Tensor Force and Inelastic Electron and Proton Scattering to Unnatural-Parity States of Stretched Configurations

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A systematic comparison of the (e,e') and (p,p') transiton strengths for the excitation of unnatural-parity states of "stretched" configurations gives definite information on the high-momentum components of the tensor part of the nucleon-nucleon force. The available data emphasize the isovector part of the tensor force.

In this Letter we report on the first systematic comparison of (e,e') and (p,p') transition strengths for the excitation of high-spin states of unnatural parity. The analysis is restricted to "stretched" configurations, i.e., $(j_a j_b^{-1})j_{\max}$, where $j_a = l_a + \frac{1}{2}$, $j_b = l_b + \frac{1}{2}$, and $j_{\max} = j_a + j_b$, because the number of terms contributing to the amplitude for both inelastic electron and inelastic proton scattering[1] is severely reduced. This means that the comparison is less model dependent and interpretation of results more transparent. Furthermore, detailed structure calculations are not necessary, since only one particle-hole configuration enters, i.e., the stretched one whose contribution is determined by simply normalizing to the (e,e') data. After the structure factor is determined, subsequent comparison to the (p,p') data leads in the specific cases reported herein to definite information on the high-momentum components of the isovector tensor part of the nucleon-nucleon force.

Table I lists a number of nuclei each having an identified excited state of "stretched" character for which inelastic-electron-scattering data exist. In each case a major component of the excited-state wave function will be the particle-hole configuration given in parentheses. These configurations are supported by a comparison of the excitation energies of these levels (column 2) and the pure single-particle, single-hole energy differences (column 3). The stretched particle-hole configurations are unique within a basis which excludes single particle-hole excitations with energies greater than or equal to $3\hbar\omega$. To the extent that the latter configurations can be ignored, the transition amplitudes from the ground state will be proportional to a single spectroscopic amplitude and a single matrix element.

For a magnetic transition the transverse electron-scattering differential cross section can be written in plane-wave Born approximation as[16,17]

$$\sigma(\theta) = \frac{4\pi\sigma_M(\theta)}{\eta} f(\theta) \left| \frac{g\hbar}{2Mc} \sum_{\lambda,t} \{ C_\lambda{}^l g_l{}^t \rho_{J\lambda}{}^{lt}(q) + C_\lambda{}^s g_s{}^t \rho_{J\lambda}{}^{st}(q) \} \right|^2, \tag{1}$$

where λ is restricted to the value $J-1$ and $J+1$, t differentiates between proton and neutron quantities, the quantity within the absolute value sign is the transverse form factor $F_T(q)$, $f(\theta) = \frac{1}{2} + \tan^2\frac{1}{2}\theta$, and

$$C_{J-1}{}^l = 2(J+1)^{-1/2}, \quad C_{J+1}{}^l = -2J^{-1/2}, \quad C_{J-1}{}^s = \tfrac{1}{2}(J+1)^{1/2}, \quad C_{J+1}{}^s = \tfrac{1}{2}J^{1/2}.$$

$\sigma_M(\theta)$ is the Mott cross section, q is the momentum transter, η is a recoil factor, $g_l{}^t$ and $g_s{}^t$ are the orbital and spin g factors, and $\rho_{J\lambda}{}^{lt}(q)$ and $\rho_{J\lambda}{}^{st}(q)$ are the usual Bessel transforms of the orbital current and spin transition densities defined by

$$\rho_{J\lambda}^{kt}(q)=\sum_{a,b} S_{ab}^{t}\hat{j}_a J^{-1}\langle j_a\| i^{\lambda}\bar{Y}_{J\lambda 1}\bar{O}_k\| j_b\rangle\langle n_a l_a| j_{\lambda}(qr)|n_b l_b\rangle . \tag{2}$$

Here S_{ab}^{t} is $\hat{J}\hat{j}_a^{-1}$ times the spectroscopic amplitude defined by Petrovich *et al.*,[18] $\bar{O}_k$ is $\bar{L}$ or $\bar{\sigma}$ according as k is l or s, and $\hat{j}=(2j+1)^{1/2}$. If the excited state is stretched, angular momentum restrictions require that all transition densities vanish[1] except the spin transition density with $\lambda=J-1$. On evaluation of this reduced matrix element, for the important special case of an isovector stretched excitation, the cross section reduces to

$$\sigma(\theta)=\frac{\sigma_M(\theta)}{\eta}f(\theta)\left|\left(\frac{g\hbar}{2Mc}\right)\left(\frac{J+1}{J}\right)^{1/2}\mu S_{ab}\hat{j}_a\hat{j}_b(j_a j_b\tfrac{1}{2}-\tfrac{1}{2}|J0)\langle n_a l_a|j_{J-1}(qr)|n_b l_b\rangle\right|^2, \tag{3}$$

where $\mu=\mu_p-\mu_n$ is the isovector magnetic moment and $S_{ab}=2^{-1/2}$ for a pure particle-hole excitation. Comparison of $\sigma(\theta)$ with the electron scattering data is a direct measure of the actual value of S_{ab}^2, since all other quantities are known.

The electron scattering results are summarized in column 4 of Table I. Most of the (e,e') data presented here have become available with the advent of the Bates accelerator. The magnetic nature and high excitation energies of these transitions requires large-angle (160°–180°) scattering at large momentum transfers and good energy resolution for reliable determination. The value of $\sigma_{\rm th}(\theta)$ used in forming these ratios was calculated assuming the particle-hole configurations given in Table I using oscillator radial wave functions with the oscillator parameter determined by fitting the measured magnetic form factor. The theoretical result for the 6^-, $T=1$ excitation in ^{24}Mg is compared with the data in Fig. 1(a).

One observation about the electron scattering results which can be made immediately is that the ratio of experimental to theoretical cross section is always less than unity. The implied spectroscopic amplitude, S_{ab}, ranges from 0.5 to 0.70 times the pure particle-hole value. It fol-

TABLE I. A tabulation of excitation energies and (e,e') and (p,p') cross sections for known unnatural-parity states of expected stretched configurations. The experimental uncertainties on $\sigma_{\rm exp}$ are typically ±15% except for ^{28}Si(e,e'), where it is larger.

Nucleus	$E_{\rm exp}$ (MeV)	$E_{\rm th}$ (MeV)	$\sigma_{\rm exp}/\sigma_{\rm th}$ (e,e')	(p,p')
^{12}C$(d_{5/2}p_{3/2}^{-1})4^-$	19.5	17.6[b]	0.27[f]	
^{16}O$(d_{5/2}p_{3/2}^{-1})4^-$	18.9	17.7[b]	0.45[g]	0.32[m]
^{24}Mg$(f_{7/2}d_{5/2}^{-5})6^-$	15.14	17.6[c]	0.27[h]	0.30[n]
^{28}Si$(f_{7/2}d_{5/2}^{-1})6^-$	14.36	16.9[c]	0.59[i]	0.29[n]
^{58}Ni$(g_{9/2}f_{7/2}^{-1})8^-$	10.30[a]	10.4[d]	0.30[j]	
^{208}Pb$(\pi i_{13/2}h_{11/2}^{-1})12^-$	7.06	7.18[e]	0.56[k]	0.20[o]
^{208}Pb$(\nu j_{15/2}i_{13/2}^{-1})12^-$	6.42	6.49[e]	0.54[k]	0.80[o]
^{208}Pb$(\nu i_{15/2}i_{13/2}^{-1})14^-$	6.75	6.49[e]	0.56[l]	0.46[o]

[a]Weighted average of several levels.
[b]Ref. 2.
[c]Ref. 3.
[d]Ref. 4.
[e]Ref. 5.
[f]Ref. 6.
[g]Ref. 7.
[h]Ref. 8.
[i]Ref. 9.
[j]Ref. 10.
[k]Ref. 11.
[l]Ref. 12.
[m]Ref. 13.
[n]Ref. 14.
[o]Ref. 15.

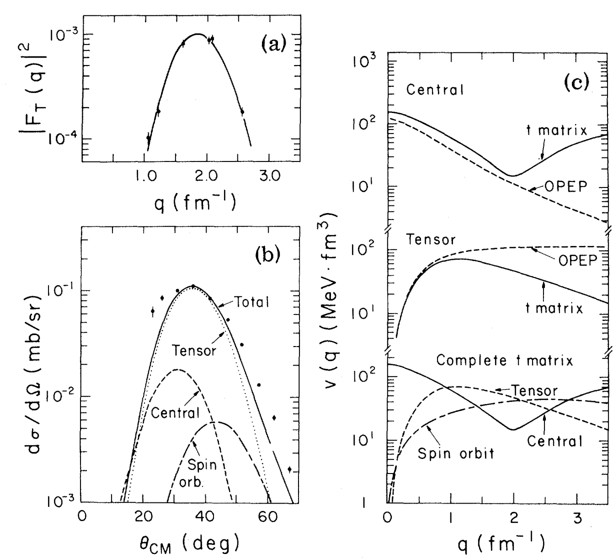

FIG. 1. (a) Inelastic electron form factor for the 6^- (15.14 MeV) state in ^{24}Mg. (b) Inelastic proton cross section for the 6^- (15.14 MeV) state in ^{24}Mg and contributions from the central, tensor, and spin-orbit components. (c) Isovector components of the central, tensor, and spin-orbit parts of nucleon-nucleon t matrix in momentum space. The central and tensor parts of the OPEP are also shown for comparison.

lows that these unnatural-parity states are not of deformed type with a large collective transition strength to the ground state, nor are the transitions of pure particle-hole character. The fact that S_{ab}^2 is less than the pure particle-hole value is attributed to more complicated configurations, which involve angular momentum couplings that cannot be connected by a one-body operator. For many of the nuclei in Table I a simple shell-model description of the nucleus is known to be inadequate.

By use of the momentum space techniques of Ref. 19, the distorted-wave approximation expression for the cross section for an unnatural-parity transition in the (p,p') reaction can be written in a form which makes the relationship with the (e,e') reaction particularly transparent. The expression is

$$\sigma(\theta)=\left(\frac{m}{2\pi\hbar^2}\right)^2\frac{k_f}{k_i}(2J+1)\Big[\sum_{\lambda,M}\Big|\sum_t\frac{2}{\pi}\int k^2\,dk\{D_{\lambda M}(k,\theta)[v_t^C(k)\rho_{J\lambda}^{st}(k)+\sum_{\lambda'}Z_{\lambda\lambda'}{}^J v_t^T(k)\rho_{J\lambda'}{}^{st}(k)+A_\lambda{}^J v_t{}^{LS}(k)\rho_J{}^{lt}(k)]\}\Big|^2+\sum_M\Big|\sum_{\lambda',t}\frac{2}{\pi}\int k^2\,dk[D_{JM}'(k,\theta)A_{\lambda'}{}^J v_t{}^{LS}(k)\rho_{J\lambda'}{}^{st}(k)]\Big|^2\Big]. \qquad (4)$$

Here $v^C(k)$, $v^T(k)$, and $v^{LS}(k)$ are the Bessel transforms of the spin-dependent central, tensor, and spin-orbit interaction components, $A_\lambda{}^J$ and $Z_{\lambda\lambda'}{}^J$, are statistical coefficients defined elsewhere,[20] $D_{\lambda M}(k,\theta)$ and $D_{JM}'(k,\theta)$ are distortion functions[19] (the prime signifies that the spin-orbit interaction differentiates the distorted waves), $\rho_{J\lambda}{}^{st}(k)$ is the spin transition density, and $\rho_J{}^{lt}(k)$ is a linear combination of the orbital current densities. The factors outside the sum on λ are the usual kinematical and statistical factors and λ and λ' can take the values $J-1$ and $J+1$ as in Eq. (1). Equation (4) is somewhat schematic in that it assumes the local form of the distorted-wave approximation, which is only valid if the effects of knockout exchange can be approximately included in the interaction. This is only well established for the central and spin-orbit interaction components.[21] This limitation does not crucially affect the present discussion.

For transitions to stretched excited states the only terms which contribute in Eq. (4) are those containing $\rho_{JJ-1}{}^{st}(k)$. This leaves a term with $\lambda=J-1$ to which the central, tensor, and spin-orbit interaction components all contribute and a term with $\lambda=J+1$ to which only the tensor component contributes. The isovector components of the t-matrix interaction of Love *et al.* and Bertsch *et al.*[22] are shown in Fig. 1(c). This is a representation of the nucleon-nucleon force appropriate for use at proton bombarding energies near 135 MeV which is the energy corresponding to available (p,p') data for transitions to stretched states. From Fig. 1(b) it is clear that the central force is weak for the important momentum region $k\approx 2\ \mathrm{fm}^{-1}$. Although the spin-orbit interaction is not weak its effect is small because $D_{JM}'(k,\theta) < D_{\lambda M}(k,\theta)$. This leaves mainly the tensor interaction; and, making use of the fact that $Z_{J+1,J-1}{}^J > Z_{J-1,J-1}{}^J$ we obtain the following approximate result for the special case of isovector stretched excitations:

$$\sigma(\theta)\simeq\left(\frac{m}{2\pi\hbar^2}\right)^2\frac{k_f}{k_i}(2J+1)\sum_M\Big|2\pi^{-1}\int k^2\,dk\,D_{J+1M}(k,\theta)3\frac{[J(J+1)]^{1/2}}{2J+1}v_1{}^T(k)S_{ab}\hat{j}_a\hat{j}_b(j_a j_b\tfrac{1}{2}-\tfrac{1}{2}\,|\,J0)\times\langle n_a l_a|j_{J-1}(kr)|n_b l_b\rangle\Big|^2, \qquad (5)$$

where $v_1{}^T(k)$ is the isovector tensor interaction. This result is to be compared directly with Eq. (3).

The available inelastic proton-scattering results are summarized in column 5 of Table I. The $\sigma_{\mathrm{th}}(\theta)$ used in forming these ratios were calculated using the same parameters as in the (e,e') calculations. The (p,p') cross sections were calculated with code[23] DWBA 70 which allows inclusion of knockout exchange terms [in contrast to the schematic expression given in Eq. (4)]. The complete t matrix of Ref. 22 was used in the calculations and the optical-potential parameters needed to generate the distorted waves were taken from an elastic proton-scattering study at $E_p\sim 135$ MeV.[24] A typical result is that for the 6^-, $T=1$ excitation in ^{24}Mg shown in Fig. 1(b).

It is expected that any inadequacy in nuclear structure is accounted for by normalizing the proton ratio $[\sigma_{\exp}(\theta)/\sigma_{\mathrm{th}}(\theta)]$ to the electron ratio. Any discrepancy beyond experimental error between the electron and proton ratio is presumably not due to structure, but is attributable to some shortcoming in the reaction theory, most likely the representation of the two-nucleon interaction.

For ^{16}O, ^{24}Mg, and ^{28}Si, the three self-conjugate nuclei where the comparison is possible, there is good agreement for ^{24}Mg and reasonable agreement for ^{16}O. There is reason to believe that the disparity with the proton results for ^{28}Si is due to overestimation of the (e, e') experimental cross section[9] because of poor energy resolution, which is in the process of being remeasured.[25]

There is also reasonable agreement between the electron and proton results for the 14^- in ^{208}Pb. The 12^- excitations in ^{208}Pb are not of the unique stretched type we have been discussing. They have been included in Table I because their nearly separate proton and neutron character allows the possibility of gaining information about the isospin mixture of the nucleon-nucleon force once the extent of configuration mixing has been determined. This problem is being considered elsewhere.[5,15] Scattering from ^{58}Ni provides another opportunity for gaining information on the isospin mixture of the nucleon-nucleon force since several 8^- states, including both T_0 and $T_0 + 1$ states, are observed in electron scattering.

In summary we find the overall comparison between the inelastic electron and proton results encouraging. The results imply that it may be possible to use the free nucleon-nucleon t matrix to describe the (p,p') reaction at 135 MeV. More specifically they suggest that the isovector part of the tensor force has the right strength and momentum dependence near $k = 2\ \mathrm{fm}^{-1}$. Its departure from one-pion exchange potential is consistent with the viewpoint that short-range contributions associated with heavier meson exchanges are important.

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