Fifth Sound in Superfluid ^{4}He and ^{3}He-^{4}He Mixtures

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A new thermal mode, fifth sound, has been observed in a superleak partially filled with superfluid helium. The velocity of the mode in ^{4}He and ^{3}He-^{4}He mixtures is in good agreement with the theoretical value $c_5^2 = (\rho_n/\rho)c_2^2$. The index of refraction is found to be a strong function of the fractional filling of the superleak.

A new thermal sound mode in superfluid helium, fifth sound, is predicted[1] to propagate in a superleak under pressure-release conditions. The velocity c_5 of the mode is given by

$$c_5^2 = (\rho_n/\rho)c_2^2, \quad (1)$$

where ρ_n/ρ is the normal-fluid fraction and c_2 is the velocity of second sound. We have now observed this mode experimentally in a packed-powder superleak which is partially filled with helium, the free surface of the liquid film which coats the powder grains providing the necessary pressure release. By subtracting a surface tension restoring force we are able to quantitatively verify the dependence of c_5 on ρ_n/ρ and c_2 in both pure ^{4}He and in ^{3}He-^{4}He mixtures, at temperatures between 2.1 and 0.1 K.

The experiment is performed in a cell containing a superleak made of aluminum oxide powder particles[2] having a nominal diameter of 1 μm. The powder is packed to a porosity $P = 0.75$ in an annular channel machined out of brass. The channel is 11.4 cm in mean circumference, 1.1 cm in width, and packed with the powder to a depth of 0.7 cm. A tightly fitting Plexiglas plate covers the top surface of the powder to prevent the strong coupling of the thermal wave to the sound in the helium vapor.[1,3] Inserted in this plate is a 50-Ω heater wire on one side and on the opposite side are three carbon resistance thermometers. The heater and thermometers are flush with the powder. A 200-Ω, $\frac{1}{8}$-W Allen-Bradley resistor with the casing sanded off is used as the detector above 1 K, while 10- and 30-Ω, $\frac{1}{8}$-W Ohmite resistors are used below 1 K. This cell is sealed with indium O-rings and immersed in a bath of helium or attached to the mixing chamber of a dilution refrigerator. The film is built up on the powder by condensing in measured amounts of helium gas from a storage tank. Complete filling of the superlead corresponds to 0.27 mole of helium. A pressure gauge on the filling line allows the adsorption isotherm properties of the powder to be monitored.

The sound velocity is measured by observing the resonant frequencies of the annulus. Up to six plane-wave harmonics are observed before height and width modes complicate the spectrum, which is observed with a Hewlett-Packard 3580-Å spectrum analyzer. The drive current to the heater wire is frequency halved with a diode. The heater power is kept as small as possible, between 50 and 200 μW rms above 1 K, and 10 μW below 1 K. We find no dependence of the resonant frequencies on heater power at these levels.

In addition to the temperature gradients which give rise to fifth sound there will be other restoring forces acting on the thickness wave propagating in the helium adsorbed on the powder. For very thin films the van der Waals forces will give rise to an appreciable third-sound contribution.[4] There will also be surface-tension restoring forces due to the curvature of the liquid surface within the powder.[5] These contributions to the observed velocity c of the sound mode add as the sum of squares,[1]

$$n^2c^2 = c_3^2 + c_\sigma^2 + c_5^2, \quad (2)$$

where c_3 is the third-sound velocity, c_σ is the surface-tension component, and n is the index of refraction ($n > 1$) which arises because of the uneven thickness of the film and the torturous path followed by the wave.

Each of these components can be seen in Fig. 1, which is a plot of the observed sound velocities as a function of the fractional filling f of the powder with pure ^{4}He. For $f < 0.05$ the third sound dominates, and the velocity drops rapidly with increasing film thickness. By comparing the value of the velocity at the superfluid onset with the values previously observed on flat substrates,[4] it is possible to deduce an index of refraction in this thin film region to be $n \cong 3.4$. In the region $0.05 < f < 0.3$ surface-tension forces are appreciable and finally dominate the velocity.[5] The upturn in the velocity at $f = 0.05$ is accompanied by a strong feature in the adsorption isotherm, where the vapor pressure remains nearly constant as the adsorbed volume increases. This is characteristic of the onset of capillary condensa-

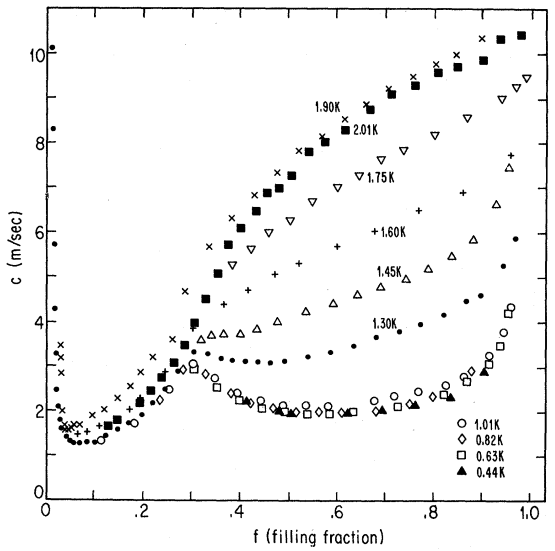

FIG. 1. Observed velocities in pure ^{4}He as a function of the fractional filling of the superleak.

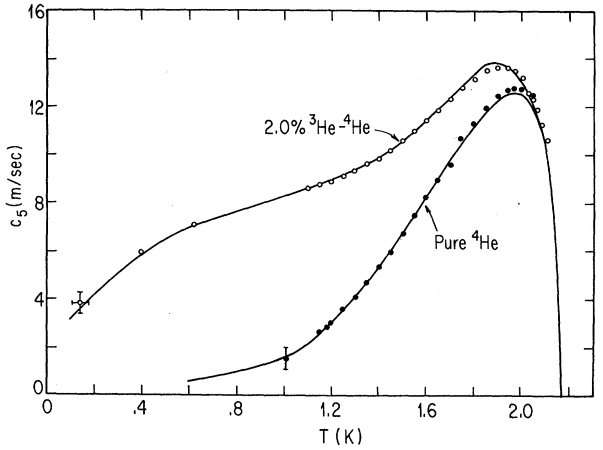

FIG. 2. Experimental values of fifth sound determined from Eq. (4) using the data of Figs. 1 and 3 plus some additional data at a filling fraction $f = 0.65$. The solid lines are the theoretical values predicted by Eq. (1). The error bars apply only to the two lowest-temperature points and are discussed in the text; the error bars for the remaining points are smaller than the dots.

tion at the points where the powder particles touch.[6] The dominant radius of curvature of the surface comes from this condensate, and it provides a positive restoring force for a thickness wave.[5,7] The break in velocity near $f = 0.3$ appears to be the point where the pores start to fill in[6] as the capillary condensate at neighboring contact points begins to overlap. The surface-tension forces are thereby reduced, lowering c_σ. The adsorption isotherm also changes at this point, with the pressure rapidly approaching the saturated vapor pressure as further liquid is added.

It is in this region, $f > 0.3$, that the fifth-sound component becomes readily apparent, the velocity increasing rapidly with increasing temperature as ρ_n/ρ increases. To extract quantitative measurements of c_5 from these data, however, it is necessary to subtract the surface-tension contribution, which is still present. In order to do this we exploit the fact that c_5 becomes quite small below 1 K, where ρ_n/ρ drops rapidly to zero. As can be seen in Fig. 1 the observed velocity becomes temperature independent below about 0.8 K. Calling this limiting velocity c_0, the surface-tension and third-sound velocities ($c_\sigma \gg c_3$ in this region) vary at higher temperatures as $(\rho_s/\rho)^{1/2}$, giving

$$(c_\sigma^2 + c_3^2)/n^2 = (\rho_s/\rho)c_0^2. \quad (3)$$

In terms of the measured values c and c_0 at a fixed filling fraction, the experimental value for c_5 can then be found from Eq. (2):

$$c_5^2 = n^2[c^2 - (\rho_s/\rho)c_0^2]. \quad (4)$$

The index of refraction n is determined by requiring c_5 at one temperature, arbitrarily chosen to be 1.45 K, to equal the theoretical prediction, Eq. (1).

The results of this analysis procedure in pure ^{4}He for $f = 0.65$ ($n = 1.46$) are shown in the lower curve of Fig. 2. The agreement with Eq. (1) (the solid line) at temperatures other than 1.45 K is clearly excellent except for a slight deviation above 1.9 K. Fits of data at other filling factors ($f = 0.55$, $n = 1.60$ and $f = 0.81$, $n = 1.28$) give results nearly identical to those of Fig. 2. The point at 1.01 K is taken from the data of Fig. 1, and the large error bars for this point are due to c and c_0 being very nearly equal.

To further verify Eq. (1) we undertook measurements in ^{3}He-^{4}He mixtures, with ^{3}He molar concentrations of 2.0% and 18.4%. The ^{3}He strongly modifies both ρ_n/ρ and c_2, and provides a stringent test of Eq. (1). Filling-curve measurements for the 2.0% solution are shown in Fig. 3. The expected qualitative features are evident: The velocities are higher than in ^{4}He because of the increased second-sound velocity, and the temperature dependence remains strong down to 0.14 K in contrast to the temperature-independent results of Fig. 1 for $T < 0.8$ K. This is because ρ_n/ρ remains finite in the mixture at low tempera-

tures, while c_2 is a strong function of temperature.[8] In Figs. 2 and 3 the temperature of the nominal 0.14-K data is uncertain to ± 0.03 K due to temperature gradients arising from the large Kapitza boundary resistance and the 10-μW heater power.

The results for the 2.0% mixture using Eq. (4) are shown in Fig. 2 for a filling fraction of $f = 0.65$ ($n = 1.42$). c_0 is taken to be the same as found in pure ^{4}He, except scaled by the factor $(\sigma_{34}/\sigma_4)^{1/2}$, where σ_{34} and σ_4 are the surface tensions for the mixture and for pure ^{4}He.[9] Above 1 K the surface-tension term is a very small correction to c_5, but at 0.14 K it becomes appreciable, and the uncertainty is estimated by the error bars for this point. The solid line, Eq. (1), obtained below 0.6 K from values of m_3* and c_2 determined by Brubaker *et al.*,[8] while above 1.2 K c_2 is found from direct measurements in our laboratory using the same 2.0% mixture. The agreement of the mixture data with the theory is again excellent, and the fact that the index of refraction is very nearly the same as that found in pure ^{4}He (to within the scatter of the data) is strong proof that Eq. (1) correctly gives both the magnitude and the temperature dependence of the new mode.

Figure 4 shows a plot of the index of refraction as a function of the filling fraction, for both pure ^{4}He and the 2.0% ^{3}He-^{4}He mixture. It appears to be nearly independent of temperature and ^{3}He concentration, a result which is also implied by the close agreement with theory found in Fig. 2. The fact that n is a function of the amount of filling is not surprising, because this geometry is a classic percolation problem,[10] with the connectivity of the sample depending on the ratio of filled to unfilled pores. When the powder is nearly filled it can be seen that n appears to be approaching the limiting value $(2-P)^{1/2} = 1.12$ found in fourth-sound studies.[11] The sharp increase in velocity seen near $f = 1$ in Figs. 1 and 3 may in fact indicate the transition to fourth sound. Velocities as high as 20 m/sec have been observed in this region by careful metering of the fill.

We have not investigated the attenuation of this mode in any detail, but we can give a few qualitative observations. At low temperatures the mode is sharply defined (with quality factors $Q > 50$) but as the λ point is approached the Q degrades. Measurements in pure ^{4}He become difficult above 1.9 K where the Q is about 10, and the broadened signal could not be seen above 2.05 K. The attenuation was not as strong in the 2.0% mixture where measurements could be taken up to 2.1 K, while in the 18.4% mixture [where c_5 also agreed with Eq. (1)] there was little attenuation to within

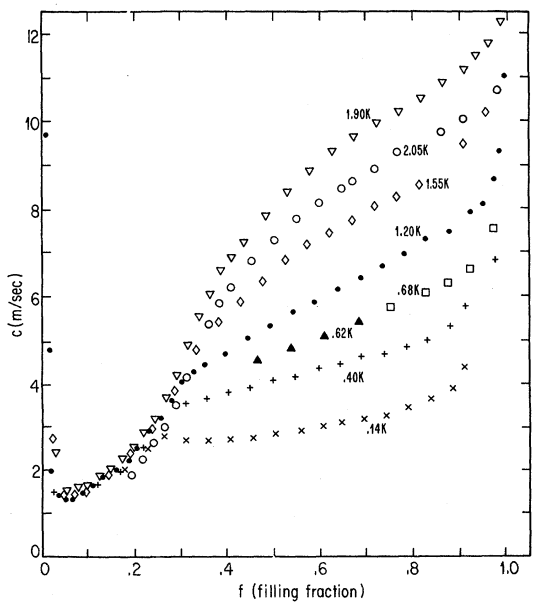

FIG. 3. Observed velocities in a 2.0% ^{3}He-^{4}He mixture as a function of the fractional filling of the superleak.

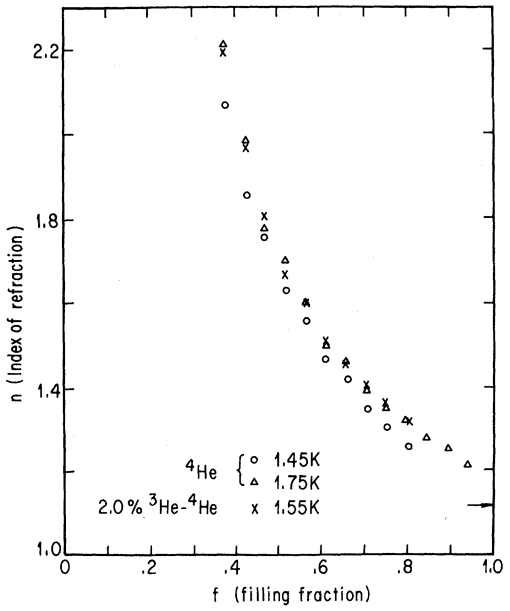

FIG. 4. Index of refraction as a function of the fractional filling of the superleak. The arrow denotes the value $(2-P)^{1/2} = 1.12$ which would be expected for fourth sound in this powder.

a few millikelvins of the λ point. The attenuation does not appear to come from unlocking of the normal fluid because of the very low frequencies used, but may arise from attenuation in the unsaturated film which connects the filled pores.

In summary, we have observed a new sound mode in superfluid helium which is in excellent agreement with theoretical predictions. This mode should have many useful applications in the study of clamped superfluids (including ^{3}He), and also appears to be a good tool for studying the nature of adsorption in porous materials.

We benefitted greatly from earlier attempts in our laboratory to observe the fifth-sound mode. Professor Julian Maynard using aluminum superconducting film transducers first experimentally demonstrated the existence of the mixed third-sound, fifth-sound, and surface-tension modes [Eq. (2)]. David Scholler in even earlier experiments, using a vibrating diaphragm transducer to investigate third sound, first demonstrated the feasibility of using resonators of the type used here. We take the opportunity here to acknowledge the importance of this ground work.

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Observation of Fifth Sound in a Planar Superfluid ^{4}He Film

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We report an observation of a superfluid helium-4 film sound mode consisting predominantly of a temperature wave, as in second sound, but with the normal-fluid component clamped. Our measurements indicate that this wave, called fifth sound, results in a contribution to the velocity of propagation which varies as $C_5 = (\rho_n/\rho)^{1/2} C_2$, as predicted by the two-fluid theory.

When third sound propagates in a thin superfluid helium-4 film, evaporation and condensation exchanges between the film and the helium vapor above the film result in heat transfers which render the wave motion isothermal rather than adiabatic. The isothermal third-sound velocity is

