the electrons $p \simeq 0.17$, corresponding to a ratio $I(2\pi\vec{G} \pm \vec{Q})/I_0$ of superstructure reflections to Bragg reflection intensities of the unmodulated lattice of the order of 7×10^{-3}. It may not be surprising, therefore, that in molecular-beam diffraction patterns CDW superstructure reflections and Bragg peaks have comparable intensities.

We believe that molecular-beam diffraction has a great potential as a direct probe of instabilities displayed by systems of conduction electrons with respect to periodic perturbations.

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Corrections to Dynamic Scaling for the Lambda Transition in Liquid Helium

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The noncritical background thermal diffusion is extracted from experimental data and found to be relatively strong. This produces large transient corrections in the critical relaxation rates. The *negative amplitude of the slow transient* strongly perturbs the entropy relaxation rate. Inclusion of *both slow and fast* transient yields an excellent fit with Ahlers's thermal conductivity data. Light scattering data are also well accounted for.

Because of the absence of inhomogeneities and elastic strains, fluids generally have sharper phase transitions than solids. Of all the fluid transitions, the λ transition in liquid ^{4}He has been investigated with perhaps the greatest precision, with an accuracy for the reduced relative temperature, $t=(T-T_\lambda)/T_\lambda$, of the order of 10^{-6}. It has been, therefore, a severe disappointment to theorists and experimentalists alike that in this "most favored" of phase transitions the agreement between theory and experiment appears to be not at all good. This situation has been reviewed by Hohenberg and Halperin.[1] Specifically, Ahlers[2] finds that the temperature dependence of the thermal conductivity above the λ point is significantly stronger than predicted by dynamic scaling theory. The purpose of this note is to demonstrate that this discrepancy is only apparent. When certain natural and necessary corrections are included in the theory, remarkably good agreement is obtained with both the thermal conductivity measurements of Ahlers[2] and the light scattering measurements of Tarvin, Vidal, and Greytak.[3] (We remain, however, unable to account for the surprisingly large second-sound damping observed by Tyson.[4])

The correction that we treat here is associated with the noncritical background contributions to $\gamma_{S,\psi}$, the entropy and order-parameter relaxation rates. The importance of background thermal conductivity has been established by Keyes and Sengers[5] for the normal fluids. Our goal here is to exhibit in full quantitative detail how background enters in the critical dynamics of the λ

transition. Some of the thermal conductivity data of Ahlers[2] is shown in Fig. 1. Using, in addition, the data of Bowers[6] and Kerrisk and Keller,[7] we have arrived at the background value of $\lambda_B = 0.12$ mW/°K cm, as shown at the right-hand side of Fig. 1, with a weak pressure dependence which we neglect. This corresponds to a background thermal diffusion coefficient[8] $B_S = 2.0\times10^{-4}$ cm^2/sec.

We now want to compare the strength of the background thermal diffusion with the prediction of dynamic scaling[9,10] that a fluctuation of wave number k is characterized by the frequency $D(k)k^2$. The nonlocal "diffusion coefficient" (actually a k-dependent function) is predicted to scale as $D(k) = ak^{-1/2}$. From our mode-coupling calculation to two-loop accuracy, the details of which will be presented elsewhere,[11,12] we find

$$a = \frac{1}{\pi}\left(\frac{J}{2}\right)^{1/2} \frac{\gamma_0 \bar{\sigma}}{(n_0 c_P)^{1/2}} = 0.63 c_P^{-1/2} \text{ cm}^{3/2} \text{ sec}^{-1} \quad (1)$$

at saturated vapor pressure. $J = J_1 + J_2 = J_1(1 + J_2/J_1)$, where $J_{1,2}$ are one- and two-loop convolution integrals that have to be evaluated numerically. We find $J_1 = 2.46$ and $J_2/J_1 = 0.39$, so that $J = 3.43$. n_0 is the helium-atom density; $\bar{\sigma}$ and c_P are the mean entropy and constant-pressure specific heat, both in units of Boltzmann's constant k_B; $\gamma_0 = k_B T_\lambda/\hbar$ (where $2\pi\hbar$ is Planck's constant) is the basic reference frequency of the system. The origin of γ_0 can be traced to the equation of motion for the phase of the order parameter. (The coupling to the temperature fluctuations is via the ratio $k_B \bar{\sigma}/\hbar$).

We now estimate k_c, the maximum wave number for which the critical dynamics will be visible above background. Equating $D(k_c) = ak_c^{-1/2} = B_S$ gives

$$k_c = (a/B_S)^2 \cong c_P^{-1} 10^7 \text{ cm}^{-1}. \quad (2)$$

As $c_P = O(10)$ (at $t = 10^{-4.5}$), Eq. (2) is down by two orders of magnitude from the scale on which the Wegner[13] correction might *a priori* be expected to set in.

A further, even more vitally important effect enters because Eq. (1) is a statement only about the geometric mean of $\gamma_{\psi,S}$, the order-parameter and entropy relaxation rates, and says nothing about their ratio, $w = \gamma_\psi/\gamma_S$. Two-loop calculations[11,14-16] exhibit, however, an incipient instability in the self-consistent determination of the rates, in which $w \to 0$. This instability is not entirely unexpected[11] in the generalized Sasvári-Schwabl-Szépfalusy model,[17] and leads to $w = 0.11$, which we round off to 0.10. The rates now separate into $\gamma_{\psi,S} = a_{\psi,S}k^{3/2}$, where $a_\psi/a_S = w = 0.1$ and $a_\psi a_S = a^2$. Division of the rates by k^2 yields $D_{\psi,S}(k) = a_{\psi,S}k^{-1/2}$, shown in Fig. 2 by the lowermost and uppermost curves, labeled "scaling." The small value of w causes the order-parameter background B_ψ to impose an even more severe restriction on the critical region than we have seen above from B_S. Because the complex order parameter is not accessible to direct experimental observation, we cannot infer the strength of B_ψ from a plot of experimental data,

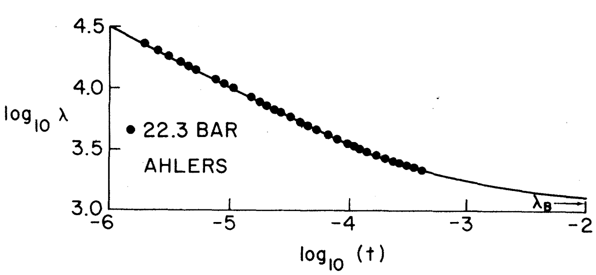

FIG. 1. Thermal conductivity (in units of ergs/°K cm sec) vs reduced temperature t for helium at a pressure of 22.3 atmospheres. The solid line fit to Ahlers's data includes *both slow and fast transients*. λ_B denotes the background value.

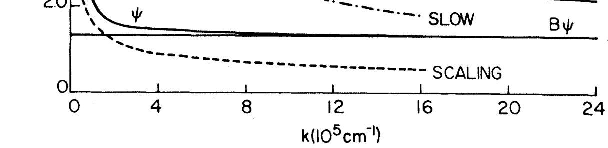

FIG. 2. Onsager coefficient vs wave number for the order-parameter (ψ) and the entropy (S) modes. The ψ background value is $B_\psi = 1.35\times10^{-4}$ cm^2/sec. The solid curves include both slow and fast transients, while the dashed curves show the pure scaling solutions without any transient corrections. The middle dot-dashed curve shows the scaling solution for the entropy diffusion coefficient corrected only for the negative slow transient.

such as we did for B_S. Therefore, we must deduce B_ψ indirectly from its effect on the observable quantity $D_S(k)$. As an equivalent source of information, we can use the thermal conductivity $\lambda = k_B c_P D_S$, as a function of t via the inverse correlation length $\kappa(t) = \kappa_0 t^{2/3}$, with $\kappa_0 = 0.7$ Å^{-1}.

Introduction of background effects into the mode-coupling theory[11] produces transient solutions to the coupled integral equations which can be described as the replacement of $a_{\psi,S}$ by $a_{\psi,S}(1 \pm b_s k^{\omega_s} + b_f k^{\omega_f})$. The eigenvectors are such that the effect on the two modes is the same for the "fast" transient and equal and opposite (as expressed by the $\pm b_s$ coefficient) for the "slow" transient. We find[11] $\omega_f = \epsilon - 0.038\epsilon^2 = 0.96$ and $\omega_s \approx w = 0.10$ for the fast and slow transients, respectively. (As emphasized by Dohm[18] and by the present authors,[11] the calculation of the slow transient has to be carried out with some care in order to ensure that w_s vanishes on the stability boundary.) This leaves only the two amplitudes $b_{s,f}$ unknown. Both must be positive, in order to lift $D_\psi(k)$ from its small scaling magnitude to the background value B_ψ. This requires, in the small-k range, where k^{ω_f} is negligibly small, that D_S be *severely suppressed*, according to

$$D_S(k) = a_S k^{-1/2}(1 - b_s k^{\omega_s}), \tag{3}$$

as illustrated by the middle (dot-dahsed) curve in Fig. 2. The label "slow" signifies that only the slow transient has been included in the calculation of this curve. The logarithmic derivative of the correction factor in Eq. (3) corresponds to an effective incremental exponent $-\Delta z$, where

$$\Delta z = w_s \frac{b_s k^{\omega_s}}{1 - b_s k^{\omega_s}} = 0.1 \frac{\text{correction}}{\text{net value}}, \tag{4}$$

or one-tenth of the ratio of the correction to the net corrected strength of the entropy diffusion coefficient. With this ratio taken to be of the order of unity, we see that we expect an approximately 20% increase in the scaling exponent $z = 0.50$. The plot of $\lambda(t)$ vs t undergoes the same steepening, which seems to account satisfactorily for the discrepancy in the scaling exponent, as reported by Ahlers.[2] *The negative sign of the correction in Eq. (3) is attributable to the small value of w, and is essential for understanding the increased exponent found by Ahlers.*

We now report briefly on an extensive quantitative application of the above ideas, which has yielded the good fit to the Ahlers's data shown in Fig. 1. The details will be presented elsewhere.[12] The two unknown transient amplitudes $b_{s,f}$ need to be determined. The value of B_S provides one piece of information, so that there is essentially one degree of freedom left. This is embodied in the "invisible" order-parameter background, B_ψ. By imposing *a priori* our value of a_S and allowing only B_ψ to vary as the single free parameter, we find excellent agreement with Ahlers's data for $B_\psi = 1.35 \times 10^{-4}$ cm^2/sec, which gives

$$\lambda = 21.0(c_P)^{1/2} t^{-1/3}(1 - 1.16t^{0.067} + 25.9t^{0.64}) \quad \mu\text{W}/^\circ\text{K cm}. \tag{5}$$

The transients cause the ratio D_ψ/D_S to increase gradually and monotonically from the scaling ratio of $w = 0.1$ for $k \ll k_c$ to the background ratio $B_\psi/B_S \cong \frac{2}{3}$ for $k \gg k_c$. This behavior is illustrated by the solid curves in Fig. 2, labeled ψ and S. Again we emphasize the salient feature of these curves: *The slow transient increases D_ψ and decreases D_S,* relative to their scaling values.

It can be seen in Fig. 2 that the approach to background takes place in the vicinity of $k = 1.79 \times 10^5$ cm^{-1}, the wave number of the fluctuations studied in the light scattering experiment of Tarvin, Vidal, and Greytak.[3] In a previous publication[19] it has been pointed out that the small value of w introduces some double-humped structure into the entropy fluctuation spectrum, as well as eliminating a considerable amount of the temperature dependence which would otherwise be expected. This latter point is illustrated by Fig. 2 of Ref. 1, where it is shown that the median frequency of the spectrum has much less temperature dependence than the zero-frequency value of the entropy relaxation rate. We now add to this situation the fact that background and the concomitant transients have to be taken into account. For temperatures well above T_λ the entropy fluctuation spectrum must be pure Lorentzian, with breadth $\Gamma_2/2\pi = B_S k^2/2\pi = 1.0$ MHz. This is precisely what is found. Thus, the decrease in linewidth which might be expected as the temperature is raised does not materialize because of background. Our predicted spectra, convolved with the instrumental resolution and interpreted in terms of the Γ_2 parameter of Tarvin, Vidal, and Greytak,[3] yields the good agreement[20] with their data (circles) shown in Fig. 3. Also shown (dashed curve) is less satisfactory prediction obtained by Hohenberg, Siggia, and Halperin[21] from conventional scaling theory, which neglects background and frequency dependence. The shift parameter at the λ point is predicted by the present work to be $\omega_2/2\pi = 1.3$ MHz, also in good agree-

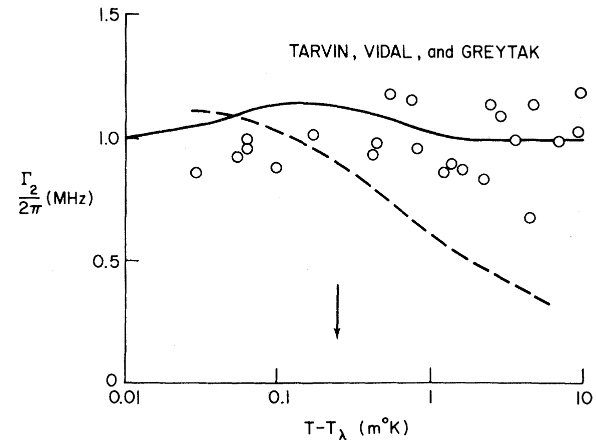

FIG. 3. Comparison of theoretical predictions with experimental data (circles) for Γ_2 vs temperature. The dashed curve shows conventional theory, while the present calculations using background and frequency dependence is represented by the solid line. The arrow indicates the temperature at which $\kappa = k$.

ment with the experimental value. Below T_λ the two Lorentzians separate into the second-sound doublet. The second-sound damping is then $\Gamma_2 = \frac{1}{2}(D_S + D_\psi)k^2$. Sufficiently far below T_λ, $D_{S,\psi}$ have to be replaced by their background values $B_{S,\psi}$. B_S is reduced somewhat by the larger specific heat for $T < T_\lambda$. Consequently we predict a smooth drop to $\Gamma_2/2\pi = 0.7$ MHz, followed at larger values of $T_\lambda - T$ by a gradual rise, resulting from the decreasing c_P. These predictions match very well the experimental situation.

To summarize, we have described two physical effects which decrease by orders of magnitude the size of wave-number space in which dynamic scaling is valid. These are the critical specific heat and the small value of the ratio of the order-parameter to entropy relaxation rate. When the negative slow transient resulting from the latter is taken into account, an excellent fit is obtained with the thermal conductivity data of Ahlers.[2] The light scattering data[3,22] are also well accounted for. We remain unable to account for the large second-sound damping observed by Tyson.[4]

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