Asymptotic Normalization of the Triton D State

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The ratio $R_2(^3\mathrm{H})$ of the D- and S-state asymptotic normalizations of the neutron-deuteron tail of the triton wave function is calculated from a solution of the Faddeev equations with the Reid soft-core potential. We find $R_2(^3\mathrm{H}) = -0.24\ \mathrm{fm}^2$. Both the phase and magnitude of the calculated ratio are in agreement with the experimental value determined from the recent measurements of tensor analyzing powers for (d,t) reactions. An estimate of R_2 for ^{3}He with a Coulomb correction gives $R_2(^3\mathrm{He}) \approx -0.24\ \mathrm{fm}^2$.

In a recent paper,[1] Knutson *et al.* reported the measurements of the tensor analyzing powers for (d,t) reactions on ^{118}Sn and ^{208}Pb and showed that their results are sensitive to the D-state components of the triton wave functions. In the analysis of their data, they used the distorted-wave Born approximation (DWBA) with a parameter D_2 which determines the effects due to the D-state part of the overlap between the deuteron and triton bound-state wave functions on the three tensor analyzing powers (T_{20}, T_{21}, and T_{22}). From the best fit to the (d,t) measurements they obtained $D_2(^3\mathrm{H}\text{-}nd) = -0.24\ \mathrm{fm}^2$. Their theoretical estimate of D_2 based on the first-order perturbation theory with 8.8% D-state probability for the triton wave function gives $D_2(^3\mathrm{H}\text{-}nd) \doteq -0.20\ \mathrm{fm}^2$. Most recently, Roman *et al.*[2] obtained a $D_2(^3\mathrm{He}\text{-}pd)$ of -0.20 to $-0.30\ \mathrm{fm}^2$ from similar measurement of $(d,{}^3\mathrm{He})$ reaction on ^{27}Al. The best fit to their data gives $D_2(^3\mathrm{He}\text{-}pd) = -0.22\ \mathrm{fm}^2$.

In this Letter, we present our calculation of the triton D-state asymptotic normalization constant, C_2 (which is related to the D_2 of Knutson *et al.*[1]), using the trinucleon wave function[3] obtained from a solution of the Faddeev equations with the Reid soft-core potential.[4] To our knowledge, this is the first exact nonperturbative calculation of the normalization constant C_2 of the neutron-deuteron tail of the triton D-state wave function which involves the realistic nuclear forces and D-state components.

The ^{3}H-nd and ^{3}He-pd coupling constants or asymptotic normalizations of the ^{3}H and ^{3}He wave functions are basic parameters of the trinucleon bound-state properties and should have comparable status as the binding energies and charge radii.[5] The previous theoretical prediction with the Reid soft-core potential for the S-state ^{3}H-nd asymptotic normalization is $|C_0(^3\mathrm{H}\text{-}nd)|^2 \approx 2.8_9$,[6] which is in reasonable agreement with the values extracted from various experimental data, $C_0^2 \approx 2.2\text{–}3.4$.[5,7]

In calculating the ^{3}H-nd coupling constant C_2, we use the nonrelativistic wave functions[3] calculated by solving the Faddeev equations in momentum space using the Reid soft-core potential,[4] effective in the $^1S_0$ and $^3S_1$-$^3D_1$ partial-wave states. The completely antisymmetric trinucleon bound-state wave function Ψ is expanded in terms of the $\mathcal{L}$-$\mathcal{S}$ basis state,[8] $\varphi_\alpha(p,q)$ [i.e., $\Psi = \sum_\alpha \varphi_\alpha(p,q)$], which is defined to be an eigenstate of the operators $\vec{p}^2$, $\vec{q}^2$, $\vec{L}^2$, $\vec{\mathcal{L}}^2 = (\vec{L}+\vec{l})^2$, $\vec{S}^2 = (\vec{s}_2+\vec{s}_3)^2$, $\vec{s}_2^{\,2}$, $\vec{s}_3^{\,2}$, $\vec{s}_1^{\,2}$, $\vec{\mathcal{S}}^2 = (\vec{S}+\vec{s}_1)^2$, $\vec{\mathcal{J}}^2 = (\vec{\mathcal{L}}+\vec{\mathcal{S}})^2$, $\mathcal{J}_z$, $\vec{t}_2^{\,2}$, $\vec{t}_3^{\,2}$, $\vec{T}^2 = (\vec{t}_2+\vec{t}_3)^2$, $\vec{t}_1^{\,2}$, $\vec{\mathcal{T}}^2 = (\vec{T}+\vec{t}_1)^2$, and $\mathcal{T}_z$. $\vec{L}$ is the relative orbital angular momentum of the (2,3) pair; $\vec{l}$ is the orbital angular momentum of nucleon 1 in the c.m. system; $\vec{s}_i$ and $\vec{t}_i$ are the spin and isospin of nucleon i. For ^{3}H and ^{3}He, $\mathcal{J} = \frac{1}{2}$, $\mathcal{T}_z = -\frac{1}{2}$ (for ^{3}H) and $\mathcal{T}_z = +\frac{1}{2}$ (for ^{3}He). The momenta $\vec{p}$ and $\vec{q}$ are defined by $\vec{p} = \frac{1}{2}(\vec{k}_2 - \vec{k}_3)$ and $\vec{q} = (\vec{k}_2 + \vec{k}_3 - 2\vec{k}_1)/2\sqrt{3}$, and are conjugate to the coordinate vectors $\vec{r} = \vec{r}_2 - \vec{r}_3$ and $\vec{\rho} = (\vec{r}_2 + \vec{r}_3 - 2\vec{r}_1)/\sqrt{3}$, respectively, in the c.m. system; these are known as the Lovelace coordinates.[9] Our ^{3}H wave function has 8.8% D-state ($\mathcal{L}=2$) probability.[3]

We define the dimensionless asymptotic normalization constants, ${}^L C_l$ for the nd tail of the c.m. ^{3}H wave function in terms of the J-j basis, which is an eigenstate of the same operators as in the $\mathcal{L}$-$\mathcal{S}$ basis except that the operators $\vec{\mathcal{L}}^2 = (\vec{L}+\vec{l})^2$ and $\vec{\mathcal{S}}^2 = (\vec{S}+\vec{s})^2$ are now replaced by $\vec{J}^2 = (\vec{L}+\vec{S})^2$ and $\vec{j}^2 = (\vec{l}+\vec{s})^2$:

$$\langle \vec{r},\vec{y}|\Psi^{\mathrm{asymp}}\rangle = -\beta^{3/2}\sum_{L,l}\sum_{\text{all } m\text{'s}} {}^L C_l \langle JM_J jm_j|\mathcal{J}\mathcal{J}_z\rangle\langle LM_L SM_S|JM_J\rangle\langle lm_l \tfrac{1}{2}m_s|jm_j\rangle \times u_L(r)Y_{LM_L}(\hat{r})|SM_S\rangle h_l^{(1)}(i\beta y)Y_{lm_l}(\hat{y})|\tfrac{1}{2}m_s\rangle, \qquad (1)$$

where we suppress the isospin part of the ^{3}H wave function. The function $h_l^{(1)}(i\beta y)$ is the spherical Hankel function of the first kind, and β is given by $\beta = [(3M/4\hbar^2)|E_t - E_d|]^{1/2}$, where $E_d = 2.225$ MeV (the deuteron binding energy) and E_t is the triton binding energy. We use the calculated value of $E_t = 6.96$ MeV[3] instead of the experimental value of 8.48 MeV. The Jacobi coordinate variables are used for $\vec{r}$ and $\vec{y}$, i.e., $\vec{r} = \vec{r}_2 - \vec{r}_3$ and $\vec{y} = \frac{1}{2}(\vec{r}_2 + \vec{r}_3) - \vec{r}_1$. The deuteron radial wave functions, $u_L(r)$, are normalized, $\int [u_0^2(r) + u_2^2(r)] r^2 dr = 1$. For the deuteron, $J = S = 1$ and $L = 0, 2$. The parity consideration restricts l to even values.

There are several methods of extracting ${}^L C_l$ from the trinucleon bound-state wave function Ψ. One method is to use the integral relations for C derived by Lehman and Gibson[10] and by Kim, Sander, and Tubis.[11] We use here a simple extrapolation method of Kim and Tubis.[6]

Since our ^{3}H momentum-space wave function Ψ is given in terms of the $\mathcal{L}$-$\mathcal{S}$ basis, Eq. (1) is transformed into the $\mathcal{L}$-$\mathcal{S}$ basis in momentum space after a change of variable $\vec{\rho} = (\frac{4}{3})^{1/2}\vec{y}$. After some algebra, we obtain the following expression:

$$\langle \vec{p}, \vec{q} | \Psi^{\text{asymp}} \rangle = \sum_{L,l} g_l(q) {}^L C_l \varphi_L(p) \sum_{\mathcal{L},\mathcal{S}} V_\alpha \sum_{\text{all } m\text{'s}} \langle L M_L l m_l | \mathcal{L} m_L \rangle \langle S M_S \tfrac{1}{2} m_s | \mathcal{S} m_\mathcal{S} \rangle \times \langle \mathcal{L} m_\mathcal{L} \mathcal{S} m_\mathcal{S} | \mathcal{J} \mathcal{J}_z \rangle Y_{L M_L}(\hat{p}) Y_{l m_l}(\hat{q}) | S m_S \rangle | \tfrac{1}{2} m_s \rangle, \tag{2}$$

where

$$g_l(q) = (3)^{1/4} (\beta/\pi)^{1/2} (q^2 + \tfrac{3}{4}\beta^2)^{-1} (2q/\sqrt{3}\beta)^l$$

and

$$V_\alpha = [(2\mathcal{L}+1)(2\mathcal{S}+1)(2J+1)(2j+1)]^{1/2} \begin{Bmatrix} L & S & J \\ l & \tfrac{1}{2} & j \\ \mathcal{L} & \mathcal{S} & \mathcal{J} \end{Bmatrix}$$

with the subscript α representing a set of quantum numbers for the $\mathcal{L}$-$\mathcal{S}$ basis state with $J = S = 1$ and $L = 0$ or 2. The momentum-space deuteron wave functions, $\varphi_L(p)$, are defined as

$$\varphi_L(p) = i^L \left(\frac{2}{\pi}\right)^{1/2} \int_0^\infty j_L(pr) u_L(r) r^2 dr.$$

The $\mathcal{L}$-$\mathcal{S}$ component (characterized by a set of quantum numbers $\{\alpha\}$) of Ψ^{asymp} can be extracted from Eq. (2) as

$$\langle p, q, \alpha | \Psi^{\text{asymp}} \rangle = \int d^3p' \int d^3q' \langle p, q, \alpha | \vec{p}', \vec{q}' \rangle \langle \vec{p}', \vec{q}' | \Psi^{\text{asymp}} \rangle = g_l(q) {}^L C_l \varphi_L(p) V_\alpha. \tag{3}$$

The comparison of Eq. (3) with the component $\langle p, q, \alpha | \Psi \rangle$ of the complete ^{3}H wave function shows that $\langle p, q, \alpha | \Psi \rangle$ has a pole at the unphysical value $q^2 = -\frac{3}{4}\beta^2$ ($= -0.1142$ fm^{-2}) with the residue proportional to

$${}^L C_l = \lim_{q^2 \to -(3/4)\beta^2} {}^L \mathcal{R}_l(p, q) \tag{4}$$

with

$${}^L \mathcal{R}_l(p, q) = \frac{\langle p, q, \alpha | \Psi \rangle (q^2 + \frac{3}{4}\beta^2)}{(3)^{1/4} (\beta/\pi)^{1/2} (2q/\sqrt{3}\beta)^l \varphi_L(p) V_\alpha}. \tag{5}$$

The complete ^{3}H wave function Ψ can be expanded in terms of the Faddeev components $\Psi^{(i)}$, i.e.,

$$\langle \vec{r}, \vec{y} | \Psi \rangle = \langle \vec{r}, \vec{y} | \Psi^{(1)} \rangle + \langle \vec{r}, \vec{y} | \Psi^{(2)} \rangle + \langle \vec{r}, \vec{y} | \Psi^{(3)} \rangle. \tag{6}$$

The second and third terms of Eq. (6) will be negligible compared to $\langle \vec{r}, \vec{y} | \Psi^{(1)} \rangle$ when y approaches ∞. Hence, we may replace $\langle p, q, \alpha | \Psi \rangle$ in Eqs. (5) by $\langle p, q, \alpha | \Psi^{(1)} \rangle$.[11]

The numerical values of ${}^0C_0$, ${}^2C_0$, ${}^0C_2$, and ${}^2C_2$ are found by fitting the corresponding ${}^L\mathcal{R}_l(p, q)$ [Eqs. (5)], for a fixed p, with a polynomial of degree N as described in Ref. 6. In Table I, we give the values of ${}^0C_2$ corresponding to fits for $N = 2$, 4, 6, 8, and 10. Our final extrapolated values are ${}^0C_0 = 1.7762 \pm 0.0025$, ${}^2C_0 = 1.7764 \pm 0.0026$, ${}^0C_2 = 0.065\,07 \pm 0.000\,17$, and ${}^2C_2 = 0.065\,10 \pm 0.000\,12$. The calculated results of both ${}^0C_0 \cong {}^2C_0 = C_0$ (^{3}H) and ${}^0C_2 \cong {}^2C_2 = C_2$ (^{3}H) are expected since we can rewrite Eq. (1) in the following form:

$$\langle \vec{r}, \vec{y} | \Psi^{\text{asymp}} \rangle = -\beta^{3/2} \sum_l \sum_{\text{all } m\text{'s}} C_l \langle J M_J j m_j | \mathcal{J} \mathcal{J}_z \rangle \langle l m_l \tfrac{1}{2} m_s | j m_j \rangle \Psi_{J M_J}{}^{\text{deut}}(\vec{r}) h_l^{(1)}(i\beta y) Y_{l m_l}(\hat{y}) | \tfrac{1}{2} m_s \rangle, \tag{7}$$

TABLE I. Values of ${}^0C_2$ corresponding to polynomial fits with ${}^0\mathcal{R}_2(p,q)$ [Eq. (5)] for $N=2$, 4, 6, 8, and 10. q_{max} is the maximum value of extrapolarization points, $q_i (i=1-N)$ used for a given N.

N / q_{max} (fm^{-1}) / p (fm^{-1})	2 / 0.115	4 / 0.0482	6 / 0.977	8 / 1.428	10 / 1.678
0.360	0.05733	0.06489	0.06494	0.06486	0.06480
2.484	0.06246	0.06548	0.06522	0.06507	0.06497
5.882	0.06154	0.06531	0.06508	0.06493	0.06483
9.477	0.06053	0.06530	0.06508	0.06493	0.06483

where Ψ^{deut} is the deuteron wave function given by

$$\Psi_{JM_J}{}^{deut}(\vec{r}) = \sum_{L=0,2} \langle LM_L SM_S | JM_J \rangle u_L(r) Y_{LM_L}(\hat{r}) | SM_S \rangle, \quad (8)$$

with $J=S=1$.

It is important to note that under the definition given by Eq. (1) the relative phase of C_0 and C_2 are unique for a given nuclear force model. In fact, the recent measurement of tensor analyzing powers for (d,t) reaction by Knutson *et al.*[1] has determined the phase as well as the magnitude of the ratio of C_2 to C_0, since their experimentally determined parameter D_2 is related to our $C_2(^3\mathrm{H})$ and $C_0(^3\mathrm{H})$ by[2]

$$D_2(^3\mathrm{H}) \approx -C_2(^3\mathrm{H})/[C_0(^3\mathrm{H})\beta^2(^3\mathrm{H})] \equiv R_2(^3\mathrm{H}). \quad (9)$$

If we use our calculated values of $C_0(^3\mathrm{H}) = 1.776$, $C_2(^3\mathrm{H}) = 0.065$, and $\beta(^3\mathrm{H}) = 0.390$ fm^{-1}, we obtain $R_2(^3\mathrm{H}) \approx -0.24$ fm^2 in agreement with the experimental value of -0.24 fm^2, Knutson *et al.*[1]

To estimate the correction for not using the experimental value of β, we replace β in the denominator of Eq. (5) by the experimental value, $\beta_{exp}(^3\mathrm{H}) = 0.449$ fm^{-1}. This replacement leads to the following results:

$$C_0(^3\mathrm{H}, \beta_{exp}) = (\beta/\beta_{exp})^{1/2} C_0(^3\mathrm{H}) = 1.657, \quad (10)$$

$$C_2(^3\mathrm{H}, \beta_{exp}) = (\beta/\beta_{exp})^{-3/2} C_2(^3\mathrm{H}) = 0.080, \quad (11)$$

and

$$R_2(^3\mathrm{H}, \beta_{exp}) = -\frac{C_2(^3\mathrm{H}, \beta_{exp})}{C_0(^3\mathrm{H}, \beta_{exp})\beta_{exp}{}^2} = -\frac{C_2(^3\mathrm{H})}{C_0(^3\mathrm{H})\beta^2}.$$

We see that R_2 is independent of this correction, i.e., $R_2(^3\mathrm{H}, \beta_{exp}) = R_2(^3\mathrm{H}) = -0.24$ fm^2.

In order to estimate the Coulomb correction for the case of ^{3}He-pd coupling constants, we replace $h_l^{(1)}(i\beta y)$ in Eq. (1) with the Whittaker function W:

$$h_l^{(1)}(i\beta y) \to w_{-\eta,l}(\beta y) = -i^l (1/\beta y) W_{-\eta, l+1/2}(2\beta y).$$

Note that, when $\eta = 0$, $w_{0,l}(\beta y) = h_l^{(1)}(i\beta y)$. For the proton-deuteron tail, $\eta = 2e^2 M/3\hbar^2 \beta_{exp}(^3\mathrm{He}) = 0.0550$ with the experimental value of $\beta_{exp}(^3\mathrm{He}) = 0.4203$ fm^{-1} and the nucleon mass M.

The above replacement leads to the following approximations:

$$C_0(^3\mathrm{He}) \approx [\beta(^3\mathrm{H})/\beta(^3\mathrm{He})]^{1/2} C_0(^3\mathrm{H})/f(0,\eta) = 1.80,$$

$$C_2(^3\mathrm{He}) \approx [\beta(^3\mathrm{H})/\beta(^3\mathrm{He})]^{-3/2} C_2(^3\mathrm{H})/f(2,\eta) = 0.060,$$

and

$$R_2(^3\mathrm{He}) \approx -F(\eta)\frac{C_2(^3\mathrm{He})}{C_0(^3\mathrm{He})\beta^2(^3\mathrm{He})} = -0.24 \text{ fm}^2,$$

where f is a Coulomb correction factor[12] given by $f(l,\eta) = l!/\Gamma(l+1+\eta)$, [$f(0,\eta) = 1.030$, and $f(2,\eta) = 0.950$], and we used the calculated values, $\beta(^3\mathrm{H}) = 0.390$ fm^{-1}, $\beta(^3\mathrm{He}) = 0.357$ fm^{-1}, and $F(0.0550) = 0.9242$ with $F(\eta) = [(1+\eta^2)(4+\eta^2)]^{1/2}/(1+\eta)(2+\eta)$. This factor $F(\eta)$ for $R_2(^3\mathrm{He})$ is obtained by Fourier transforming the expression for $D_2(^3\mathrm{H})$ given by Knutson *et al.*[1] after substituting the plane-wave state for the neutron with the Coulomb distorted wave for the proton.[2] The binding-energy correction similar to Eqs. (10) and (11) leads to the following results:

$$C_0(^3\mathrm{He}, \beta_{exp}) \approx [\beta(^3\mathrm{He})/\beta_{exp}(^3\mathrm{He})]^{1/2} C_0(^3\mathrm{He}) \simeq 1.66,$$

$$C_2(^3\mathrm{He}, \beta_{exp}) \approx [\beta(^3\mathrm{He})/\beta_{exp}(^3\mathrm{He})]^{-3/2} C_2(^3\mathrm{He}) = 0.077,$$

$$R_2(^3\mathrm{He}), \beta_{exp}) = R_2(^3\mathrm{He}) = -0.24 \text{ fm}^2.$$

Our result of $|C_0(^3\mathrm{H}, \beta_{exp})|^2 = 2.75$ is consistent with the experimental value of Bornand *et al.*,[13] $|C_0(^3\mathrm{H})|^2 = 2.6 \pm 0.3$. The estimated value of $|C_0(^3\mathrm{He}, \beta_{exp})|^2 = 2.76$ is consistent with the experimental value, $|C_0(^3\mathrm{He})|^2 = 2.8 \pm 0.3$, of Bolsterli and Hale,[7,14] but inconsistent with $|C_0(^3\mathrm{He})|^2 = 3.5$

± 0.4 of Bornand *et al.*[13] Our result $C_0(^3\mathrm{He}) \approx C_0(^3\mathrm{He})$ is in support of the similar conclusion previously stated by Kim and Tubis.[5]

In this paper, we have shown that the normalization constant of the *nd* tail of the *D*-state ^{3}H function, which Knutson *et al.*[1] have shown to have direct experimental significance, is easily calculated value with the Reid soft-core potential is consistent in both magnitude and phase with the experimental value of Knutson *et al.*[1] Since this constant is one of the basic trinucleon bound-state parameters, we strongly advocate that new independent measurements of this constant be made for both ^{3}H and ^{3}He. In particular, it would be very desirable to determine this constant more accurately, using the analytic properties of the scattering amplitudes, from experimental measurements of *d*-*n*, *t*-*n*, *d*-*p*, and ^{3}He-*p* elastic scattering and polarization cross sections.[7,12,14,25]

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Pion Sources in Relativistic Heavy-Ion Collisions

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Implications of the most recent pion-production data are discussed. It is observed that they can be used to answer some of the crucial questions raised in understanding the basic reaction mechanism of relativistic heavy-ion collisions. As a by-product of this discussion, it is shown that the volume of the pion sources in violent collisions can be determined from *single*-particle inclusive data. Further hadron-nucleus and nucleus-nucleus collision experiments are suggested.

In heavy-ion collisions at incident energies above a few hundred MeV per nucleon, production processes become important, and the overwhelming part of the produced particles are pions. Hence, in order to understand the basic reaction mechanism of such collisions, it is necessary to know "How are the pions produced at these energies?"

In this paper, I discuss the implications of the most recent pion-production results,[1-3] especially in connection with the following problems:

(a) What do we know about the space-time evolution of the produced pions? For example, are the pions created while the participating nucleons of the projectile nucleus are still inside the target nucleus? Hadron-nucleus collision experiments[4] at very high energies strongly suggest that the production time is so long that the nucleons inside the nucleus along the path of the incident hadron can be envisaged as acting *collectively*, and can in first-order approximation be considered as a *single* object—an effective tar-

