Quantum-Chromodynamics Perturbation Expansions in a Coupling Constant Renormalized by Momentum-Space Subtraction

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We relate two popular methods of renormalization; minimal subtraction and momentum-space subtraction. It is shown that $\Lambda_{\rm mom}/\Lambda_{\rm min}=5.73$, where Λ is the mass used to parametrize the quantum-chromodynamic running coupling constant. Perturbation expansions are compared in the two methods. Our results support the conjecture that momentum-space subtraction leads to better convergence and that, therefore, $\Lambda_{\rm mom}$ is the parameter which should be measured by experiments.

Until very recently, theoretical calculations of deep-inelastic scaling violations have been done only to leading order[1] in the quantum-chromodynamic (QCD) coupling constant, g. During the past year some of these results have been extended to the next order[2-4] where, for the first time, we are confronted with problems of convention dependence.[2,3] In particular, different definitions of g related to one another by the transformation

$$g=g'[1+ag'^2+O(g'^4)] \qquad (1)$$

lead to different perturbation expansions. To be specific, consider as an example the (nonsinglet) structure-function moments, M_n, which are of the form

$$M_n=g^{a_n}[1+b_ng^2+O(g^4)]A_n. \qquad (2)$$

Under the transformation of g given by Eq. (1) we find

$$M_n=g'^{a_n}[1+(a_na+b_n)g'^2+O(g'^4)]A_n. \qquad (3)$$

Thus we see that the coefficients of the nonleading orders depend on the definition of g.

In what follows we examine the relationship between g as first defined by minimal subtraction[5] and, secondly, by momentum-space subtraction. The former, $g_{\rm min}$, is technically convenient in QCD perturbation theory but has the drawback that because it is not a "physical" subtraction scheme, expansions in $g_{\rm min}$ might be expected to converge poorly. If so, low-order predictions will be inaccurate if $g_{\rm min}$ is not small. Momentum-space subtraction, on the other hand, is not regularization-scheme dependent and, since some radiative corrections are incorporated into the definition of $g_{\rm mom}$, reasonable convergence can be expected from expansion in that coupling.[6] We will outline a one-loop calculation we have done[7] in which we derive $g_{\rm mom}$ in terms of $g_{\rm min}$. We then use the moment calculations of Bardeen and co-workers[3,4] to show [using Eqs. (1)–(3)] that $g_{\rm mom}$ indeed leads to better convergence than $g_{\rm min}$. In actual fact, QCD predictions are usually written as expansions in $1/\ln(Q^2/\Lambda^2)$, where Λ is a mass used to parametrize the running coupling constant. Λ, like g, depends on the renormalization scheme and we will argue that a reliable expansion can be made in $1/\ln(Q^2/\Lambda_{\rm mom}{}^2)$.

In order to define the couplings we must first write down the QCD Lagrangian. Written in terms of renormalized quantities, it is[8,9]

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{4}Z_3(\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu)^2-\tfrac{1}{2}Z_1g(\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu)\cdot(\vec{A}^\mu\times\vec{A}^\nu)\\&-\tfrac{1}{4}Z_4g^2(\vec{A}_\mu\times\vec{A}_\nu)^2-(2\alpha)^{-1}(\partial_\mu\vec{A}^\mu)^2-\tilde{Z}_3(\partial_\mu\vec{\eta}^{\,+})\cdot D^\mu\vec{\eta}+iZ_2\bar{\Psi}\not{D}\Psi\end{aligned} \qquad (4)$$

$\vec{A}_\mu$ is an SU(3) color field; $\vec{\eta}$ are ghost fields and ψ represents n_f flavors of quarks. D_μ and $\not{D}$ are the appropriate gauge-covariant derivatives and α is the renormalized gauge parameter. g is related to the bare coupling constant, g_B, by $g=(Z_1^{-1}Z_3^{3/2})g_B$ and different ways of defining the Z's are what lead to the different definitions of g. We dimensionally regularize, following the rules of 't Hooft and Veltman.[10] Then the gluon propagator $\pi_{ab}{}^{\mu\nu}(p)$, computed through order g^2 in $4+\epsilon$ dimensions, is

$$\Pi_{ab}{}^{\mu\nu}(p)=-i\delta_{ab}\left\{\left[\left(g_{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)\frac{1}{p^2}\right]h(p^2)+\alpha\frac{p^\mu p^\nu}{p^2}+O(\epsilon)\right\}\mu^\epsilon, \qquad (5)$$

where

$$h(p^2)=1+\frac{3g^2}{16\pi^2}\left\{-\frac{13}{6}\left[\frac{2}{\epsilon}+\gamma_E-\ln(4\pi)+\ln\left(\frac{-p^2}{\mu^2}\right)\right]+\frac{97}{36}+\alpha\left(\left[\frac{2}{\epsilon}+\gamma_E-\ln(4\pi)+\ln\left(\frac{-p^2}{\mu^2}\right)(2+\tfrac{1}{2})^{-1}\right]+\alpha^2(\tfrac{1}{4}\right)\right\}$$
$$+\frac{g^2}{16\pi^2}\left(\frac{n_F}{2}\right)\left\{\frac{4}{3}\left[\frac{2}{\epsilon}+\gamma_E-\ln(4\pi)+\ln\left(\frac{-p^2}{\mu^2}\right)\right]-\frac{20}{9}\right\}-(Z_3\mu^{-\epsilon}-1); \qquad (6)$$

μ is a mass parameter chosen arbitrarily,[5] and γ_E is Euler's constant (=0.5772...). In Eq. (6), $Z_3\mu^{-\epsilon}-1$ is the counterterm expanded through $O(g^2)$. $Z_3^{\min}$ is defined to cancel *only* the pole part of π. As we see, it depends implicitly on μ. $Z_3^{\text{mom}}(\mu)$ is, on the other hand, defined to cancel the entire g^2 correction to $\pi_{ab}{}^{\mu\nu}(p^2=-\mu^2)$. We define the Z_1's in a similar fashion. We have computed[7] the three-gluon vertex at the symmetric point[9] ($p^2=q^2=r^2=-\mu^2$); the coefficient of the bare vertex to order g^2 is

$$G_0=1+(Z_1\mu^{-\epsilon}-1)+\frac{g^2}{4\pi}\left\{\frac{0.68}{\epsilon}-0.57+\alpha\left(\frac{-0.36}{\epsilon}+0.05\right)+\alpha^2(-0.10)+\alpha^3(0.01)-n_f\left(\frac{0.11}{\epsilon}-0.10\right)+O(\epsilon)\right\}.$$

Z_1^{mom} is defined to cancel the radiative corrections and $Z_1^{\min}$ is defined to simply cancel the pole term. The $\ln(4\pi)$ and γ_E which appear in $\pi_{\mu\nu}$ and (implicitly) in G_0 are artifacts of dimensional regularization and, in the momentum-space subtraction scheme, they are canceled by the Z's.[11]

Having defined the Z's we can compute $A(\alpha, n_f)$ in the equation

$$g_{\text{mom}}^2(\mu)=g_{\min}^2(\mu)\{1+A(\alpha,n_f)[g_{\min}^2(\mu)/4\pi]+\dots\}.$$

Some examples (we will do all our computations for four flavors, the number appropriate to the momentum range $10\leq Q^2\leq 100$) are $A(0,4)=2.32$, $A(1,4)=2.07$, and $A(3,4)=1.90$. We see that the gauge dependence is weak. Furthermore, one might argue that from the point of view of optimizing the convergence of expansions, Landau gauge ($\alpha=0$) is to be preferred over other covariant gauges. For $\alpha=0$, the longitudinal piece of the propagator is absent through all orders so that fewer degrees of freedom are present in (off-shell) Feynman diagrams. We will do the rest of our analysis in Landau gauge.

The running coupling constant[12] $g(\mu)$ is scale dependent and obeys the renormalization-group equation[5,13]

$$\mu\, dg(\mu)/d\mu=-\beta_0 g^3-\beta_1 g^5+O(g^7)$$

(β_0 and β_1 are independent of renormalization prescription).[13] This allows a convenient expansion of $g^2(\mu)$ in terms of a mass Λ:

$$\frac{g_R^2(\mu)}{4\pi}=\frac{g_{0,R}^2(\mu)}{4\pi}-\frac{4\pi\beta_1}{\beta_0}\left(\frac{g_{0,R}^2(\mu)}{4\pi}\right)^2\ln\ln(\mu^2/\Lambda_R^2)+O(g_{0,R}^6(\mu)), \qquad (7)$$

where $g_{0,R}^2(\mu)=1/\beta_0\ln(\mu^2/\Lambda_R^2)$ and the subscript "R" refers to the renormalization method—for instance "mom" or "min." The coefficient of the second term is $-(4\pi\beta_1/\beta_0)\ln\ln(\mu^2/\Lambda_R^2)$ which, for the momentum range of interest is ~ -0.7. Because that coefficient is fairly small [a third of $A(0,4)$] we expect that $g_{0,\text{mom}}$ will be about as good an expansion parameter as g_{mom}.[14] Expansions in $g_{0,R}$ of physically measurable quantities have the property that their renormalization-prescription dependence is generated through all orders by rescalings of Λ_R which are exactly deducible from one-loop calculations.[15] Using Eq. (7) and the value of $A(0,4)$, we find that $\Lambda_{\text{mom}}/\Lambda_{\min}=5.73$.

We now compare the moment expansions of Bardeen and co-workers[3,4] using the different coupling constants. For the nonsinglet structure function moments, M_n, we have[14]

$$M_n(Q^2)=A_n[g_{\min}^2(Q)]^{a_n}\left[1+b_{\min}^n\left(\frac{g_{\min}^2(Q)}{4\pi}\right)+O(g_{\min}^4)\right]$$
$$=A_n[g_{\text{mom}}^2(Q)]^{a_n}\left[1+b_{\text{mom}}^n\left(\frac{g_{\text{mom}}^2(Q)}{4\pi}\right)+O(g_{\text{mom}}^4)\right], \qquad (8)$$

where $b_{\text{mom}}^n=b_{\min}^n-a_nA(0,4)$. a_n and $b_{\min}^n$ are computed in Refs. 1–3 and in Table I are compare $b_{\min}^n$ and b_{mom}^n for low moments ($n\leq 8$) (higher moments are unreliable because of the effects of high twist operators[2]). For these we see that b_{mom}^n are much smaller than $b_{\min}^n$, which (when $g^2/4\pi\sim 0.3$)

TABLE I. Coefficients of next-to-leading order corrections to moments of nonsinglet structure functions (e.g., $\nu W_2^{ep} - \nu W_2^{en}$) (Ref. 3). The coefficients are defined in the text [Eqs. (8) and (9)].

n	$b_{\rm min}^n$	$b_{\rm mom}^n$	$c_{\rm min}^n$	$c_{\rm mom}^n$
2	0.72	−0.27	0.41	−0.58
3	1.27	−0.27	0.78	−0.76
4	1.73	−0.21	1.11	−0.82
5	2.12	−0.13	1.41	−0.84
6	2.46	−0.04	1.67	−0.84
7	2.77	0.05	1.90	−0.81
8	3.04	0.14	2.12	−0.79

TABLE II. Coefficients of next-to-leading order corrections to moments of the photon-photon structure function F_2^γ (Ref. 4; we use their results for four flavors). The coefficients are defined in the text [Eqs. (10) and (11)].

n	$B_{\rm min}^n$	$B_{\rm mom}^n$	$D_{\rm min}^n$	$D_{\rm mom}^n$
4	−2.65	−0.33	−1.45	0.31
6	−2.79	−0.48	−0.98	0.12
8	−2.91	−0.60	−0.76	0.05
10	−3.01	−0.70	−0.61	0.01
12	−3.09	−0.77	−0.52	−0.01

are alarmingly large.[16] We can also look at $M_n(Q^2)$ expanded in terms of $g_0^2(Q) = 1/4\pi\beta \ln(Q^2/\Lambda^2)$. From Eqs. (7) and (8),

$$M_n(Q^2) = A_n\left[1 + \frac{C^n(Q^2)}{4\pi\beta_0 \ln(Q^2/\Lambda^2)}\right]\left[\ln\left(\frac{Q^2}{\Lambda^2}\right)\right]^{-a_n}, \quad (9)$$

where $C^n(Q^2) = C'^n + C''^n \ln\ln(Q^2/\Lambda^2)$. As before, C^n and Λ depend on the subtraction scheme. The values of $C_{\rm min}^n$ and $C_{\rm mom}^n$ are presented also in Table I. In calculating C^n we have taken $\ln\ln(Q^2/\Lambda^2) = 1.5$ but the slow variation of this quantity over the range of interest hardly affects our results. We see that the expansion in terms of $\Lambda_{\rm mom}$ appears, as expected, to be reasonably trustworthy wheras that in $\Lambda_{\rm min}$ does not. Finally, we analyze expansions of the moments, $F_{2,n}^\gamma$, of the photon-photon structure function,[4,14]

$$F_{2,n}^\gamma(Q^2) = \frac{\alpha_{\rm QED}^2 d_n}{g^2(Q)}\left[1 + B^n\left(\frac{g^2(Q)}{4\pi}\right) + O(g^4)\right] \quad (10)$$

$$= \alpha_{\rm QED}^2\left[d_n' \ln\frac{Q^2}{\Lambda^2} + D^n + O\left(\frac{1}{\ln(Q^2/\Lambda^2)}\right)\right]. \quad (11)$$

Again, B^n and D^n depend on the renormalization scheme and, using the results of Ref. 4 for $B_{\rm min}^n$ and $D_{\rm min}^n$, we compare in Table II $B_{\rm min,mom}^n$ and $D_{\rm min,mom}^n$. As before, the coefficients are smallest for $g_{\rm mom}$ and $\Lambda_{\rm mom}$.

We have now demonstrated a fair amount of evidence in favor of using momentum-space subtraction which, *a priori*, we expect to give good convergence. The important point to stress is that this *a priori* expectation implies that higher *uncalculated* orders of QCD expansions will be relatively small. Therefore, predictions at the present order may be expected to be most trustworthy when given in terms of $g_{\rm mom}$ (or $\Lambda_{\rm mom}$). This, then, is the parameter which should be measured in experiments.

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(1972). Following Refs. 2–4, we use a slight variation of their rules and let $(2\pi)^4 \to (2\pi)^n$ in Fourier transforms.

[11]For other momentum-space subtraction definitions see Ref. 7. Results turn out to be relatively insensitive to which of these definitions one chooses.

[12]A more commonly used notation for $g(\mu)$ is $\bar{g}(t)$, where $t = \ln(\mu/\mu_0)$ and μ_0 is held fixed.

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[14]It is tacitly assumed in these formulas that the optimal choice of the effective coupling is to define it at $\mu^2 = Q^2$. The validity of this assumption has been studied by S. Wolfram, California Institute of Technology Report No. 68-690, 1978 (to be published).

[15]Equation (7) can be solved: $\Lambda = \mu \exp\{-\int g(\mu) [dg'/\beta(g')\}$ which implies $\mu\, d\Lambda/d\mu = 0$. Therefore, $\Lambda_{R1}/\Lambda_{R2}$ is a pure number which can be evaluated in the limit $g(\mu \to \infty) = 0$. Thus, $\Lambda_{\rm mom}/\Lambda_{\rm min} = \exp[A(\alpha, n_f)/8\pi\beta_0]$.

[16]Bardeen *et al.* (Refs. 3 and 4) have noted that rescaling $\Lambda_{\rm min} \to \Lambda_{\rm min} \exp\frac{1}{2}(\ln 4\pi - \gamma_E) = 2.66\Lambda_{\rm min}$ leads to more acceptable values for these coefficients.

Search for New Particles at the Alternating-Gradient–Synchrotron Beam Dump

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This paper presents results of a beam-dump experiment performed at the Brookhaven alternating-gradient synchrotron to search for prompt sources of neutrinos and axionlike particles. We observe no excess of ν_e or $\bar{\nu}_\mu$ events, and no excess in neutral-current events over that expected from neutrinos from π and K decays. We report on limits of prompt particle-production cross sections and lifetimes.

We report on a beam-dump experiment performed at the Brookhaven National Laboratory (BNL) alternating-gradient synchrotron (AGS) neutrino beam to search for prompt sources of neutrinos and new penetrating neutral particles. Prompt sources of electron neutrinos have been observed at the CERN Super Proton Synchrotron.[1]

The detector consisted of 22 6-ft×6-ft thin-plate optical spark chambers interspersed with twenty planes of plastic scintillator (Fig. 1). The fiducial volume was five tons. The fiducial volume was followed by a muon identifier consisting of magnetized toroids and 8-ft×8-ft aluminum spark chambers.

The BNL neutrino beam was run in two configurations. In the first (bare target) configuration, protons impinged on a 6-in.×12-in.×12-in. copper target. Pions and kaons produced in the target decayed in a 200-ft drift space behind the target. Remaining charged particles were then attenuated by a 100-ft iron shield. In the second (beam dump) configuration, protons were transported to a 24-in.×12-in.×12-in. copper target immediately in front of the iron shield. The strong suppression of neutrinos from π and K decays in this configuration increases sensitivity to new sources of neutrinos or other neutral penetrating particles. The bare-target run allows a direct comparison of interactions observed in the beam-dump run with neutrino interactions from π and K decays. We report on data for 1.9×10^{17} protons on target in the bare-target configuration and 4.7×10^{18} protons on target in the beam-dump configuration. The two exposures were adjusted to yield approximately equal numbers of charged-current neutrino interactions from π and K decays. The calculated energy spectra of neutrinos was similar in the two configurations. We compare the beam-dump data to the bare-target data for anomalous ν_μ and $\bar{\nu}_\mu$ charged-current rate,