individual rotational states. For zero distance between target and carbon foil, their relative intensities directly reflect the side-feeding intensities as discussed in Ref. 1. Their uncertainties are included in the error of the quadrupole moment which is determined by a fit of the cascade calculation to the data of Fig. 3 yielding a value of 29 ± 3 b.

This result can be found in Table I summarizing all quadrupole moments of fission-isomeric states presently known. The value for ^{239}Pu has been obtained from a reanalysis of the first charge-plunger measurement[2] using the spectroscopic information provided by the direct observation of rotational transitions in the second minimum.[3] For completeness the result for ^{236}Pu is also included.[11] All measured quadrupole moments systematically exceed the quadrupole moments of the nuclear ground states by about a factor 3 and are in good agreement with theoretical predictions.[12,13] If the shape of the nucleus in the isomeric state is approximated by a spheroid, the measured quadrupole moments correspond to an axis ratio of about 2:1 compared to about 1.3:1 for the ground states. These results thus provide consistent experimental proof for the large deformation of fission isomers and the existence of a second minimum in the fission potential.

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Incomplete Fusion in ^{12}C + ^{160}Gd Collisions Interpreted in Terms of a Generalized Concept of Critical Angular Momentum

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The energy dependence of incomplete-fusion reactions ^{160}Gd(^{12}C,α) and ^{160}Gd(^{12}C,2α) has been studied by means of α-γ coincidences in the range of bombarding energies 7.5–16.7 MeV/A. Excitation functions are interpreted in terms of a generalized concept of critical angular momentum. An energy threshold (of about 15 MeV/A) is predicted at which projectile fragmentation processes set in rapidly at the cost of the decreasing cross section for binary massive-transfer reactions.

For very mass-asymmetric systems the transfer of nucleons in heavy-ion reactions is directed almost exclusively from the light to the heavy nucleus. These "massive-transfer" reactions are frequently referred to as "deep-inelastic" processes because of their large cross sections and very negative Q values, though the underlying reaction mechanism might be quite different.

We report in this Letter on an investigation of the ^{160}Gd + ^{12}C reaction in the energy range from 7.5 to 16.7 MeV/A. Aside from complete fusion, and quasi-inelastic and deep-inelastic scattering, we find the incomplete-fusion reactions (^{12}C,α) and (^{12}C,2α) and projectile fragmentation (^{12}C,3α) to be the dominant reaction channels. We show that incomplete-fusion reactions appear to be a natural extension of the fusion process for those collisions for which angular momentum limitations do not allow complete fusion. Above 15 MeV/A, projectile fragmentation sets in rapidly at the cost of the decreasing cross section for fusion and incomplete-fusion reactions. As we will show in this Letter, the competition between the different reaction modes can be explained by a generalized concept of critical angular momentum.

Experiments were performed using $^{12}C^{4+}$ beams of 90, 120, 160, and 200 MeV from the Kernfysisch Versneller Instituut variable-energy cyclotron bombarding a 1.35-mg/cm^2-thick target of metallic ^{160}Gd enriched to 98.6%. Angular distributions of charged particles, α-α coincidences to determine α-particle multiplicities, and charged-particle-γ coincidences were measured. Charged particles were detected with a standard solid-state ΔE-E telescope(s). In the particle-γ coincidence measurements the particle telescope was placed at a forward angle (usually at 20°), and a γ-ray Ge(Li) detector was positioned at 90° to the beam direction at a distance of 4 cm from the target. By recording particle-γ coincidences we were able to identify simultaneously projectile residues (mostly "fast" α particles) and target-residue nuclei. ^{160}Gd was chosen as target because of the well-known and characteristic γ transitions in the possible residual nuclei. Since the details of our extensive study will be published in a forthcoming paper, we only briefly summarize the main results and conclusions before we concentrate on the discussion of the energy dependence and the competition between the dominant reaction channels.

The main reaction channels that were identified via particle-γ coincidences are ^{160}Gd(^{12}C, αxn)-$^{168-x}$Er and ^{160}Gd(^{12}C, $2\alpha xn$)$^{164-x}$Dy. The coincident α particles from the (^{12}C, αxn) and (^{12}C, $2\alpha xn$) reactions have, in the average, the beam-velocity energies. Angular distributions of these α particles are forward peaked, approximately with the same shape as for α particles detected inclusively. The absence of side feeding to the lowest members of the yrast bands in the target-residue nuclei indicates that low partial waves are strongly hindered in the (^{12}C, αxn) and (^{12}C, $2\alpha xn$) reactions. This feature of the incomplete-fusion reactions has also been reported by Inamura *et al.*[1] and by Zolnowski *et al.*[2]

We interprete the observed reactions ^{160}Gd(^{12}C, αxn)$^{168-x}$Er and ^{160}Gd(^{12}C, $2\alpha xn$)^{164}Dy as the capture of a "^{8}Be" fragment in the first reaction and the capture of an α particle in the second, leading to highly excited ^{168}Er* and ^{164}Dy* "compound" nuclei, respectively, which subsequently deexcite by a statistical neutron-evaporation cascade. (Such a mechanism can be considered as a heavy-ion analog to the Oppenheimer-Phillips stripping of the deuteron.) The kinematics of these incomplete-fusion reactions and their localization in l space (deduced from the side-feeding distributions) indicate that they occur in the peripheral collisions, probably for l values just above the critical angular momentum for complete fusion.

Absolute cross sections for the incomplete-fusion reactions (^{12}C, α) and (^{12}C, 2α) were obtained by summing up the integrated cross sections for all reaction subchannels with appreciable yield:

$$\begin{aligned}\sigma(^{12}\mathrm{C},\alpha) &= \textstyle\sum_x \sigma(^{12}\mathrm{C},\alpha xn),\\ \sigma(^{12}\mathrm{C},2\alpha) &= \textstyle\sum_x \sigma(^{12}\mathrm{C},2\alpha xn).\end{aligned} \tag{1}$$

These integrated cross sections are shown in Fig. 1(a) together with the total inclusive cross sections, $\sigma_\alpha^{\mathrm{incl}}$ It is seen that only a fraction of the singles α particles [(20–40)% in the studied energy range] can be explained as resulting from the (^{12}C, α) and (^{12}C, 2α) reactions. Three-α-particle breakup of ^{12}C is the likely candidate to explain the missing cross section. We have estimated expected magnitudes of $\sigma(^{12}\mathrm{C},3\alpha)$ by assuming that $\sigma_\alpha^{\mathrm{incl}}$ consists of three main processes:

$$\sigma_\alpha^{\mathrm{incl}} = \sigma(^{12}\mathrm{C},\alpha) + 2\sigma(^{12}\mathrm{C},2\alpha) + 3\sigma(^{12}\mathrm{C},3\alpha). \tag{2}$$

In order to verify this hypothesis we performed a series of α-α coincidence experiments.[3] These measurements yielded a value $\langle M_\alpha \rangle = 1.92 \pm 0.15$ for the average α multiplicity at $E(^{12}\mathrm{C}) = 120$ MeV. Neglecting processes with emission of more than three α particles in a single ^{12}C + ^{160}Gd collision, i.e., and assuming

$$\langle M_\alpha \rangle = \frac{\sigma_\alpha^{\mathrm{incl}}}{\sigma(^{12}\mathrm{C},\alpha) + \sigma(^{12}\mathrm{C},2\alpha) + \sigma(^{12}\mathrm{C},3\alpha)}, \tag{3}$$

we obtain for $\sigma(^{12}\mathrm{C},3\alpha) = 190 \pm 40$ mb, whereas relation (2) gives $\sigma(^{12}\mathrm{C},3\alpha) = 175 \pm 20$ mb. The consistency between these two independent results is support for assumption (2).

In spite of the large cross section $\sigma(^{12}\mathrm{C},3\alpha)$, and high γ-α efficiency (enhanced by the multiplicity $M_\alpha = 3$) we did not observe any γ transitions in ^{160}Gd or $^{160-x}$Gd in the γ-α coincidence spectra. From this we conclude that, in the average, the 3α breakup of ^{12}C does not involve excitation of the target nucleus.

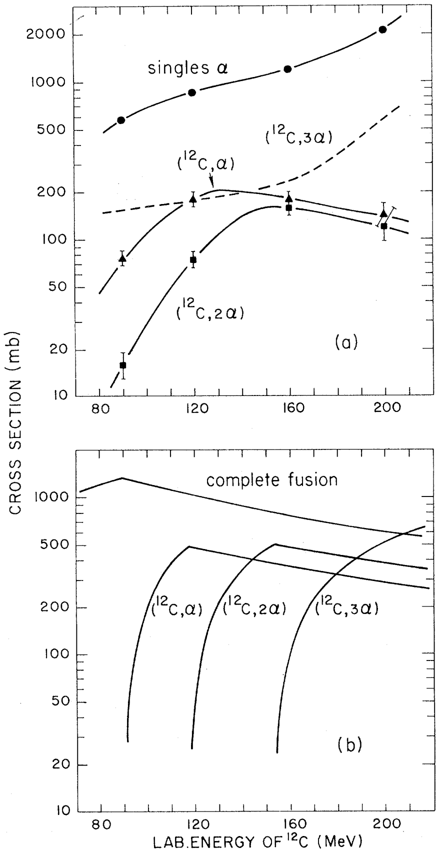

FIG. 1. (a) Energy dependence of the cross sections $\sigma_\alpha{}^{\text{incl}}$, $\sigma(^{12}\text{C},\alpha)$, $\sigma(^{12}\text{C},2\alpha)$, and $\sigma(^{12}\text{C},3\alpha)$ in $^{12}\text{C}+{}^{160}\text{Gd}$ collisions. The cross sections $\sigma(^{12}\text{C},3\alpha)$ were deduced assuming relation (2). The lines are drawn to guide the eye. (b) Energy dependence of the cross sections for complete fusion, reactions $(^{12}\text{C},\alpha)$, $(^{12}\text{C},2\alpha)$, and $(^{12}\text{C},3\alpha)$ resulting from a simple model based on a generalized concept of critical angular momentum (see text).

A striking feature of the excitation functions for the $(^{12}\text{C},\alpha)$ and $(^{12}\text{C},2\alpha)$ reactions [Fig. 1(a)] is the presence of well-defined energy thresholds, that differ for both reactions, and that are localized well above the Coulomb barrier for the $^{12}\text{C}+{}^{160}\text{Gd}$ system (~ 50 MeV in lab). Also the $\sigma(^{12}\text{C},3\alpha)$ cross section which does not change significantly in the energy range from 90 to 160 MeV, shows a fast rise at higher bombarding energies. We interprete these results in terms of a generalized concept of critical angular momentum. We suggest that angular momentum limitations in the entrance channel govern the competition between complete-fusion, incomplete-fusion, and projectile-fragmentation processes.

To quantitatively describe the observed energy dependence of the cross sections we propose the following model. We consider the class of collisions between $l=0$ and the "hard-grazing" l value (l_{hg}), characterized by the distance of closest approach being equal to the sum of the half-density radii, $R_{\min}=C_1+C_2$. For all those collisions the nuclear density in the contact region reaches the bulk nuclear density. The balance of the nuclear, the Coulomb, and the centrifugal forces in this particular configuration does determine whether the colliding system fuses completely, whether only a fragment of the projectile is captured, or whether none of the projectile fragments are captured. We assume that *the heaviest fragment is preferentially captured* (if allowed by the angular momentum limitations). Moreover, it is assumed that every virtual *fragment of the projectile carries a part of the total angular momentum that is proportional to its mass number.* Thus, in the sharp-cutoff approximation the part of the reaction cross section that is limited by the hard-grazing angular momentum,

$$\sigma_R' \simeq \pi \bar{\lambda}^2 \sum_{l=0}^{l_{\text{hg}}} (2l+1), \tag{4}$$

is divided into l bins dominated by different reactions:

complete fusion,

$$0<l<l_{\text{cr}(\text{C+Gd})}; \tag{5}$$

"^{8}Be" capture, $(^{12}\text{C},\alpha)$,

$$l_{\text{cr}(\text{C+Gd})}<l<\tfrac{12}{8}l_{\text{cr}(\text{Be+Gd})}; \tag{6}$$

α capture, $(^{12}\text{C},2\alpha)$,

$$\tfrac{12}{8}l_{\text{cr}(\text{Be+Gd})}<l<\tfrac{12}{4}l_{\text{cr}(\alpha+\text{Gd})}; \tag{7}$$

3α breakup, $(^{12}\text{C},3\alpha)$,

$$\tfrac{12}{4}l_{\text{cr}(\alpha+\text{Gd})}<l<l_{\text{hg}}. \tag{8}$$

In the notation used here $l_{\text{cr}(\text{C+Gd})}$, $l_{\text{cr}(\text{Be+Gd})}$, and $l_{\text{cr}(\alpha+\text{Gd})}$ are the critical angular momenta for $^{12}\text{C}+{}^{160}\text{Gd}$, $^8\text{Be}+{}^{160}\text{Gd}$, and $\alpha+{}^{160}\text{Gd}$ systems, respectively, and $\bar{\lambda}$ is the entrance-channel wavelength, with $\bar{\lambda}^2=\hbar^2/2\mu E$.

In Fig. 1(b) we show the energy dependence of the cross sections for the $^{12}\text{C}+{}^{160}\text{Gd}$ complete-fusion, the $^{160}\text{Gd}(^{12}\text{C},\alpha)$ and $^{160}\text{Gd}(^{12}\text{C},2\alpha)$ incomplete-fusion, and the $^{160}\text{Gd}(^{12}\text{C},3\alpha)$ projectile-breakup reactions calculated according to the model outlined above. The calculated curves in Fig. 1(b) correspond to $l_{\text{cr}(\text{C+Gd})}=43\hbar$, $l_{\text{cr}(\text{Be+Gd})}=35\hbar$, and $l_{\text{cr}(\alpha+\text{Gd})}=21\hbar$. The hard-grazing angular momenta were calculated at the half-density radius[4,5] using the nuclear potential derived from boundary conditions[6] following the liquid-drop model. (For justification of this potential for fusion reactions, see Siwek-Wilczyńska and Wilczyński.[7])

The calculated curves show a good qualitative agreement with the experimental data. The assumed sharp-cutoff approximation results in the steep rise of the calculated cross sections just above the corresponding energy thresholds. Relaxing this assumption a bit, i.e., assuming a more realistic smooth transition between the l bins (5)-(8), one could easily reproduce the observed shapes of the incomplete-fusion excitation functions. Absolute cross sections of the (^{12}C, α) and (^{12}C, 2α) reactions exhaust about 40% of the geometrical cross section in the corresponding l windows. The rest of the cross section in these l windows must be taken by other channels, e.g., by inelastic processes involving excitations of ^{160}Gd while ^{12}C remains bound with respect to $\alpha + {}^8$Be breakup. Only the fast rise of the (^{12}C, 3α) cross section (above 160 MeV) is explained by our model. Probably the breakup in more-distant collisions ($R_{\min} > C_1 + C_2$) is responsible for the flat part below the threshold.

It is interesting to note that the critical angular momenta deduce in this work agree very well with results obtained from completely different experiments. The comparison is shown in Table I. Included are also the l_{cr} values obtained from a simple estimate[10] based on the liquid-drop model:

$$l_{\mathrm{cr}}^{\;2} \simeq \frac{\mu(C_1+C_2)^3}{\hbar^2}\left[4\pi\gamma\,\frac{C_1C_2}{C_1+C_2} - \frac{Z_1Z_2e^2}{(C_1+C_2)^2}\right]. \quad (9)$$

Here $\gamma \simeq 0.95$ MeV/fm^2 is the surface-tension coefficient, μ is the reduced mass, Z_1 and Z_2 are the atomic numbers, and C_1 and C_2 are the half-density radii.

TABLE I. Critical angular momenta for ^{12}C + ^{160}Gd, ^{8}Be + ^{160}Gd, and ^{4}He + ^{160}Gd systems (in units of $\hbar$).

System	This work	Other data	Eq. (9)
^{12}C + ^{160}Gd	43 ± 3	46 ± 3[a]	42
^{8}Be + ^{160}Gd	35 ± 2	...	32
^{4}He + ^{160}Gd	21 ± 3	25 ± 2[b]	22[c]

[a] Deduced from the complete-fusion cross section for ^{12}C + ^{158}Gd (Ref. 8).

[b] Deduced from the energy dependence of the ^{160}Gd(α, xn) cross section (Ref. 9).

[c] For $C_\alpha = 1.5$ fm.

Formula (9) was shown[10] to correctly reproduce the essential trends in the dependence of the fusion cross section on the charge and mass numbers of the constituents. As can be seen from Table I, its validity seems to extend even to as light a projectile as ^{4}He. The sequence of the critical angular momenta for incomplete-fusion reactions, $(M_P/M_i) l_{\mathrm{cr}(i+T)}$, predicted by Eq. (9) is always increasing with the decreasing mass M_i of the captured fragment [M_P is the mass of the projectile, $l_{\mathrm{cr}(i+T)}$ is the critical angular momentum for the target (T) plus the fragment (i) system]. Therefore, above a certain critical energy (which is predicted to be about 15 MeV/A for typical colliding systems) none of the projectile fragments (except for single nucleons or at most ^{3}He) can be captured in peripheral collisions. Consequently, above 15 MeV/A the cross sections for all possible binary incomplete-fusion reactions must decrease with increasing bombarding energy, thus making room for three-body (or multibody) fragmentation processes. Results of our study as well as of experiments carried out at 20 MeV/A by Gelbke *et al.*[11] confirm this prediction.

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