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Soft *CP*-Invariance Violation at High Temperature

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We study the temperature dependence of soft *CP*-invariance violation in a class of gauge theories based on the gauge group $SU(2)_L \times U(1)$ and show that there exist physically acceptable domains of coupling parameters of the theory where *CP*-invariance violation persists at high temperatures. Such models have the potential to explain matter-antimatter asymmetry in the universe while at the same time providing a cure for the strong *CP* problem.

The high-temperature behavior of gauge and global symmetries has been studied extensively in recent years,[1] with the emphasis on the spontaneously broken gauge theories at weak and electromagnetic interactions. In analogy with superconductors, where the transition from superconducting to normal phase occurs at high temperature, it was noted that the broken gauge symmetry could be restored beyond a certain critical temperature. The same phenomenon is expected to happen for all softly broken symmetries of nature.

In the present Letter, we analyze this question carefully in the context of the specific example where a discrete *CP* symmetry is softly broken and study whether *CP* invariance is necessarily restored at high temperature. The reason this particular example is of interest is the following: While on phenomenological grounds there appears to be no way to distinguish[2] between models with hard and soft[3] *CP*-invariance violation, the latter seems to be necessary to understand the smallness of the quantum-chromodynamics-induced strong *CP*-invariance-violating phase.[4,5] On the other hand, attempts[6] have recently been made to understand the matter-antimatter asymmetry in the universe in the context of gauge theories with baryon-number and *CP*-invariance-violating interactions. According to these ideas the baryon asymmetry was supposed to have originated at extremely high temperatures, generally of order of energies where presumably weak, electromagnetic, and strong interactions became identical, i.e., $\sim 10^{16}$ GeV. If soft *CP*-invariance violation disappears at high temperatures as is generally thought to happen, even qualitative understanding of the observed matter-antimatter asymmetry would seem to require[7] hard *CP*-invariance violation.

Our analysis shows, however, that there exist a class of gauge models where soft *CP* noninvariance *does persist* at high temperatures, contrary to the common belief. In such a theory, there is the potential to understand both the resolution of the strong CP problem as well as the question of matter-antimatter asymmetry. We illustrate our idea with the help of a simple, yet realistic, model of soft CP-invariance violation based on the gauge group $SU(2)_L \otimes U(1)$. Application to models which also cure the strong CP problem is then straightforward.

Our example is a simple extension of the standard[8] $SU(2)_L \otimes U(1)$ model with three Higgs doublets (φ_1, φ_2, and χ). Our strategy is first to analyze the Higgs potential at zero temperature in order to obtain the domain of coupling parameters where the spontaneously borken symmetry represents a stable minimum of the theory. Then we calculate the one-loop–induced temperature-dependent terms in the potential in order to study the high-temperature behavior of soft CP-invariance violation, as dictated by the allowed range of parameters obtained from the previous analysis. To simplify our analysis, and without any loss of generality, we require the potential to be symmetric under $\varphi_1 \leftrightarrow \varphi_2$ and $\chi \to -\chi$. The potential is then given by

$$\begin{aligned} V(\varphi_1,\varphi_2,\chi) = &-\mu_1^2(\varphi_1^\dagger\varphi_1+\varphi_2^\dagger\varphi_2)+\lambda_1[(\varphi_1^\dagger\varphi_1)^2+(\varphi_2^\dagger\varphi_2)^2]+2\lambda_3(\varphi_1^\dagger\varphi_1)(\varphi_2^\dagger\varphi_2) \\ &+2\lambda_4(\varphi_1^\dagger\varphi_2)(\varphi_2^\dagger\varphi_1)+\lambda_5[(\varphi_1^\dagger\varphi_2)^2+\text{H.c.}]+\lambda_6(\varphi_1^\dagger\varphi_1+\varphi_2^\dagger\varphi_2)(\varphi_1^\dagger\varphi_2+\varphi_2^\dagger\varphi_1)-\mu_2^2\chi^\dagger\chi \\ &+\delta(\chi^\dagger\chi)^2+2\alpha(\chi^\dagger\chi)(\varphi_1^\dagger\varphi_1+\varphi_2^\dagger\varphi_2)+2\beta(\varphi_1^\dagger\chi)(\chi^\dagger\varphi_1)+(\varphi_2^\dagger\chi)(\chi^\dagger\varphi_2) \\ &+\gamma[(\varphi_1^\dagger\chi)(\chi^\dagger\varphi_2)+(\varphi_2^\dagger\chi)(\chi^\dagger\varphi_1)]+\gamma'(\chi^\dagger\chi)(\varphi_1^\dagger\varphi_2+\varphi_2^\dagger\varphi_1)+\beta'[(\varphi_1^\dagger\chi)^2+(\varphi_2^\dagger\chi)^2+\text{H.c.}]\ldots, \end{aligned} \tag{1}$$

where $\mu_i^2>0$ ($i=1,2$) to ensure spontaneous symmetry breaking. We further restrict[9] $\gamma,\gamma',\beta'\simeq 0$. We then look for a minimum of the potential of the following type:

$$\langle\varphi_i\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \rho_i\exp(i\theta_i)\end{pmatrix},\quad \langle\chi\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix},$$

with

$$\theta_1=0,\quad \theta_2\equiv\theta. \tag{2}$$

The conditions for extrema (with nonvanishing θ) are

$$\mu_1^2=(\lambda_1+\lambda_3+\lambda_4-\lambda_5-\lambda_6^2/2\lambda_5)\rho^2+(\alpha+\beta)v^2, \tag{3a}$$

$$\mu_2^2=\delta v^2+2(\alpha+\beta)\rho^2, \tag{3b}$$

$$\cos\theta=-\lambda_6/2\lambda_5, \tag{3c}$$

where we have used (3c) to obtain 3(a).

Positivity of the Higgs-boson mass matrix at the minimum imposes the following further constraints on the Higgs-boson self-couplings: (a) for charged Higgs bosons,

$$(\lambda_5-\lambda_4-\beta)\rho^2-\beta v^2>0,\quad \tfrac{1}{2}(\beta^2v^2)-(\lambda_5-\lambda_4)\beta\rho^2>0, \tag{4}$$

which is satisfied by

$$\lambda_5>\lambda_4,\quad \beta<0; \tag{5}$$

(b) for neutral Higgs bosons,

$$\begin{aligned} &\lambda_1>0,\quad \lambda_5>0,\quad \lambda_1-\lambda_6^2/4\lambda_5>0,\quad \delta>0,\quad \lambda_1-\lambda_3-\lambda_4+\lambda_5>0, \\ &\lambda_1+\lambda_3+\lambda_4-\lambda_5-\lambda_6^2/2\lambda_5>0,\quad \delta(\lambda-\lambda_6^2/4\lambda_5)-(\alpha+\beta)^2>0,\quad \delta(\lambda+\lambda_3+\lambda_4-\lambda_5-\lambda_6^2/2\lambda_5)-2(\alpha+\beta)^2>0. \end{aligned} \tag{6}$$

It is easily seen that the last set of conditions does not affect the previously obtained ones, Eq. (5). Furthermore, if we assume $v<\rho$, we can chose also $\alpha+\beta<0$ (we take both $\alpha,\beta<0$).

We should remark that once we have $\langle\varphi_i\rangle\neq 0$, $\theta\neq 0$, then the CP-invariance violation in this theory is induced in a usual way in gauge theories through complex quark-mass matrices.

Now, let us calculate a one-loop–induced temperature-dependent term in the potential. Using standard methods,[1] we obtain (for the general analysis of temperature-dependent effects, see Weinberg[1])

$V_{\rm eff}(\varphi_1,\varphi_2,\chi)=V(\varphi_1,\varphi_2,\chi)+V_T$, where

$$V_T=\tfrac{1}{2}(kT)^2\{[\lambda_1+\tfrac{1}{3}(2\lambda_3+\lambda_4)+\tfrac{1}{3}(2\alpha+\beta)+\tfrac{1}{16}(3g^2+g'^2)](\varphi_1^\dagger\varphi_1+\varphi_2^\dagger\varphi_2)$$
$$+[\delta+\tfrac{1}{3}(4\alpha+2\beta)+\tfrac{1}{16}(3g^2+g'^2)]\chi^\dagger\chi+\tfrac{1}{2}\lambda_6(\varphi_1^\dagger\varphi_2+\varphi_2^\dagger\varphi_1)\}, \quad (7)$$

where k is the Boltzmann constant. We find using (5) and (1)–(4) that, if $2\alpha+\beta<0$ and if

$$\delta>|2\alpha+\beta|>3[\lambda_1+\tfrac{1}{3}(2\lambda_3+\lambda_4)+\tfrac{1}{16}(3g^2+g'^2)], \quad (8)$$

then the coefficient of $(\varphi_1^\dagger\varphi_1+\varphi_2^\dagger\varphi_2)(kT)^2$ is negative, whereas that of $(\chi^\dagger\chi)(kT)^2$ is positive. Therefore, above a critical temperature T_c {determined by

$$\mu_2^2=(kT_c)^2[\delta+\tfrac{1}{3}(4\alpha+2\beta)+\tfrac{1}{16}(3g^2+\rho'^2)]\},$$

we have $\langle\chi\rangle=0$, but $\langle\varphi_1\rangle$ and $\langle\varphi_2\rangle$ remain nonzero. They, however, increase with temperature, i.e., $\rho(T)=cT$. The CP phase θ also remains nonzero at the minimum at all temperatures, provided $c\gg1$ which is guaranteed by our choice of the range of parameters.

The noteworthy feature of our result is that the SU(2) ⊗ U(1) gauge symmetry will not be restored at high temperature. Namely, in this model $\langle\varphi_i\rangle$ never vanish, so that $U_{\rm em}(1)$ is the symmetry of the theory at all temperatures[10] [strictly speaking, for $\beta'=0$ the theory has extra U(1) symmetry which is restored at high temperature, and so again we emphasize that β' is small, but nonzero; for the simpler and more clear example of the fact that no symmetry may be restored see below and to be published].

We therefore find that *there always exist a domain of the Higgs couplings for which the soft CP-invariance violation remains at high temperature.* We now comment on several implications of our work.

(a) For our strategy to work, some of the Higgs self-couplings must be of order of or bigger than $\frac{1}{16}(3g^2+g'^2)\simeq 4\pi\alpha$. Thus our theory predicts several heavy Higgs bosons ($m_{\rm H}^2\gtrsim m_W^2$). However, there may still be some light neutral Higgs scalars.

(b) Note that had we dropped the χ doublet and had only φ_1 and φ_2, α and β would be zero. Then Eqs. (1)–(5) (with $v=0$) would imply that the coefficient of the term $(kT)^2(\varphi_1^\dagger\varphi_1+\varphi_2^\dagger\varphi_2)$ is always positive. Therefore, by increasing the temperature sufficiently $\langle\varphi_i\rangle$ would vanish and the theory would eventually become CP conserving. This situation does not change even if we relax the discrete symmetry $\varphi_1\leftrightarrow\varphi_2$. Namely, then one can have $\langle\varphi_1\rangle\neq0$, $\langle\varphi_2\rangle=0$ (or vice versa) at high temperature, but never both $\langle\varphi_1\rangle\neq0$, $\langle\varphi_2\rangle\neq0$ which is a necessary ingredient to ensure CP-invariance violation. However, this simpler model can serve as an illustration of the fact that the gauge symmetry may remain broken at high temperature.

(c) We have carried out a similar analysis for the gauge model $SU(2)_L\otimes SU(2)_R\otimes U(1)$, which accommodates naturally the strong CP-invariance-violating phase.[4] This will be presented in a longer paper now in preparation.

(d) Since the vacuum expectation values increase with temperature, the masses of the fermions and bosons in the theory will also increase with temperature. However, since Yukawa couplings are $\sim gm_q/m_W$ ($\approx10^{-4}$–10^{-5}) at zero temperature, at $T\sim10^{15}$ GeV the fermions are much lighter than superheavy gauge bosons, X, of grand unified theories. Therefore, the origin of the baryon asymmetry in the universe could possible be sought in such a theory through the decay of the X bosons. This, however, requires a more detailed analysis, which is beyond the scope of this Letter.

(e) A few years ago, Linde[11] argued that a phenomenon similar to the one discussed here could occur in weak-interaction models with neutral currents, provided the universe has a large net neutrino density (compared to baryon density, for example). Our analysis shows that although Linde's mechanism, in principle, helps the kind of phenomena we discussed, it is not necessary. This is only welcome since the neutrino density may not be large enough to ensure the validity of Linde's arguments. Our mechanism, on the other hand, requires only the existence of several reasonably heavy ($m_{\rm H}\sim m_W$) Higgs scalars.

We also remark that objections have been raised[12] on cosmological grounds against models with spontaneous CP-invariance violation which are CP invariant at high temperature. Such objections would obviously no longer apply for the theories of the kind discussed in this Letter.

Finally we should point out that the unusual temperature behavior we discussed is not at all a unique property of our particular example. There exists a much wider class of such theories, some of which we will discuss in a forthcoming paper.[13]

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[9]Including γ, γ', and β' does not affect our discussion as long as we keep them small (say, $\sim g^4$). Actually, arbitrary γ, γ', and β' only give us more freedom; however, we keep them small in order to be able to present a complete quantitative analysis.

[10]It was noted by Weinberg (Ref. 1) that a gauge symmetry may be only partially restored at high temperatures in the case when the gauge group is $O(n_1)\times O(n_2)$ (he did not notice, however, that it may not be restored at all, as happens in our example). For the related discussion and interesting physical analysis we refer the reader to his paper in Ref. 1.

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Superweak Gauge Theory of *CP*-Invariance Violation

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We describe an extension of the Weinberg-Salam model to the group $SU(2)\otimes U(1)\otimes U(1)'$ in which *CP* invariance is spontaneously broken and physical *CP*-invariance violation is mediated by the new massive Z' boson.

The $SU(2)\otimes U(1)$ model of Weinberg and Salam[1] has been spectacularly successful in describing most aspects of weak and electromagnetic interactions. However, the best way to incorporate *CP*-invariance violation into the model has never been clear. The simplest possibility[2] in the six-quark model is to allow arbitrary *CP*-nonconserving phases in the Yukawa couplings; after diagonalizing the quark mass matrix a single *CP*-invariance-violating phase will then show up in the charged *W* boson couplings. This model by Kobayashi and Maskawa (KM) was believed to provide a satisfactory description of *CP*-invariance violation as long as the relevant phase and mixing angles are sufficiently small. However, the discovery[3] of the existence of θ vacua in quantum chromodynamics (QCD) has thrown new obstacles in the way of theories of *CP*-invariance violation. Whereas it had once been thought that the QCD Lagrangian naturally conserves *P* and *T*, it has been found that a term of the form

$$\mathcal{L}_I = (\theta/32\pi^2)G_a^{\mu\nu}{}^*G_{\mu\nu a} \qquad (1)$$

may in general appear. This term violates both