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Superweak Gauge Theory of *CP*-Invariance Violation

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We describe an extension of the Weinberg-Salam model to the group $SU(2)\otimes U(1)\otimes U(1)'$ in which *CP* invariance is spontaneously broken and physical *CP*-invariance violation is mediated by the new massive Z' boson.

The $SU(2)\otimes U(1)$ model of Weinberg and Salam[1] has been spectacularly successful in describing most aspects of weak and electromagnetic interactions. However, the best way to incorporate *CP*-invariance violation into the model has never been clear. The simplest possibility[2] in the six-quark model is to allow arbitrary *CP*-nonconserving phases in the Yukawa couplings; after diagonalizing the quark mass matrix a single *CP*-invariance-violating phase will then show up in the charged W boson couplings. This model by Kobayashi and Maskawa (KM) was believed to provide a satisfactory description of *CP*-invariance violation as long as the relevant phase and mixing angles are sufficiently small. However, the discovery[3] of the existence of θ vacua in quantum chromodynamics (QCD) has thrown new obstacles in the way of theories of *CP*-invariance violation. Whereas it had once been thought that the QCD Lagrangian naturally conserves P and T, it has been found that a term of the form

$$\mathcal{L}_I=(\theta/32\pi^2)G_a^{\mu\nu}{}^*G_{\mu\nu a} \tag{1}$$

may in general appear. This term violates both

P and T invaraince unless $\theta = 0$ or π. In order to be consistent with experiment, particularly present bounds[4] on the neutron electric dipole moment, one must assume that the parameter θ in Eq. (1) is less than or about 10^{-8}. (There may be additional suppressions of CP-invariance-violating effects due to the small mass of the u and d quarks, in which case a larger value of θ may be permissible.)[5]

These limits actually apply to $\theta_{\text{eff}} = \theta_{\text{QCD}} + \theta_{\text{QFD}}$, where θ_{QCD} is the bare parameter in the Lagrangian and

$$\theta_{\text{QFD}} = \arg\det M_Q \tag{2}$$

is the argument of the determinant of the quark mass matrix (i.e., θ is rotated by the chiral transformation involved in the diagonalization of M_Q). In the KM model and other models of hard (dimension four) CP-invariance violation, neither θ_{QCD} nor θ_{QFD} need be small. In fact, θ_{eff} is divergent when loops are taken into consideration. While it is possible simply to set the renormalized $\theta_{\text{eff}} = 0$, this is not especially elegant. For this reason, several authors have proposed models in which θ is rendered unobservable by an exact U(1) global symmetry of $\mathcal{L}$. This leads to the existence of massless quarks[6] or axions[7] depending on the realization of the U(1) symmetry.

Another possibility is to consider models in which CP is an exact symmetry of $\mathcal{L}$ (or its dimension-four terms at least) and that CP violation is spontaneous (or soft). This ensures that $\theta_{\text{QCD}} = 0$ and θ_{QFD} is finite and calculable. Nevertheless, in most theories of this θ_{QFD} turns out to be of order 10^{-3}, five orders of magnitude too big. Thus, we must have recourse to some additional symmetry to naturally force $\arg\det M_Q$ to vanish at tree level. Several published models employ discrete symmetries for this purpose.[8]

In particular, Bég and Tsao[8] have extended the group $G = SU(2)_L \otimes SU(2)_R \otimes U(1)$ to the group G $\otimes U(1)$,[1] where the new Z' boson mediates CP-invariance violation. This is a superweak theory[9] in which CP is spontaneously violated and θ is naturally sufficiently small. In the present paper, we describe a similar extension of the standard model[1] to $[SU(2) \otimes U(1)] \otimes U(1)'$. This new group $U(1)'$ will justify its existence by playing three roles: First, it distinguishes between the generations of fermions; second, it restricts the form of the quark mass matrix M_Q to make $\arg\det M_Q = 0$ at tree level a natural condition; and third, its gauge boson, denoted Z', mediates the superweak CP-invariance-violating interaction. The model we shall describe has, then, the virtues that it preserves the successes of the standard model, requires no new discrete symmetries, and is relatively economical in new symmetries and particles. The price paid is that Z' is quite heavy. We will describe the model in the next section.

The weak gauge group of our model is $[SU(2) \otimes U(1)] \otimes U(1)'$. The fermion multiplet structure under the subgroup $SU(2) \otimes U(1)$ is exactly as in the six-quark standard[1,2] model, with left-handed components in SU(2) doublets and right-handed components in singlets. The weak hypercharge $(Q = I_3 + Y)$ assignments are also as in the standard model. We will denote the quantum number of the group $U(1)'$ by Y'. In Table I are shown the quantum-number assignments of the quarks, leptons, and Higgs fields. Note that all quarks (leptons) of a given generation have been assigned the same Y' and that the total Y' of the quarks (leptons) is zero. Together with the fact that the sum of $(Y')^3$ for the leptons vanishes these conditions ensure anomaly cancellation. Of course there is great freedom in choosing the Y' assignments. We have chosen particular values of Y' for definiteness. It will be noticed that there are four Higgs doublets, φ, χ_1, χ_2, and η, and a Higgs singlet, σ. The doublets φ, χ_1, and χ_2 develop vacuum expectation values (VEV's) and generate the $W^\pm$, Z, and fermion masses as in the standard model. [There are three such doublets to allow a CP-invariance-violating relative phase angle between their VEV's. Two would not suffice as their relative phase could be absorbed by a global $U(1)'$ rotation.] φ has $Y' = 0$ and has generation-diagonal Yukawa couplings. χ_1 and χ_2 are responsible for Cabibbo mixing. The tree-level Yukawa couplings responsible for quark masses are

$$\mathcal{L}_{\text{Yukawa}} = (\bar{u}^0\ \bar{c}^0)_L \begin{pmatrix} f_u\tilde{\varphi}^0 & \sum_j h_{uj}\tilde{\chi}_j{}^0 \\ 0 & f_c\tilde{\varphi}^0 \end{pmatrix} \begin{pmatrix} u^0 \\ c^0 \end{pmatrix}_R + (\bar{d}^0\ \bar{s}^0)_L \begin{pmatrix} f_d\varphi^0 & 0 \\ \sum_j h_{dj}\chi_j{}^0 & f_s\varphi^0 \end{pmatrix} \begin{pmatrix} d^0 \\ s^0 \end{pmatrix}_R$$

$$+ f_t\bar{t}_L\tilde{\varphi}^0 t_R + f_b\bar{b}_L\varphi^0 b_R + \text{H.c.}, \quad \tilde{\varphi} \equiv i\tau_2\varphi^*. \tag{3}$$

TABLE I. Quantum-number assignments of the fields of the theory.

	I	Y	Y'		I	Y	Y'
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	1/2	1/6	1	χ_j	1/2	1/2	1
$\begin{pmatrix} c \\ s \end{pmatrix}_L$	1/2	1/6	2	η	1/2	1/2	−4
$\begin{pmatrix} t \\ b \end{pmatrix}_L$	1/2	1/6	−3	σ	0	0	1
u_R	0	2/3	1	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	1/2	−1/2	2
c_R	0	2/3	2	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	1/2	−1/2	−2
t_R	0	2/3	−3	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	1/2	−1/2	0
d_R	0	−1/3	1	e_R	0	−1	2
s_R	0	−1/3	2	μ_R	0	−1	−2
b_R	0	−1/3	−3	τ_R	0	−1	0
φ	1/2	1/2	0				

This is the most general form consistent with the gauge symmetries and CP invariance. There is no Cabibbo mixing of the t and b quarks with lighter quarks. (This feature is not essential, however. If t and b mix, then box diagrams will also contribute to CP-invariance violation.) Hence b decay must go via the exchange of some Higgs other than the φ, χ_1, or χ_2. This is the reason for the η doublet. We have $\langle\eta\rangle = 0$. The b decays only semileptonically as by $b \to d + \eta \to d + \mu^- + e^+$.[10] The singlet Higgs σ is assumed to develop a very large VEV, and is responsible for the large Z' mass. Note that the mixing of the Z and Z' is of order $(M_Z/M_{Z'})^2$.

The form of the Higgs potential is assumed also to be the most general consistent with gauge symmetry and CP invariance. For example, terms such as $(\chi_1^\dagger\chi_2)^2 + \text{H.c.}$ are present and lead to a nontrivial phase angle between $\langle\chi_1^0\rangle$ and $\langle\chi_2^0\rangle$. Also terms of the form $\sigma\chi_j^\dagger\varphi + \text{H.c.}$ are present and prevent the appearance of a Goldstone particle. Such is the model whose consequences for CP phenomenology we will now outline.

In tree approximation in the broken-symmetry theory the determinant of the quark mass matrix is automatically real. This is due to the triangular form of the matrices in Eq. (3), in turn due to the restrictions imposed by the U(1)′ gauge symmetry. The first nonvanishing contribution to $\arg\det M_Q$ comes from Higgs loops—discussed later.

Let us call $\langle\varphi^0\rangle \equiv v$, $\langle\sum_j h_{uj}\chi_j^0\rangle \equiv -|\Delta_u| e^{i(\theta+\delta)}$, and $\langle\sum_j h_{dj}\chi_j^0\rangle \equiv |\Delta_d| e^{i\theta}$, where v is conventionally chosen real [this can be done by a global U(1) rotation]. Notice that the angle θ can be changed by global U(1)′ rotation and is hence not observable. We make no assumption about the magnitude of δ; it may be of order unity, for example. Changing to a basis where the quark mass matrices are real and diagonal (denoted by tildes) gives

$$\begin{aligned} \tilde{M}_u &= U(\theta_{uL}, \theta+\delta) M_u U^\dagger(\theta_{uR}, \theta+\delta), \\ \tilde{M}_d &= U(\theta_{dL}, \theta) M_d U^\dagger(\theta_{dR}, \theta), \\ U(\alpha,\beta) &\equiv \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\beta} \end{pmatrix}, \\ U_{\text{Cabibbo}} &= U(\theta_{uL}, \theta+\delta) U^\dagger(\theta_{dL}, \theta), \end{aligned} \tag{4}$$

where u and d refer to the charge $\frac{2}{3}$ and $-\frac{1}{3}$ quarks, respectively. By straightforward algebra one finds (assuming $|\Delta_u| < m_c$, $|\Delta_d| < m_s$) that

$$\begin{aligned} &\theta_{dL} \cong |\Delta_d| m_d/(m_s)^2 \ll 1, \quad \theta_{dR} \cong |\Delta_d|/m_s, \\ &\theta_{uL} \cong -|\Delta_u|/m_c, \quad \theta_{uR} \cong -|\Delta_u| m_u/(m_c)^2 \ll 1. \end{aligned} \tag{5}$$

Since θ_{dL} is small, the Cabibbo angle is approximately equal to $-\theta_{uL}$ so that

$$|\Delta_u| \cong (\theta_C) m_c \cong 250 \text{ MeV} \tag{6}$$

If δ is nontrivial then the Cabibbo matrix is complex. Since we do not have any mixing of t and b with lighter quarks, we may absorb the phases of the Cabibbo matrix by a redefinition of the quark fields which leaves the quark mass matrices real. In this new quark basis the $W^\pm$ and Z couplings are real at tree level. But the Z' couplings become complex in this basis. (So also do the Yukawa couplings.) Thus

$$\begin{aligned} \mathcal{L}_{Z'} = &-i\hat{g}(\bar{d}\bar{s})_x \not{Z}' \begin{pmatrix} 1+\sin^2\theta_{dx} & -\sin\theta_{dx}\cos\theta_{dx}\exp(-i\theta_{CP}{}^d) \\ -\sin\theta_{dx}\cos\theta_{dx}\exp(i\theta_{CP}{}^d) & 1+\cos^2\theta_{dx} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}_x \\ &-i\hat{g}(\bar{u}\bar{c})_x \not{Z}' \begin{pmatrix} 1+\sin^2\theta_{ux} & -\sin\theta_{ux}\cos\theta_{ux}\exp(-i\theta_{CP}{}^u) \\ -\sin\theta_{ux}\cos\theta_{ux}\exp(i\theta_{CP}{}^u) & 1+\cos^2\theta_{ux} \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}_x, \end{aligned}$$

where

$$x = L \text{ or } R, \quad \theta_{CP}{}^d \cong -\delta, \quad \theta_{CP}{}^u = O(\theta_{dL}) \ll 1, \tag{7}$$

where $\hat{g}$ denotes the U(1)′ gauge coupling constant. Thus there is an effective four-quark interaction

$$-\mathfrak{L}_{\text{eff}}=\epsilon_L[\bar{s}_L\gamma_\mu d_L][s_L\gamma^\mu d_L]+\epsilon_R[\bar{s}_R\gamma_\mu d_R][\bar{S}_R\gamma^\mu d_R]+2\epsilon_{LR}[\bar{s}_L\gamma_\mu d_L][\bar{s}_R\gamma^\mu d_R]+\text{H.c.} \tag{8}$$

which leads to K_L-K_S mixing. The real part of ϵ_L arises mainly from the usual box diagram, but the imaginary parts of ϵ_L, ϵ_R, and ϵ_{LR} arise predominantly from Z' exchange (i.e., first order in the superweak interaction) so that

$$\begin{aligned}
\operatorname{Im}\epsilon_R &\cong \frac{\hat{g}^2}{(M_{Z'})^2}\sin^2\theta_{dR}\sin\theta_{CP}{}^d,\\
\operatorname{Im}\epsilon_{LR} &\cong \frac{\hat{g}^2}{(M_{Z'})^2}\sin\theta_{dL}\sin\theta_{dR}\sin\theta_{CP}{}^d \ll \operatorname{Im}\epsilon_R,\\
\operatorname{Im}\epsilon_L &\cong \frac{\hat{g}^2}{(M_{Z'})^2}\sin^2\theta_{dL}\sin\theta_{CP}{}^d \ll \operatorname{Im}\epsilon_R.
\end{aligned} \tag{9}$$

The phenomenology of the neutral-kaon system requires that

$$\operatorname{Im}\epsilon_R \cong (\hat{g}^2/M_{Z'}{}^2)\sin^2\theta_{dR}\sin\theta_{CP}{}^d \approx 4\times10^{-10}G_{\text{F}}. \tag{10}$$

Therefore, if $|\theta_{CP}{}^d|\cong|\delta|$ is of order unity, and if we define $G_{\text{F}}'/\sqrt{2}\equiv\hat{g}^2/4M_{Z'}{}^2$, then we have

$$G_{\text{F}}'\sin^2\theta_{dR}\cong G_{\text{F}}'(\Delta_d/m_d)^2\approx 2\times10^{-10}G_{\text{F}}. \tag{11}$$

For example, if $|\Delta_d\approx 10$ MeV we must have $G_{\text{F}}'\approx10^{-7}G_{\text{F}}$. Or with $|\Delta_d|\approx 1$ MeV we must have $G_{\text{F}}'\approx10^{-5}G_{\text{F}}$. For $\hat{g}=g$, these values correspond to $M_{z'}/M_w\sim 4000$ and 400, respectively. Smaller values for Δ_d or $\hat{g}$ yield smaller values for $M_{z'}$.

There are also Higgs exchange contributions to the imaginary part of M_{12}. Their effective interactions go roughly as $(\lambda/M_{\text{H}}{}^4)(\Delta_d)^2$ where λ is some quartic Higgs coupling. This is, crudely speaking, of order $G_{\text{F}}(\Delta_d/M_{\text{H}})^2$. If Higgs exchange effects are to be negligible compared to Z' exchange, then this quantity must be less than $4\times10^{-10}G_{\text{F}}$. So for $\Delta_d\approx 10$ MeV this implies $m_{\text{H}}\approx 500$ GeV. For $\Delta_d\approx 1$ MeV the condition is that $m_{\text{H}}\approx 50$ GeV.

There are also $\Delta S=1$ interactions induced by Z' and Higgs exchange which contribute to $K_L\to2\pi$ and to $K_L\to\mu^+\mu$, etc. These are negligible for the values of G_{F}' and m_{H} being considered.

Let us now estimate in a rough way the one-loop contributions to $\arg\det M_Q$. These come from Higgs loop diagrams. These diagrams depend, of course, on a plethora of parameters. To estimate the typical size of the effect we assume that none of the relevent parameters in the Higgs potential is unnaturally large or small. Then one finds contributions to $\arg\det M_Q$ of the order $(G_{\text{F}}/16\pi^2)(\Delta_d)^2$, $(G_{\text{F}}/16\pi^2)\Delta_u\Delta_d m_c/m_d$, and $(G_{\text{F}}/16\pi^2)(\Delta_u)^2$. For $\Delta_d\approx$ a few MeV one therefore expects to find $\theta_{\text{QFD}}=O(10^{-8})$ which is acceptable.

The neutron electron dipole moment arises principally from the effects of θ_{QFD}. The direct contribution from superweak CP-invariance-violating interactions is or order 10^{-29} $e\cdot$cm which is well below the present experimental bounds.

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Nonperturbative Determination of $\alpha(q^2)$ and Its Experimental Implications

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The lowest-order perturbative expression for $\alpha(q^2)$, the quantum-chromodynamic running coupling constant, is inadequate for describing the moments of nucleon structure functions at present energies. I propose to determine $\alpha(q^2)$ with use of a new Borel-like summation technique described by Khuri. The $\alpha(q^2)$ which emerges from this procedure is free of the Landau ghost and it is consistent with experiments in both spacelike and timelike regions.

Recently, scaling violation in nucleon structure function has been studied in detail.[1] The first indications are that the observed scaling violation is in quantitative agreement with theoretical predictions based on quantum-chromodynamic (QCD) perturbation theory. This is an exciting result. It is, therefore, important to ask the following question: How valid is the perturbation theory in the range of momentum transfer (1 GeV$^2 \lesssim Q^2 \lesssim 10$ GeV^2) most relevant in this study?

Consider the β function including the two-loop effects[2]:

$$\beta_2(g) = -\beta_0 g^3/16\pi^2 - \beta_1 g^5/(16\pi^2)^2, \tag{1}$$

where $\beta_0 = 11 - \frac{2}{3}N_f$, $\beta_1 = 102 - \frac{38}{3}N_f$, and N_f is the number of quark flavors. The effective coupling corresponding to β_2 is[3]

$$\alpha_2(Q^2) = [\beta_0/4\pi)\ln(Q^2/\Lambda^2) + \eta f(Q^2)]^{-1},$$

where (2)

$$f(Q^2) = \ln[1 + \beta_0{}^2/\beta_1 \ln(Q^2/\Lambda^2) + f(Q^2)].$$

$Q^2 = -q^2$ in the spacelike region, $\eta = \beta_1/4\pi\beta_0$, and Λ is an adjustable scale parameter. In Fig. 1, we compare $\alpha_1(Q^2) = [\beta_0/4\pi \ln(Q^2/\Lambda^2)]^{-1}$ (curve A), and $\alpha_2(Q^2)$ (curve B). Clearly, more convergent $\alpha(Q^2)$ is necessary in order to compare the theory and experiments.

As a possible solution[4,5] to this problem, I propose to use a Borel-like summation technique[6,7] in computing the β function. Consider a perturbative expression for the β function

$$g\beta(g) = \sum_{n=2}^{\infty} a_n (g^2)^n. \tag{3}$$

It is widely suspected that the radius of convergence of this power series is zero. In order to

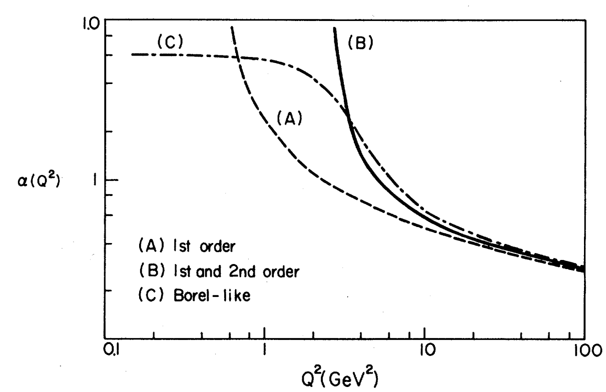

FIG. 1. The running coupling constants based on one leading-order term (curve A) and two leading-order terms (curve B), in the perturbation expansion for the β function are compared. Curve C is obtained from the Borel-like summation. All three curves are normalized at so that $\alpha(100\ \text{GeV}^2) = 0.27$ corresponding to $\Lambda = 0.75$ GeV (see Ref. 1) for curve A.