Production of Fluctuations in the Early Universe by the Quark-Nucleon Phase Transition

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A model of the early universe is proposed which initially contains a zero-temperature quark phase. The entropy of the cosmic background radiation is generated by an irreversible transition to the usual radiation-dominated nucleon gas. At recombination time the model has density fluctuations with a characteristic mass of 10^6 solar masses.

The standard model of the universe[1] is at early times close to a homogeneous Friedmann model filled with a hot dense gas in thermal equilibrium. It is necessary, however, to postulate a spectrum of preexisting fluctuations in order to form the stars, galaxies, and clusters of galaxies into which the matter of the universe is presently organized. This Letter proposes a model based on a few assumptions about the quark-nucleon phase transition. In this model the entropy of the cosmic background radiation and the density fluctuations essential to the formation of condensed objects are generated by the irreversible transition from the quark phase to the nucleon phase. The model begins as a perfectly homogeneous zero-temperature Friedmann model; then the spatial symmetry is broken by the quantum fluctuations which nucleate the lower-energy nucleon phase at random points.

The line element of the proposed model is the Robertson-Walker metric with a flat three-space,

$$ds^2 = -dt^2 + A^2(t)(dr^2 + r^2 d^2\Omega), \qquad (1)$$

where the dimensionless scale factor $A(t)$ is taken to be unity at the time t_0 when the baryon number density $b(t_0) = b_0 = 1\ \text{fm}^{-3}$. At zero temperature and high densities, the quark model of strongly interacting particles predicts a variation of energy with density like that of a relativistic gas,[2] while the energy of nuclear matter increases more rapidly as the density increases. The quark phase is then the more stable one at high densities[3] and early times.

The baryon density at which the energies of the two phases are equal is not precisely known; it will be taken as $b = b_0 = 1\ \text{fm}^{-3}$. In the Massachusetts Institute of Technology bag model[4] the energy of a hadron has a term equal to the bag constant $B = 59$ MeV fm^{-3} times the volume. I assume that the quark phase consists of a lattice of quarks connected by gluon strings and has the smaller energy density $B(b/b_0)^{2/3}$. Use of B as the coefficient gives the proper energy when the baryon density $b = b_0$, the inverse volume of a nucleon. The coefficient may also be derived from the equations of Johnson and Thorn[5] in which case it is proportional to $gB^{1/2}$ where g is the quantum chromodynamics coupling constant. The order of magnitude of the coefficient is the same in either case, and so the simpler expression will be used.

The energy-momentum tensor of the quark phase in the limit of low density is diagonal in its rest frame. Its diagonal components are

$$T_Q = \text{diag}[B(A^{-2} + \delta A^{-3}), -\tfrac{1}{3}BA^{-2}, -\tfrac{1}{3}BA^{-2}, -\tfrac{1}{3}BA^{-2}], \qquad (2)$$

where $B = 59$ MeV fm^{-3} is the value of the bag constant obtained by fitting the Massachusetts Institute of Technology model to the masses of the hadrons,[4] and δ is a small dimensionless number proportional to the rest mass of the two quarks having zero strangeness. The spatial components of this energy-momentum tensor represent the pressure whose value is determined from the assumed energy density.

The justification for using the limiting form for the energy-momentum tensors as the baryon density goes to zero, is that most of the quark fluid will make the transition to the nucleon phase at late times when the scale factor $A(t)$ is large and the baryon density $b(t)$ is correspondingly small. When the parameters of the model are fixed, it will be seen that this is indeed the case.

My most crucial assumption is the presence, in the range of densities relevant to the model, of two coexisting phases of matter, a quark phase consisting of a uniform Fermi sea of quarks with their interacting gluon fields and a conventional nucleon phase containing no free quarks. Furthermore, it is assumed that the phase transition is of first order so that the quark phase will be "superextended" as the universe expands beyond the transition point. Then the nucleon phase must be nucleated by quantum fluctuations in the form of small bubbles located at random isolated points.

The scale factor $A(t)$ increases with time according to the usual equation,

$$dA/dt = A[(8\pi/3c^2)G|T_{Q00}|]^{1/2} \simeq h_0, \quad A \simeq 1 + h_0(t-t_0), \tag{3}$$

where $h_0 = (8\pi GB/3c^2)^{1/2} = 7700\ \mathrm{s}^{-1}$ is the Hubble constant at $t=t_0$. Shortly after the baryon density passes the transition point, bubbles of nucleon gas are nucleated with a probability per unit volume, p, which is taken as a parameter of the model. It is assumed that nucleation probability much later than t_0 is negligible.

After the bubbles are nucleated, they will grow as spherical detonation waves.[6] The properties of the nucleon gas immediately behind the detonation front can be found from the energy-momentum tensors of the quark phase and the nucleon phase by invoking the conservation laws across the detonation front and using the Chapman-Jouguet condition that the velocity of the fluid behind the detonation front, in the inertial frame in which the front is stationary, is equal to the local velocity of sound.[7]

The energy-momentum tensor of the nucleon phase in the limit of low density is

$$T_N = \mathrm{diag}[(aT^4 + M_N n), \tfrac{1}{3}aT^4, \tfrac{1}{3}aT^4, \tfrac{1}{3}aT^4], \tag{4}$$

where T is the temperature and n and M_N the nucleon number density and mass. In the inertial frame of the detonation front the material constituting the quark phase has velocity v_1 which is equal to the velocity of the detonation wave. The energy-momentum conservation laws are found by equating the 01 and 11 components of the energy-momentum tensor in the rest frame of the detonation wave. Those components of the Lorentz transformation and the energy-momentum tensor for the quark phase are

$$L = \begin{pmatrix} c_1 & s_1 \\ s_1 & c_1 \end{pmatrix}, \quad T_Q' = B\begin{pmatrix} (c_1^2 - \frac{1}{3}s_1^2)A^{-2} + c_1^2\delta A^{-3} & s_1c_1(\frac{2}{3}A^{-2} + \delta A^{-3}) \\ s_1c_1(\frac{2}{3}A^{-2} + \delta A^{-3}) & (s_1^2 - \frac{1}{3}c_1^2)A^{-2} + s_1^2\delta A^{-3} \end{pmatrix}, \tag{5}$$

where the elements of the Lorentz transformation are $c_1 = \cosh\alpha_1$ and $s_1 = \sinh\alpha_1$ with the pseudoangle α_1 determined by $v_1 = c\tanh\alpha_1$. A similar result follows for the nucleon phase with velocity v_2.

The equations for the conservation of momentum and energy across the detonation front are

$$s_1c_1B(\tfrac{2}{3}A^{-2} + \delta A^{-3}) = s_2c_2(\tfrac{4}{3}aT^4 + M_N n), \tag{6a}$$

$$B[(s_1^2 - \tfrac{1}{3}c_1^2)A^{-2} + s_1^2\delta A^{-3}] = (\tfrac{1}{3}c_2^2 + s_2^2)aT^4 + s_2^2M_N n. \tag{6b}$$

The jump conditions across the detonation front are Eqs. (6a) and (6b) and the Chapman-Jouguet condition that the velocity of the fluid behind the front must equal the local velocity of sound. In the limit of large A, the velocity of sound of the nucleon gas is $c/\sqrt{3}$, and the temperature T behind the detonation wave and the velocity v_1 of the front are

$$aT^4 = 4.24BA^{-2}, \quad y_1 = 0.934c. \tag{7}$$

Ultimately the spherical detonation waves will coalesce and all of the quark phase will have undergone an irreversible transition to high-temperature nucleon gas. Within each bubble there is a spherical divergent flow whose velocity fields will generate secondary shock waves and turbulence. These will decay and leave behind a stationary isothermal gas. The variations of baryon density, however, will be preserved by the damping of the motion of the nucleons by the radiation. The maxima of the nucleon density at the center of the bubbles will become condensation centers after the matter and radiation are decoupled by the recombination of the electrons with the protons and other light nuclei.

The smaller the value of the nucleation probability p is, the greater the typical distance between the centers of the bubbles and the later the time of transition for a typical portion of the quark phase are. The result is that the average entropy per baryon generated by the model increases as the value of p approaches zero. The appropriate value of the nucleation probability p and the mass of baryons contained in a typical bubble will be determined by equating the ratio of photons to nucleons of the model to that presently observed.

The entropy generated by the complex processes of nucleation, bubble growth, coalescence, and approach to equilibrium can be estimated by assuming that the energy of the quark phase creates an equilibrium radiation-dominated fluid at rest behind the detonation front. In the actual case the temperature behind the front will be less than this value because some of the reaction energy will appear as the kinetic energy of flow which will be added to the energy of the radiation by the action of various dissipation processes at later times. Under this assumption, the temperature generated behind the front at a time t is $T(t)=(B/aA^2)^{1/4}$.

In terms of the time of nucleation t_0 and the time of coalescence t_1 the total number of photons in a typical bubble will be

$$N_P=4\pi v_1\int_{t_0}^{t_1}20.3T^3(t)A^2r_b^{\,2}(t)dt\simeq(8\pi/3)20.3(B/ah_0^{\,2})^{3/4}v_1^{\,3}t_1^{\,3/2}\ln^2(h_0t_1), \quad (8)$$

where the time dependence of the scale factor A was taken from Eq. (3), with only the dominant term h_0t_1 retained, and $r_b(t)$ is the radius of a bubble in terms of the r coordinate of Eq. (1). The time dependence of the bubble radius is found by noting that the velocity of the detonation front in terms of the r coordinate of Eq. (1) is simply v_1/A and by taking the time integral of that velocity to obtain $r_b(t)=(v_1/h_0)\ln[1+h_0(t-t_0)]$.

The number of baryons in a typical bubble is simply b_0/p. To express this in terms of the coalescence time t_1, we write the probability p as

$$p=3/4\pi r^3(t_1)\simeq 3h_0^{\,3}/4\pi v_1^{\,3}\ln^3(h_0t_1). \quad (9)$$

By equating the photon-baryon ratio predicted by the model to the present ratio, $3.5\times10^7\Omega^{-1}(100/H)^2$, one finds the transcendental equation for the coalescence time t_1,

$$(h_0t_1)^{3/2}/\ln(h_0t_1)=3.5\times10^7\Omega^{-1}(100/H)^2b_0[2(20.3)(B/a)^{3/4}]^{-1}. \quad (10)$$

The solution of this equation for a present mass density of $\frac{1}{30}$ of critical, $\Omega=\frac{1}{30}$, and a Hubble constant of 100 km s^{-1} Mpc^{-1} gives $A(t_1)\simeq h_0t_1\approx1.6\times10^7$ and a coalescence time t_1 of 2100 s. These values justify the use of the low-density limit for the energy-momentum tensors and the assumption that the scale factor $A(t)$ was large when most of the matter of the universe made its transition from the quark to the nucleon phase.

The mass of nucleons in a bubble is given by

$$M_B=M_Nb_0/p=(4\pi M_Nb_0v_1^{\,3}/3h_0^{\,3})\ln^3(h_0t_1)$$
$$=0.78\times10^6M_\odot(v_1/0.934c)^3(\ln h_0t_1/16.6)^3[(59\text{ MeV fm}^{-3})/B]^{3/2}. \quad (11)$$

The result is rather insensitive to the value of the right-hand side of Eq. (10); a factor of 10 in that expression changes M_B by only 25%.

In this model nucleosynthesis takes behind the detonation wave as the matter expands and cools. To find the final concentration of the elements one must add up the amounts formed in different portions of material which pass through the detonation waves at different times. Some idea of the process can be formed from the median conditions of transition. Equation (11) tells us that the median time t_m is the solution of

$$\ln(h_0t_1)=2^{1/3}\ln(h_0t_m).$$

The result is $t_m=68$ s with a temperature behind the detonation front of 3.7×10^9 K. The median transition conditions are seen to resemble those of the standard model.

The density of nucleons in any bubble is a strongly decreasing function of the distance from the center of the bubble since the points closer to the center made the transition from the quark phase earlier when the baryon density was higher. Therefore the above mass of nucleons in a typical bubble $M_B\approx10^6M_\odot$ is the scale of strong density fluctuations generated by this model. This number is roughly equal to that of the Jeans mass at recombination time.[8] Because the centers of the bubbles have a random spatial distribution, there will be present low-contrast density fluctuations with much larger characteristic masses. This model will certainly form condensed objects.[9] It is not clear, however, whether it can generate the observed distribution of galaxies which has fluctuations over very large distances.[10]

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[1]Steven Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).

[2]For a recent calculation and references to earlier

work, see V. Baluni, Phys. Lett. 72B, 381 (1978).

[3]This question has been of interest in connection with the possible existence of quark stars with properties not too far from those of neutron stars. Some references are G. Baym and S. A. Chin, Phys. Lett. 62B, 241 (1976); G. Chapline and M. Nauenberg, Ann. N. Y. Acad. Sci. 302, 191 (1977); M. B. Kislinger and P. D. Morley, Phys. Lett. 67B, 371 (1977); B. Freedman and L. McLerran, Phys. Rev. D 17, 1109 (1978); W. B. Fechner and P. C. Joss, Nature (London) 274, 347 (1978).

[4]A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Wiesskopf, Phys. Rev. D 9, 3471 (1974); A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D 10, 2599 (1974). I found the review article by R. L. Jaffee, Nature (London) 268, 201 (1977), a helpful beginner's introduction to the ideas of quark confinement and to the bag model. The value of the bag constant is from T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975).

[5]K. Johnson and C. B. Thorn, Phys. Rev. D 13, 1934 (1976).

[6]One might assume, on the other hand, that the change from the quark to nucleon phase is effected by a deflagration wave, i.e., a wave of exothermic reaction in which the specific volume increases. The existence of such a wave is, however, ruled out by the conservation laws when the pressure of the initial phase is negative. See, for example, section 123 of L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, Oxford, 1959).

[7]L. D. Laudau and E. M. Lifshitz, Ref. 4, p. 483.

[8]Steven Weinberg, Ref. 4, p. 563.

[9]For a review of the theories of the formation of galaxies from fluctuations, see J. Richard Gott, III, in Annu. Rev. Astron. Astrophys. 15, 235 (1977).

[10]The literature is very extensive. For the latest of a long series of papers by Peebles and collaborators, see J. N. Fry and P. J. E. Peebles, Appl. J. 221, 19 (1978).

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