PHYSICAL REVIEW LETTERS

VOLUME 42 11 JUNE 1979 NUMBER 24

New Measurement of the Proton Gyromagnetic Ratio and a Derived Value of the Fine-Structure Constant Accurate to a Part in 10^7

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(Received 1 March 1979)

A new value for the fine-structure constant has been obtained from a new measurement of the gyromagnetic ratio of the proton. The result, $\gamma_p'(\text{low})_{\text{NBS}} = 2.675\,132\,29(57)\times 10^8\ \text{s}^{-1}\ \text{T}_{\text{NBS}}^{-1}$ (0.21 ppm), is used to derive $\alpha^{-1} = 137.035\,963(15)$ (0.11 ppm). This value of α^{-1} is (0.33 ± 0.14) ppm less than the value of α^{-1} derived from measurements of the anomalous magnetic moment of the electron, a_e, and its current best quantum electrodynamics theoretical estimate.

The major purpose of a new proton gyromagnetic ratio (γ_p') measurement is to help test the validity of the conventional QED (quantum electrodynamic) theory.[1] The important role of γ_p' in testing QED comes from the need for a value of the fine-structure constant, α, which is essentially independent of QED. Such a value may be obtained from the now well-known equation[2]

$$\alpha^{-2} = \frac{c}{4R_\infty}\,\frac{1}{(\Omega_{\text{NBS}}/\Omega)}\,\frac{\mu_p'}{\mu_{\text{B}}}\,\frac{(2e/h)_{\text{NBS}}}{\gamma_p'(\text{low})_{\text{NBS}}}. \qquad (1)$$

Here c is the speed of light in vacuum; R_∞ the Rydberg constant for infinite mass; $\Omega_{\text{NBS}}/\Omega$, the ratio of the National Bureau of Standards (NBS) as-maintained ohm to the absolute or SI (International System) ohm; μ_p'/μ_{B}, the magnetic moment of the proton in units of the Bohr magneton (throughout, the prime indicates protons in a spherical sample of pure H_2O at 25°C); $(2e/h)_{\text{NBS}}$, the ratio of twice the elementary charge to the Planck constant measured in terms of the NBS as-maintained volt using the ac Josephson effect; and $\gamma_p'(\text{low})_{\text{NBS}}$, the gyromagnetic ratio of the proton measured in terms of the NBS as-maintained ampere by the so-called low-field method.

The importance of Eq. (1) was first demonstrated in the late 1960's when the value of α derived from it was used to resolve the discrepancy between the theoretical and experimental values for the ground-state hyperfine splitting in hydrogen.[3] Subsequently, the Josephson-effect measurement of $2e/h$ advanced quickly, and by the early 1970's the primary limitation for obtaining a more accurate value of α from Eq. (1) was the uncertainty associated with the experimental determination of the proton gyromagnetic ratio. At that time, the most accurate measurement of $\gamma_p'(\text{low})$ had an uncertainty of two parts per million (ppm) and was obtained in this laboratory.[4] Recognizing the importance of an improved value of α, we undertook a new experiment; the goal of achieving an order-of-magnitude increase in accuracy has now been reached and is the subject of this Letter.

We first discuss the results and then give the experimental details. Improved methods for measuring the dimensions of a precision solenoid have yielded the following:

$$\gamma_p'(\text{low})_{\text{NBS}} = 2.675\,132\,29(57)\times10^{8}\ \text{s}^{-1}\,\text{T}_{\text{NBS}}^{-1} \quad (0.21\ \text{ppm}). \qquad (2)$$

When this is combined with the present best values for the other fundamental constants entering Eq. (1) (see Table I), the resulting value of α is

$$\alpha^{-1} = 137.035\,963(15) \quad (0.11\ \text{ppm}). \qquad (3)$$

While an accurate value of α independent of QED is needed to compare the experimentally determined and QED-theoretical values of such quantities as Lamb shifts and the ground-state hyperfine splittings in hydrogen, muonium, and positronium, the quantity which can presently benefit most from the part-in-10^7 accuracy of Eq. (3) is the anomalous magnetic moment of the electron, a_e. Van Dyck, Schwingberg, and Dehmelt[5] have experimentally obtained

$$a_e(\text{expt.}) = 1.159\,652\,200(40)\times10^{-3} \quad (0.034\ \text{ppm}). \qquad (4)$$

The QED-theoretical expression for a_e may be

TABLE I. Values assumed for constants of Eq. (1).

Constant	Value	Uncertainty [a] (ppm)
R_∞	$10\,973\,731.476\ \text{m}^{-1}$	0.003 [b]
c	$299\,792\,458\ \text{m s}^{-1}$	0.004 [c]
$\Omega_{\text{NBS}}/\Omega$	$1-0.819\times10^{-6}$	0.057 [d]
μ_p'/μ_B	$1.520\,993\,134\times10^{-3}$	0.014 [e]
$(2e/h)_{\text{NBS}}$	$4.835\,934\,20\times10^{14}\ \text{Hz V}_{\text{NBS}}^{-1}$	0.030 [f]
$\gamma_p'(\text{low})_{\text{NBS}}$	$2.675\,132\,29\times10^{8}\ \text{s}^{-1}\ \text{T}_{\text{NBS}}^{-1}$	0.21

[a] 1-standard-deviation (68%-confidence-level) estimates.

[b] J. E. M. Goldsmith, E. W. Weber, and T. W. Hänsch, Phys. Rev. Lett. 41, 1525 (1978).

[c] Value recommended by the Comité Consultatif pour la Définition du Mètre (CCDM); see J. Terrien, Metrologia 10, 75 (1974).

[d] R. D. Cutkosky, IEEE Trans. Instrum. Meas. 23, 305 (1974). Note that this includes an additional 0.05-ppm uncertainty due to the possible drift of the NBS ohm since its last absolute realization in 1974. A new realization is in progress.

[e] Corrected to 25°C, and for the new value of a_e, see W. D. Phillips, W. E. Cooke, and D. Kleppner, Metrologia 13, 179 (1977).

[f] Value used to maintain NBS volt; see B. F. Field, T. F. Finnegan, and J. Toots, Metrologia 9, 155 (1973).

written as[1]

$$a_e(\text{theory}) = \tfrac{1}{2}(\alpha/\pi) - 0.328\,478\,445(\alpha/\pi)^2 + C_3(\alpha/\pi)^3 + C_4(\alpha/\pi)^4 \cdots. \qquad (5)$$

Neglecting C_4 and other terms which are negligible at the present level of accuracy and using the values in Eqs. (3) and (4), we obtain an experimental value for C_3:

$$C_3(\text{expt.}) = 1.154(11). \qquad (6)$$

Calculations of all 72 sixth-order Feynman diagrams have resulted in the following generally accepted theoretical value[6] for C_3:

$$C_3(\text{theory}) = 1.184(7). \qquad (7)$$

The disagreement between Eqs. (6) and (7) is deemed acceptable in view of the unfinished nature of the theoretical calculation of a_e.

The experimental determination of γ_p' by the "low-field" method requires the measurement of both the proton precession frequency via NMR techniques, and the magnetic field which produces the precession. The key problem is the accurate dimensional measurement of a precision solenoid so that the magnetic field produced by a known current can be calculated. The solenoid is single layered and wound on a hand-lapped, fused silica form. Its critical dimensions are measured by a magnetic induction technique which locates the current in the windings.[6,7] Coils A and A' in Fig. 1 form a linear differen-

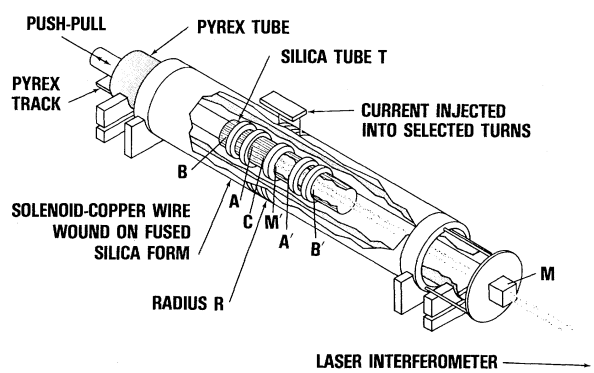

FIG. 1. Solenoid dimensional measurement system used to determine the axial position and radial variations of the wires. The five coils A, A', B, B', and C are attached to a silica tube T and can be pushed or pulled along the axis of the solenoid. Coils A and A' locate the axial position of the injected current and coils B, B', and C form a diameter-to-voltage transducer. Mirrors M and M' are part of a linear interferometer. The Pyrex tube is evacuated.

tial transformer which locates the axial position of the current injected into selected turns of the solenoid. Coils A and A' are connected so that their output voltages cancel when centered on the activated turns of the solenoid, and a servo system locks the coil assembly to the null point with a precision of better than 0.05 μm. A mirror (corner cube, M') located in the center of the coil assembly is part of a laser interferometer system with the reference mirror (M) connected to the end of the solenoid. Thus, with the aid of a computer-automated system, the relative position of successive turns can be recorded. We chose to activate 10 turns at one time, and then move the current injector to a successive 10 turns until information about all 1000 turns of the solenoid was obtained. Coils B, B', and C form a radius-to-voltage transducer which measures the variations in the radius of the injected current. The voltage induced in coil C is inversely proportional to the radius of the activated turns of the solenoid. To detect small changes we first "buck out" most of the voltage in coil C by using the two coils B and B'. At the same time that we are bucking out the voltage in coil C, we are also increasing the sensitivity of this three-coil system to changes in the radius R of the solenoid, because the voltage in coils B and B' increases when the radius R increases. This three-coil radius-to-voltage transducer is then calibrated by having a few turns of wire on both ends of the solenoid that are 25 μm larger at one end and 25 μm smaller at the other end. With this system, the axial position and the radius variations of the turns of the solenoid were measured to an accuracy of 0.05 μm. No turn deviated from that expected of a perfect solenoid by more than 2 μm.

Our solenoid is 1 m long and 0.280 m in diameter. The diameter was compared to a quartz end standard that was measured with a laser interferometer. A 0.8-μm (2.9 ppm) uncertainty in the diameter produces a 0.25-ppm error in γ_p'. With the high accuracy of the radius variation and pitch measurements, the diameter measurement became the limiting part of the γ_p' experiment. The basic problem in determining the diameter is that even if we make a perfect measurement of the position of the surface of the copper wire, it is still difficult to know where the current flows in the wire. Because of our concern over a possible systematic error in this diameter measurement, a second approach was developed. The technique is to find a coil geometry which produces a magnetic field that is nearly independent of the average diameter. While a longer solenoid is a solution, we were able to find a suitable configuration for the existing solenoid in which current could be taken out of selected turns. Using a computer we searched for a configuration where the sum of the diameter-variation weighting function is small and the second-, fourth-, and sixth-order gradients are compensated. The final configuration uses five 1-A current sources, one of which is used to pass current through the entire solenoid in the normal fashion. The other four are connected to selected turns on the opposite side from the main return lead so that the net current in these selected turns is zero. This five-current system produces a 0.8-mT field (uniform to 0.1 ppm over the H_2O sample), instead of the 1.2-mT field of the conventional solenoid, but this 0.8-mT field is 8.6 times less sensitive to the average diameter of the solenoid.

The 1-A current placed in the solenoid is measured by comparing the voltage across a 1-Ω precision resistor in series with the solenoid with a Weston cell calibrated in terms of the United States legal volt, which in turn is maintained via the Josephson effect. A cable is used to transfer this current to the NBS nonmagnetic facility,[8] thus providing us with the closest possible tie to the Josephson voltage standard. The observed scatter in the NMR measurement reflects the precision in making the voltage comparison. At the nonmagnetic facility a rubidium magnetometer system is used to reduce Earth's magnetic field fluctuations.[9]

The final uncertainty of this experiment is limited by a number of sources of systematic error (see Table II). It is not entirely evident from the table but the statistical standard deviation of the mean, σ_m, of the three major quantities measured was small; the NMR frequency measurements, and the pitch and radius variation measurements of the critical central wires, had a combined σ_m of only 0.024 ppm. Such precision was very helpful when potential sources of systematic error were investigated. The final pitch and radius variation uncertainties given in the table are closer to the statistical standard deviation of the measurements rather than to that of their mean because we feel that such an uncertainty is more representative of the true accuracy of the measurements. The position of the auxiliary currents is the largest source of error in the five-current configuration. The magnitude of the largest correction applied to the dimension-

TABLE II. Condensed summary of uncertainties in γ_p', in ppm [1-standard-deviation (68%-confidence-level) estimates].

Quantity	Normal solenoid	Five-current solenoid
NMR frequency measurements	0.021	0.015
NMR systematics [a]	0.068	0.087
Solenoid dimensional measurements:		
Pitch variation	0.044	0.147
Radius variations	0.037	0.054
Diameter	0.250	0.029
Systematics [b]	0.071	0.090
Return leads	0.020	0.041
Laser calibration	0.040	0.040
Resistor calibration [c]	0.045	0.082
Susceptibility corrections	0.037	0.030
RSS total	0.29 ppm	0.23 ppm
Least-squares fit (see text)	0.21 ppm	

[a] Root sum of squares (RSS) of a number of independent sources of uncertainty, e.g., line shape (0.06 ppm), sample shape (0.006), and current leakage (0.02).

[b] RSS includes, e.g., Pyrex track straightness (0.04), capacitive effects (0.02), and current injector straightness (0.03).

[c] Includes power coefficient uncertainty, but time dependence of NBS ohm is contained in Ω_{NBS}/Ω; see Table I.

al measurements of the solenoid was about 0.2 ppm. The computed contribution for an ideal return lead and for the helical nature of the solenoid windings[10] was −2.06 ppm for the five-current configuration, while the correction for the measured return-lead misalignment was only 0.24 ppm. The correction for the power coefficient of the resistor was −0.32 ppm. The largest susceptibility correction was −0.13 ppm.

For the conventional solenoid (1.2-mT field), we obtained a value for γ_p' of 2.675 132 73(76) (0.29 ppm) and for five-current solenoid, 2.675 132 09(61) (0.23 ppm). This good agreement between the two current configurations indicates no systematic error in the mechanical diameter measurement. Since the two γ_p' measurements could also be combined to yield a value independent of the one mechanical diameter measurement, it is proper to include the two γ_p' measurements and the mechanical measurement in a least-squares fit which gives the "best value" for γ_p' and takes account of the correlation between uncertainties.[11] This is what was done to arrive at the final result given in Eq. (2).

Three major changes incorporated into the experiment since our preliminary 1975 result was reported[8] have led to the increased accuracy of our final result. First, a new current-distribution geometry was developed in which the same solenoid is used, but selected turns have a negative current added which is so chosen that the resulting magnetic field is nearly independent of the average diameter of the solenoid. Second, the path of the laser interferometer which measures the relative position of each turn has been evacuated, so that the correction for the index of refraction of air is eliminated. Third, the solenoid was directly cooled with an inert fluorocarbon that has a very low electrical conductivity. This liquid eliminates the temperature gradients along the solenoid. These three improvements, along with a more complete evaluation of the systematics, have reduced the uncertainty of γ_p' to 0.21 ppm, which in turn provides a 0.11-ppm value for the fine-structure constant, Eq. (3). In a new γ_p' experiment now underway, we plan to reduce the present known sources of uncertainty to the point where yet another order-of-magnitude improvement in γ_p' and α should result.[12]

The assistance of H. Nakamura, W. D. Phillips, W. S. Trimmer, K. Weyand, and E. J. Wicklund is greatly appreciated. Ideas and suggestions by E. R. Cohen, R. D. Cutkosky, R. L. Driscoll, J. E. Faller, and B. N. Taylor were essential for the successful completion of this work. Close cooperation by those providing the special calibration needs is one reward of working in a national standards laboratory. T. E. Wells provided resistor calibrations. R. F. Dziuba, J. Toots, and B. F. Field provided voltage calibrations against

$2e/h$. H. P. Layer calibrated the laser. G. A. Candella measured the magnetic susceptibilities.

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Aharonov-Bohm Effect from the "Hydrodynamical" Viewpoint

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(Received 26 February 1979)

It is shown that the adoption of the hydrodynamical viewpoint in quantum mechanics provides a way to explain experimental results while giving electromagnetic potentials no more physical significance than they have in classical physics.

In the "hydrodynamical" formulation of quantum mechanics,[1] the Schrödinger equation is replaced by a set of nonlinear equations for the density of probability ρ and the density current $\vec{J}$. This formulation entered the controversial history of the Aharonov-Bohm effect[2] because fields, and not potentials, appear in the hydrodynamical equations so that, in the time evolution of ρ and $\vec{J}$ described by these equations, there is no room for potential effects, in striking contrast with the theory of Aharonov-Bohm based upon the Schrödinger equation.[3] In the theoretical prediction of the Aharonov-Bohm effect one deals with electrons moving in a multiply connected configuration space. The purpose of this paper is to point out that the relationship between the Schrödinger equation and hydrodynamical equations undergoes a significant modification when one considers multiply connected domains and that no paradox occurs once the correct relationship between the two formulations is taken into account. Let us briefly review some facts about quantum mechanics in multiply connected regions such as the exterior of an impenetrable solenoid. In this region there is a class of transformations of the electromagnetic potentials which are permissible, in the sense that they leave unchanged both the behavior at infinity and the fields in the allowed domain, and which are not eliminated by the usual gauge $\operatorname{div}\vec{A}=0$. Let one such transform carry $\vec{A}$ into $\vec{A}'$: The Schrödinger equation is not invariant under this transform unless $(e/c)\oint_\gamma(\vec{A}-\vec{A}')\cdot d\vec{s}=nh$, the contour embracing once the forbidden region; in this case to $\vec{A}$ and $\vec{A}'$ correspond unitarily equivalent Hamiltonians. Thus, for a given field, one obtains as many nonequivalent Hamiltonians. Thus, for a given field, one obtains as many nonequivalent Hamiltonians as

