Hence according to Eq. (23) the second-order energy shift is given by

$$E_2 = -\int_{x_1}^{x_2} [2/\rho(x)]\, dx\,. \tag{46}$$

On comparing this with the second-order energy shift as calculated in the Rayleigh-Schrödinger theory we obtain the following sum rule:

$$\left[\frac{1}{\psi_0(x_1)\psi_0(x_2)}\right]^2 \sum_n \frac{[\psi_0(x_1)\psi_n(x_2) - \psi_n(x_2)\psi_n(x_1)]^2}{E_n - E_0} = \int_{x_1}^{x_2} \frac{2dx}{[\psi_0(x)]^2}\,. \tag{47}$$

We see that in Eq. (46), the magnitude of the second-order shift is large if the probability density in between x_1 and x_2 is small. We give a physical interpretation to this result in the following particular example. Consider a Hamiltonian whose unperturbed potential is given by

$$V_0 = (x - x_1)^2 (x - x_2)^2. \tag{48}$$

The height of the barrier between the two valleys is equal to $[(x_1 - x_2)/2]^4$. The higher this barrier is, the smaller the wave function in between x_1 and x_2 is. In the limit that this barrier becomes infinite, the system will decouple into two separate oscillators centered around x_1 and x_2, with identical energy levels. A finite but high barrier means that almost degenerate energy levels with the same or opposite parity exist. The perturbation as given by Eq. (44) breaks the symmetry. This parity-nonconserving perturbation connects these almost degenerate states, thereby leading to a large energy shift.

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[1]See, for example, G. Baym, *Lectures on Quantum Mechanics* (Benjamin, New York, 1969).

[2]The fact the second-order energy shift can be obtained without the use of Green's functions or sums over intermediate states have been emphasized previously by Sternheimer, Dalgarno and Lewis, and Dalgarno and Stewart [R. Sternheimer, Phys. Rev. 84, 244 (1951); A. Dalgarno and J. T. Lewis, Proc. Roy. Soc. London 233, 70 (1955); A. Dalgarno and A. L. Stewart, Proc. Roy. Soc. London 238, 269 (1969)]. However, none of these authors expressed their solutions in quadrature forms as we have done here. Instead, their methods involve the solutions of inhomogenous differential equations. Our present work also goes beyond the works of these authors in our discussion on the higher-order corrections.

Constraints on Weak-Current Angles of the Six-Quark Model from the K^0-$\bar{K}^0$ System

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From calculations of the K_L-K_S mass difference and CP nonconservation based on an effective quark Lagrangian, we limit the ranges of the charged—weak-current mixing angles θ_2 and θ_3, and phase parameter δ. The relative strength of $b \to u$ and $b \to c$ couplings is determined versus θ_3.

In the sequential six-quark $SU(2)_L \otimes U(1)$ model, the left-handed $(t, b')_L$ is added to the doublets $(u, d')_L$, $(c, s')_L$ of the standard model.[1,2] The 3×3 unitary matrix U which relates the gauge-group eigenstates $(d', s', b')_L$ to the mass eigenstates (d, s, b) contains three rotation angles θ_i and a CP-nonconserving phase δ. In the form introduced by Kobayashi-Maskawa,[3] the matrix U can be written

$$U = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 + s_2s_3e^{i\delta} & c_1c_2s_3 - s_2c_3e^{i\delta} \\ -s_1s_2 & c_1s_2c_3 - c_2s_3e^{i\delta} & c_1s_2s_3 + c_2c_3e^{i\delta} \end{pmatrix}, \tag{1}$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$. By suitable choices of the signs of the quark fields, we can restrict the angles to the ranges[4] $0 \leq \theta_i \leq \pi/2$ and $-\pi \leq \delta \leq \pi$. The charged weak current is

$$J_\mu = 2(\bar{u}\,\bar{c}\,\bar{t})_L \gamma_\mu U \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L . \tag{2}$$

Since the θ_i and δ parameters govern the lifetimes, decay branching ratios, mixing, and CP nonconservation of heavy quark systems,[5-7] phenomenological determinations of their values from existing data are of considerable interest. Analyses based on $d \to u$, $s \to u$, and $\mu \to \nu$ data give[8] $|s_1| \simeq 0.23$ and set the limit $|s_3| < 0.5$. From an approximate calculation of the K_L-K_S mass difference, it was estimated[6] that $|s_2| < 0.3$ for $m_t > 10$ GeV.

In this Letter we do a careful quantitative theoretical analysis of the K_L-K_S system, without making the usual $m_q/m_W \ll 1$ approximations to the single-loop quark transition amplitudes. We allow for the possibility of a bag-model correction factor[9] $B \simeq 0.4$ to the vacuum insertion approximation in the $K^0, \bar{K}^0$ matrix element of the effective four-quark Lagrangian. We assume that the observed CP nonconservation arises solely from the phase angle δ. Requiring that the calculation of the $K^0 \to \bar{K}^0$ transition reproduces the observed mass difference and CP nonconservation, we obtain constraints on allowed values of the mixing and phase parameters.

The K_S-K_L mass difference $\delta m = m(K_S) - m(K_L)$ is computed from the K^0-$\bar{K}^0$ transition amplitude:

$$\delta m = 2\,\mathrm{Re}\langle \bar{K}_0 | [-\mathcal{L}_{\mathrm{eff}}(\bar{s}d \to s\bar{d})] | K_0 \rangle, \tag{3}$$

where $\mathcal{L}_{\mathrm{eff}}$ is the effective four-quark amplitude at the single-loop level. The transition amplitude is[10]

$$\langle \bar{K}^0 | (-\mathcal{L}_{\mathrm{eff}}) | K^0 \rangle = \frac{-B f_K^2 m_K (G_{\mathrm{F}}/\sqrt{2})(\alpha/4\pi)}{3 x_{\mathrm{W}}} \sum_{i,j=u,c,t} \lambda_i \lambda_j A_{ij}, \tag{4}$$

where $\lambda_i = U_{is} U_{id}^*$ and A_{ij} represents the single-loop integral with intermediate quarks q_i and q_j. In Eq. (4) $f_K \simeq 1.23 m_\pi$ is the kaon decay constant, and x_{W} is the Weinberg-angle factor $x_{\mathrm{W}} = \sin^2\theta_{\mathrm{W}} \simeq 0.2$. $B \simeq 1$ represents the vacuum insertion approximation, and $B \simeq 0.4$ is the estimate of the bag-model correction. We consider both values in our analysis. In the limit where external momenta are small, compared to M_W and to masses of the heavy quarks that appear in the internal lines, the amplitude A_{ij} is given by[11]

$$A_{ij} = \frac{1}{(1-x_i)(1-x_j)} + \frac{1}{x_i - x_j}\left[\frac{x_i^2 \ln x_i}{(1-x_i)^2} - \frac{x_j^2 \ln x_j}{(1-x_j)^2}\right], \tag{5}$$

where $x_i = m_i^2/m_W^2$ and $m_W = [\pi\alpha/(\sqrt{2}\, G_{\mathrm{F}}\, x_{\mathrm{W}})]^{1/2} \simeq 84$ GeV. In an expansion in powers of x_i and x_j, the terms independent of x_i and x_j cancel in Eq. (4) because of the unitarity relation $\sum_i \lambda_i = 0$. The leading terms in the summation are

$$\sum_i \lambda_i^2 x_i + \sum_{i \neq j} \lambda_i \lambda_j \frac{x_i x_j}{x_i - x_j} \ln\frac{x_i}{x_j} .$$

Previous analyses[6,9] were based on this approximation which may be inaccurate for heavy quarks. In our calculations we use the exact expression of Eq. (5).

In the decay eigenstates $|K_{S,L}\rangle = (|K_{1,2}\rangle + \rho|K_{2,1}\rangle)/[(1+|\rho|^2)]^{1/2}$, the CP parameter is[12]

$$\rho = \frac{i\,\mathrm{Im}\,m_{12} + \frac{1}{2}\mathrm{Im}\Gamma_{12}}{i\frac{1}{2}\delta\Gamma - \delta m}, \tag{6}$$

where $\delta\Gamma = \Gamma_S - \Gamma_L$. Studies of $K \to 2\pi$ decays give information on ρ in conjunction with the $\pi\pi$ amplitudes

$$A_I \equiv \exp(-i\delta_I)\langle \pi\pi, I | T | K^0 \rangle, \tag{7}$$

where δ_I is the $\pi\pi$ phase shift for isospin I. With the Kobayashi-Maskawa convention,[3] $\mathrm{Im}A_2 = 0$; it is convenient to work with this natural convention of the model rather than transforming to the Wu-Yang convention, for which $\mathrm{Im}A_0 = 0$. The CP-nonconserving observables in $K \to 2\pi$ decays are then[12]

$$\eta_{+-} = \rho + i(\mathrm{Im}A_0/\mathrm{Re}A_0)(1 + \omega/\sqrt{2})^{-1}, \tag{8}$$

$$\eta_{00} = \rho + i(\mathrm{Im}A_0/\mathrm{Re}A_0)(1 - \sqrt{2}\,\omega)^{-1}, \tag{9}$$

where

$$\omega = (\mathrm{Re}A_2/\mathrm{Re}A_0)\exp[i(\delta_2 - \delta_0)]. \tag{10}$$

The experimental phase[13] and magnitude[14] of ω are $\delta_2 - \delta_0 = -53.2° \pm 5.2°$ and $|\omega| = 0.0448 \pm 0.0002$.

In the approximation that the $I = 0$, 2π intermediate state dominates, $\mathrm{Im}\Gamma_{12}$ is given by[12]

$$\mathrm{Im}\Gamma_{12} = (\mathrm{Im}A_0/\mathrm{Re}A_0)\Gamma_S. \quad (11)$$

In the Kobayashi-Maskawa model $\mathrm{Im}A_0$ arises only from diagrams with heavy-quark intermediate states and so is expected to be suppressed by the Zweig-Iizuka rule.[5,6] The exact amount of suppression is difficult to estimate because of the uncertain role of gluon-exchange penguin diagrams.[4,15] The penguin diagrams[4] give $\mathrm{Im}\Gamma_{12} < 0$. From the experimental bound[13] $|\eta_{+-} - \eta_{00}|/|2\eta_{+-} + \eta_{00}| \lesssim 1/50$, the upper bound on penguin-diagram contributions is[4]

$$|\mathrm{Im}\Gamma_{12}/\mathrm{Im}\,m_{12}| < 0.4. \quad (12)$$

Taking the real part of Eq. (6), we find

$$\mathrm{Im}\,m_{12} = \mathrm{Re}\rho\left(\frac{\delta\Gamma}{2\delta m} + \frac{2\delta m}{\delta\Gamma}\right)\delta m + \mathrm{Im}\Gamma_{12}\left(\frac{\delta m}{\delta\Gamma}\right), \quad (13)$$

where $\delta m = -0.477\Gamma_S = -0.352\times 10^{-14}$ GeV and $\delta\Gamma \simeq \Gamma_S$. We neglect the $\mathrm{Im}\Gamma_{12}$ contribution to Eq. (13). Inserting the experimental value $\mathrm{Re}\rho = (1.62 \pm 0.088)\times 10^{-3}$ from $K_L \to \pi e \nu$ asymmetry measurements,[16] Eq. (13) becomes

$$\mathrm{Im}\,m_{12} = -3.25\times 10^{-3}\delta m. \quad (14)$$

The CP-nonconserving mass term is related to the K^0-$\bar{K}^0$ transition amplitude by

$$\mathrm{Im}\,m_{12} = \mathrm{Im}\langle \bar{K}^0|(-\mathcal{L}_{\mathrm{eff}})|K^0\rangle. \quad (15)$$

Consistency of Eqs. (3), (4), and (15) with the experimental determinations of δm and $\mathrm{Im}\,m_{12}$, places two restrictions on the parameters of Eq. (1). Since s_1 is already known, we can solve for the allowed values of s_2 and δ as a function of s_3, given the constituent quark masses[11,17] which enter the amplitude A_{ij} of Eq. (5). We take $m_u = 0.3$ GeV, $m_c = 1.5$ GeV, and consider three choices for the mass of the t quark: $m_t = 14$, 30, and 60 GeV.

Figure 1 shows the solutions for s_2 and $s_\delta \equiv \sin\delta$ vs s_3. For CP nonconservation in the six-quark model s_2, s_3, and s_δ must all be nonvanishing. The minimum value of s_2 consistent with the observed $\mathrm{Im}\,m_{12}$ occurs for very small s_3 (of order 10^{-3}) and $\delta = 90°$. The allowed range of s_2,

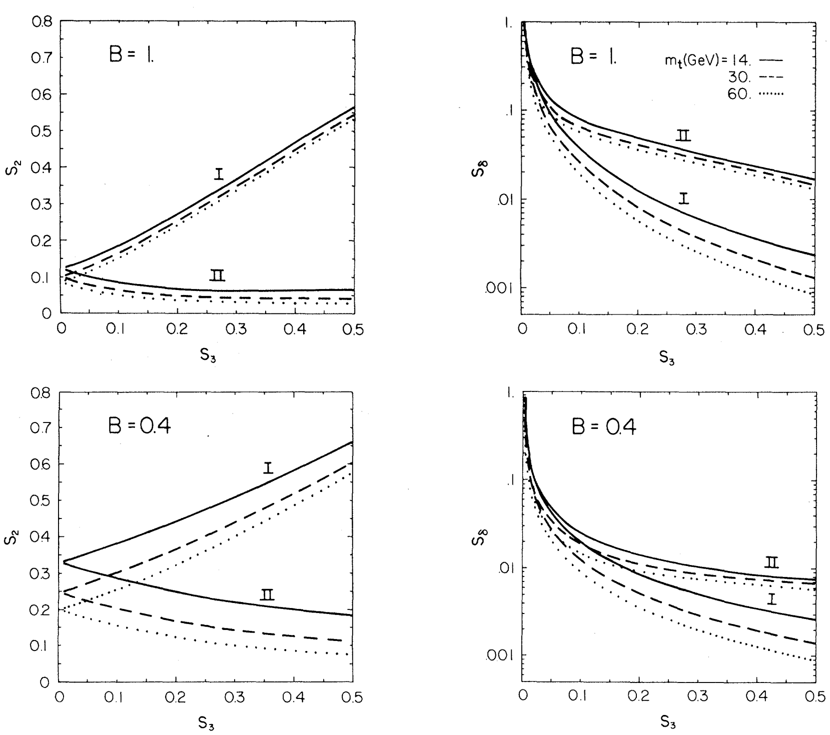

FIG. 1. Solutions for s_2 and s_δ vs s_3, for t-quark masses of 14 GeV (solid curves), 30 GeV (dashed curves), and 60 GeV (dotted curves). $B = 1$ corresponds to the vacuum insertion approximation and $B = 0.4$ to the bag model. The phase δ lies in the first quadrant for solution I and in the second quadrant for solution II.

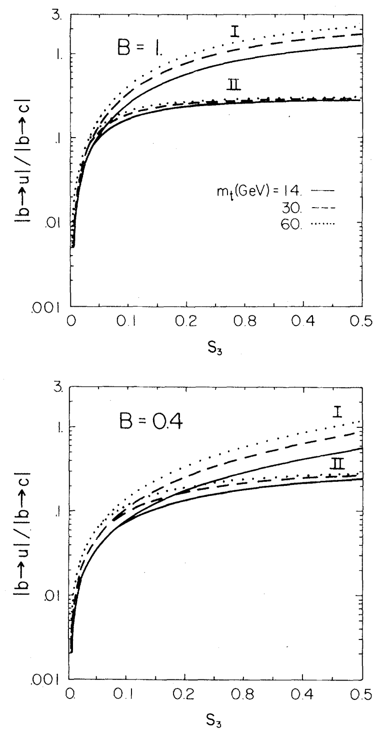

FIG. 2. Ratio of $b\to u$ and $b\to c$ couplings vs s_3. Labeling of curves corresponds to that of Fig. 1.

for $m_t \leq 30$ GeV, is

$$0.11 < s_2 < 0.66 \quad \text{for } B = 0.4, \qquad (16)$$
$$0.04 < s_2 \leq 0.57 \quad \text{for } B = 1.$$

Here the s_2 upper bounds correspond to the $s_3 \leq 0.50$ universality bound. The s_2 upper bound for $B=1$ is considerably larger than that given by the approximate calculation of Ref. 6.

The $|b\to u|/|b\to c|$ coupling ratio is plotted versus s_3 in Fig. 2. We see that the possibility that the b quark decays only via the mode $b\to u$ is ruled out. The bounds on this ratio for $m_t < 30$ GeV are

$$|(b\to u)/(b\to c)| \leq 0.9 \quad \text{for } B = 0.4, \qquad (17)$$
$$|(b\to u)/(b\to c)| \leq 1.8 \quad \text{for } B = 1.$$

Thus $b\to c\to s,d$ cascade modes occur at a substantial level.

The limits on the angles θ_2 and θ_3, and on δ, can be used to bound the lifetime of the b quark.[18] However, such lifetime limits will be dependent on assumptions about nonleptonic enhancement.

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