_0,$$

and $b_N$ is fully determined in terms of $a_1,\ldots,a_N$. From $a_2$ and $a_3$ given in (1), two leading terms, $b_2$ and $b_3$ in (6), are determined. We find that although $b_4$ is not known, $\mathcal{G}_3$ has a zero at $g^2/4\pi=1.1$ and remains negligible (compared to $\mathcal{G}_1$ and $\mathcal{G}_2$) for $g^2/4\pi\lesssim 1.5$. Thus two terms in (6) will give an accurate determination of $g\beta(g)$ for $g^2/4\pi\lesssim 1.5$. In Fig. 1, curve $C$, I show the resulting[9] $\alpha(Q^2)$. Although my determination of $\alpha(Q^2)$ cannot be trusted when $\alpha(Q^2)\gtrsim 1.5$. It should be noted that $\alpha(Q^2)$ no longer diverges unlike $\alpha_1(Q^2)$ and $\alpha_2(Q^2)$. *The undesirable Landau[10] ghost present in the perturbation theory is absent in this formalism.*

The running coupling constant determined above can be tested in both timelike and spacelike $q^2$ regions.

(A) *Spacelike region.*—With use of the notation of Bardeen *et al.*,[11] the moments of $F_3$ structure function are given by

$$M_n=A_nC_n[\alpha(Q^2)]\exp\left\{-\tfrac{1}{2}\int_{\alpha(Q_0^2)}^{\alpha(Q^2)}[\gamma^n(\alpha)/g\beta(g)]\,d\alpha\right\}. \quad (7)$$

While terms of $O(\alpha^2)$ and $O(\alpha^3)$ are known for $g\beta(g)$, only the terms of $O(\alpha^0)$ and $O(\alpha)$ are known for $C_n$ and $\gamma^n$. With these two terms, we cannot expect any improvement in the convergence through the Borel-like summation. Because of this ambiguity, I examine the data in two ways. (I) I kept two leading terms for $C_n$ and $\gamma^n$ and used my determination of $\beta(g)$ and $\alpha(Q^2)$ in (7). The result with $\alpha(100\ \mathrm{GeV}^2)=0.20$ is shown in Fig. 2. (II) I kept only the leading term $C_n,\gamma^n$ and used $\beta(g)$ and $\alpha(Q^2)$ determined in (7). A similar fit as that shown in Fig. 2 is obtained for $\alpha(100\ \mathrm{GeV}^2)=0.225$. The difference between the two values of $\alpha$ is a rough measure of uncertainty due to ambiguities in $C_n$ and $\gamma^n$.

As noted above, *Nachtmann moments cannot be determined (in the range 1 GeV$^2\lesssim Q^2\lesssim$ 10 GeV$^2$) by only the leading term of the perturbative expansion.* I have proposed a new interpretation of the beautiful experimental result[1] by showing that Borel-like summation can be used to determine $\beta(q^2)$. Resulting $\alpha(Q^2)$ restores the relavance of the agreement between theory and experiment.

(B) *Timelike region.*—Assumptions (i)–(iv) require $\alpha(q^2)$ to be an analytic function of $q^2$ and, once $\alpha(q^2)$ is determined in the spacelike region, it is also determined in the timelike region. Thus my phenomenological analysis is not complete unless it is accompanied by a consistency check of our $\alpha(q^2)$ in the $e^+$-$e^-$ colliding-beam process.

I have found that for a timelike region $q^2\gtrsim 1$

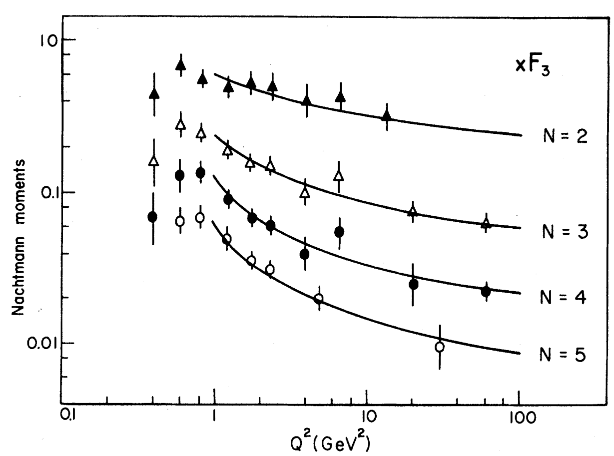


FIG. 2. Our fit to moments of $F_3$. $\alpha(100\ \mathrm{GeV}^2)=0.2$.

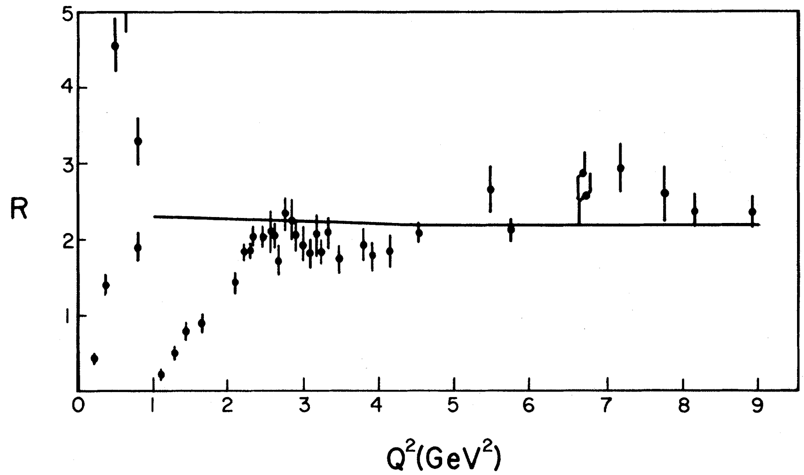


FIG. 3. $R=\sigma(e^+e^-\rightarrow \text{hadron})/\sigma(e^+e^-\rightarrow\mu^+\mu^-)$ based on my $\alpha(q^2)$. Note that the scale breaking in the timelike region can be predicted from that of the spacelike region.

$\text{GeV}^2$, $\alpha(q^2)$ can be approximated to within a few percent by $\alpha_2(q^2)$ provided that $Q^2$ is replaced by $e^{\pm i\pi}Q^2$. This is because $|\alpha(e^{\pm i\pi}Q^2)|$ is small because of the phase factor, $\pi$, and the perturbation expansion converges faster in the timelike region than in the spacelike region. The above result, $\alpha(100\ \text{GeV}^2)=0.2$, corresponds to $\Lambda=0.56$ GeV in $\alpha_2(q^2)$. This gives $|\alpha(-m_\psi^2)|=0.22$ which agrees well with the value given by the charmonium model for the $\psi$ decays,[12] $|\alpha(-m_\psi^2)|=0.2$. The QCD correction to $R$, the ratio of $\sigma(e^+e^-\rightarrow\mu^+\mu^-)$ can be approximated by[13]

$$R=\sum_q 3e_q^2\frac{3-\beta_q^2}{2}\beta_q\left\{1+\frac{1}{\pi}\frac{4}{\beta_0}\,\mathrm{Im}\left[\ln\frac{\alpha(q^2)}{1+\eta\alpha(q^2)}\right]\left[\frac{2\pi^2}{3\beta_q}-\frac{3+\beta_q}{3}\left(\frac{\pi^2}{2}-\frac{3}{4}\right)\right]\right\}, \tag{8}$$

where $\beta_q^2=1-4m_q^2/Q^2$. For 1 $\text{GeV}^2<Q^2<10$ $\text{GeV}^2$, $R$ is insensitive to the precise value of $m_u$, $m_d$, and $m_s$. In Fig. 3 I show $R$ using $m_u=m_d=250$ MeV and $m_s=360$ MeV.

I stress that aside from a weak dependence on the quark masses, the QCD expression for $R$ has no adjustable parameter once $\alpha(q^2)$ is fixed by the neutrino experiment. While the result shown in Fig. 3 is not conclusive, it is certainly encouraging.

Importance of the higher-order effects in the running coupling constant $\alpha(q^2)$ is obvious from Fig. 1. I have determined $\alpha(q^2)$ using the Borel-like summation technique which allows us to go beyond the perturbation theory. The resulting $\alpha(q^2)$ is an analytic function on the complex $q^2$ plane free of the Landau ghost. It is in good agreement with the data in both spacelike and timelike regions. I stress that the determination of $\alpha(q^2)$ depends on the validity of four assumptions stated above, and that these assumptions are the simplest ones consistent with known properties of the Borel function. The phenomenological successes described here can be interpreted as a strong evidence for validity of both QCD and the above assumptions.

I urge that further investigations on the assumed properties of the Borel function and precise determination of $R$ especially in the range $Q^2=1-10$ $\text{GeV}^2$ be undertaken.

It is a pleasure to acknowledge many useful discussions with N. Khuri.

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[5]This technique was extended and applied to $\varphi^4$ field theory in four dimensions with encouraging results: N. N. Khuri, Phys. Lett. 82B, 83 (1979). I follow the Borel-like summation technique introduced by Khuri.

[6]It is well known that QCD Green's functions are not Borel summable [see G. 't Hooft, in Proceedings of the School of Physics "Ettore Majorana," "Subnuclear Physics," Erice, Sicily, 1977 (to be published)]. This is because of at least two problems: (1) The Borel function is known to have singularities along the real positive axis; (2) QCD Green's functions have accumulation of singularities at the origin of the complex coupling constant plane. The first problem is avoided by the technique introduced in Ref. 5. The second problem comes from the $q^2$ dependence of the Green's function and it does not apply to the $\beta$ function.

[7]The detail of this analysis is presented by A. I. Sanda, Rockefeller University Report No. COO-2232B-178 (to be published).

[8]'t Hooft, Ref. 6.

[9]$\alpha_2(Q^2)$ is a good approximation to $\alpha(Q^2)$ because of the the smallness of $J_3$.

[10]See, for example, S. Schweber, *Introduction to Relativistic Quantum Field Theory* (Harper and Row, New York, 1962).

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# Spin and Spin-Isospin Distribution in Some Medium-Heavy Nuclei

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Pairs of odd-even nuclei with the wave functions of the valence nucleons different simply by an interchange of neutrons and protons (pseudo mirror pairs) have been identified and mirror symmetry in the subspace of valence nucleons conjectured. Isoscalar and isovector magnetic moments have been studied, for a given shell and for an increasing number of odd nucleons. Impressive regularities become evident.

The peculiar convenience of the isospin formalism for discussing nuclear magnetic moments and $\beta$ decay has been noticed long ago.[1] If all the $2T+1$ magnetic moments of an isospin multiplet are known, they should be completely determined by the two constants $\mu_s$ and $\mu_v$ in the relation

$$\mu(T,T_3)=\mu_s(T)+\mu_v(T)T_3,$$

where $\mu_s(T)$ is the isoscalar magnetic moment and $\mu_v(T)$ is the isovector magnetic moment. The sum of the moments of two states of an isospin multiplet with $T_3$ and $-T_3$ *(mirror pairs)* is thus equal to $2\mu_s$ and the difference is equal to $2\mu_v T_3$; then the magnetic moments of *two* members of an isospin multiplet completely determine the others.

If we ignore mesonic effects and relativistic corrections, we have

$$\langle \textstyle\sum_i \sigma_{iz}\rangle = \frac{[\mu(T,T_3)+\mu(T,-T_3)]-J}{g_p+g_n-1}, \tag{1}$$

$$\langle \textstyle\sum_i \sigma_{iz}\tau_{i3}\rangle = \frac{[\mu(T,T_3)-\mu(T,-T_3)]-\langle\sum_i j_{iz}\tau_{i3}\rangle}{g_p-g_n-1}. \tag{2}$$

In the following we will use the symbols $\langle\sigma\rangle$ and $\langle\sigma\tau\rangle$ for the left-hand sides of formulas (1) and (2).

In general these expressions are independent and there are *no further relations among them.* Only for definite models, which imply certain prescriptions for the nuclear wave function, can we write down relations involving these quantities. Such relations offer tests for the consisten-

 