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Dynamic Evolution of a *Z* Pinch

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Three-dimensional, electromagnetic computer simulation experiments are presented showing the evolution of plasma columns subjected to external electric and magnetic fields. The results of two experiments are presented here. In the first, self-confinement is achieved; in the second, as instability arises which drives the plasma into a helical configuration.

In this Letter we report some numerical plasma simulation experiments which have shown the formation of a *Z* pinch with its subsequent instability and transition into a helical configuration. These observations were made with a particle code called SPLASH which is three-dimensional, relativistic, and fully electromagnetic. With this code we have conducted a study of *Z*-pinch plasma columns using a variety of initial velocity and spatial distributions in the presence of external electric and magnetic fields of various strengths. Two such simulations will be reported here. Our initial interest was guided by a desire to study self-confinement according to the Bennett relation, as well as plasma transport across *B*. Our observations agree qualitatively with recent theoretical predictions by Montgomery, Turner, and Vahala using magnetohydrodynamic theory,[1] in that we found large-scale helical and possibly force-free structures to develop as a result of a current threshold instability.

The first experiment to be described was performed by initializing the plasma with zero drift in a columnar shape of Gaussian radial density profile. A large external magnetic field ($\beta \approx 3\%$) is applied uniformly throughout the plasma column and the simulation region with the field lines parallel to the axis of the column. Additionally, a strong uniform external electric field is applied along the column which serves to accelerate the ions and electrons in opposite directions, thus establishing significant currents ($B_0/B_{\rm self} \approx 4.5$). The simulation is periodic in space so that a particle which moves out one side of the simulation domain is returned at the opposite side with its same velocity.

After an initial acceleration phase the external electric field was turned off. In both of the simulations to be described here, external fields are simply applied everywhere. They are not generated from charge-current distributions on the boundaries of the simulation region. Hence transients or penetration processes associated with sudden changes of the external fields are not simulated realistically. Some time after turning off the external electric field, we turned off the external magnetic field as well, allowing the particles to drift in their own self-consistent electric and magnetic fields. At this point comparisons were made between measured values of the self-consistent $B_\theta(r)$, and values calculated for a plasma column with open-ended transverse boundaries. The excellent agreement between these values gave assurance that the "image" columns introduced by the periodic nature of the simulation did not generate significant spurious fields. It should be added that simulations preformed on mirror-confined plasmas using the

SPLASH code have confirmed that the periodic boundaries do not cause first-order effects. (Note that the observed phenomena are describable as an $M=1$ mode whereas the periodicity of the simulation would generate $M=4$ and multiples thereof. The mirror simulations resulted in several modes which were not multiples of $M=4$.) During the remainder of the run it was observed that the column was reasonably well contained. Apparently the Bennett current had been equaled or exceeded. Very little ion transport and essentially no electron transport was observed perpendicular to the column axis. Most interestingly, however, the plasma particles were observed to exhibit a uniform bulk rotation as if the interior of the plasma were force free. Since the external fields were so strong, and the width of the column contained so very few Debye lengths, subtle instabilities might have been suppressed.

For the second run the column width was increased. At the same time, the strength of the external fields was decreased ($\beta \approx 10\%$), allowing a more sensitive dependence of any possible instabilities on a current threshold. The external fields were left on for the duration of the run. After an initial current buildup, accompanied by visible pinching, a sudden and very dramatic $m=1$ helical mode developed. At this point the ratio of externally applied magnetic field to self-generated magnetic field was about 2.5:1. The effect of the instability was a transport of plasma across B, resulting in a column of radius $2R_0$, where R_0 is the original radius. There was an increase in the electrostatic energy of the plasma column some of which is subsequently radiated away. There was also a significant drop in the magnetostatic energy (see Figs. 1–4). The frequency of the radiation is the bulk rotation frequency which for this run is less than ω_{pe}. Since there are still relatively few Debye lengths across the column ($R_0/\lambda_D \approx 12$), it is probable that the entire column participates in the radiation process and not just the surface. As explained below, the electrons were relativistic which favored synchrotron radiation.

The development and growth of the helix is most graphically displayed in the various movies generated by the SPLASH code, but Figs. 5 and 6 convey, perhaps, the salient points. Figure 5 shows the electron density profiles before the helix develops. In Fig. 6, which shows the kink structure at the height of the instability, the structure in z is very apparent. It also required a movie to see that after the onset of the helix the plasma, and most especially the electrons, seems to participate in bulk rotation similar to that observed in the first run.

At this point, we present the results of our computer experiments merely as observations, and we make no attempts at developing theories to explain them, beyond those already available (Bennett[2] and Ref. 1). We are constantly adding to our diagnostics, so that if and when more detailed theories are developed to "explain" the re-

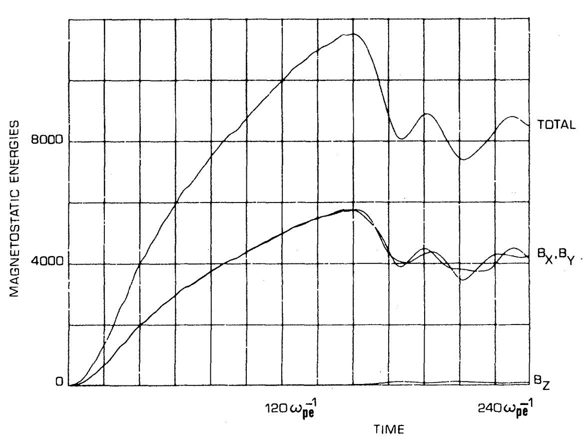

FIG. 1. The magnetostatic energy reaches its peak at $t=160\omega_{pe}^{-1}$. The onset of the instability converts this into electrostatic energy (see Fig. 2). Note that the same arbitrary energy units are used on all four field energy plots.

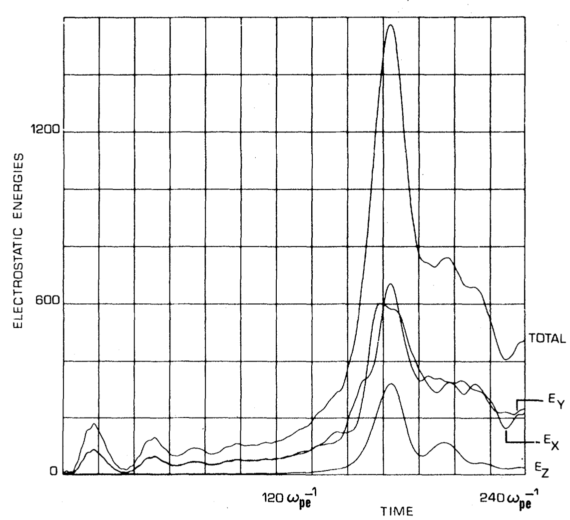

FIG. 2. The peak in electrostatic energy occurs at $t=184\omega_{pe}^{-1}$. This energy is subsequently radiated away (see Figs. 3 and 4) as electromagnetic waves.

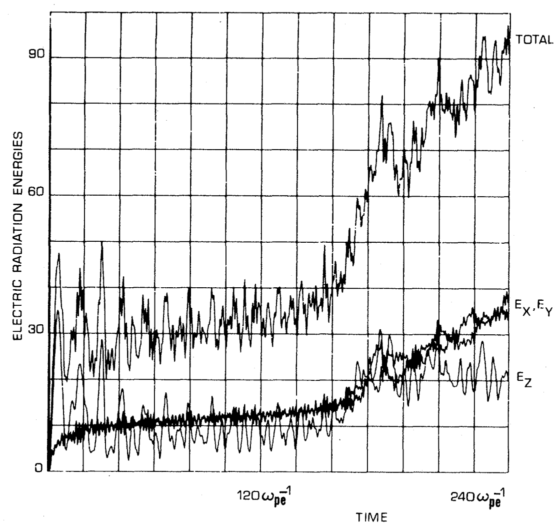

FIG. 3. The large buildup of magnetostatic and electrostatic energy is radiated away after the formation of the helix. Here we show the electric part of this radiation.

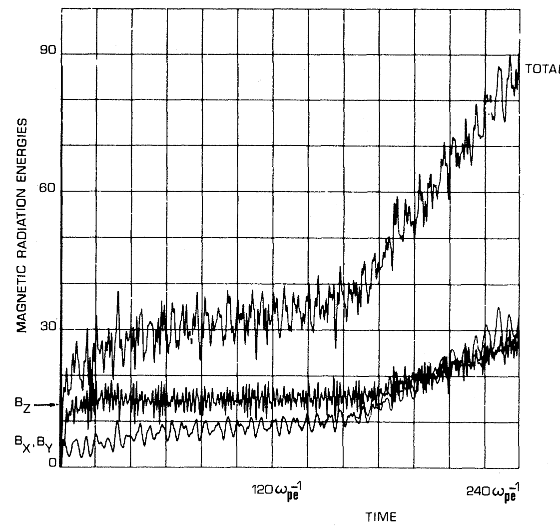

FIG. 4. The magnetic part of the radiation (see Fig. 3).

ported phenomena, every aspect of such theories can be checked.

At this point we prefer to refrain from quoting quantitative measurements of growth rates and other spatial or temporal data: Computer economy forces us to compress scales from realistic proportions to "compact" values. Thus, m_i/m_e is brought down to 16, electron temperatures are brought into the 100-eV range, so as to get v_{th} and c comparable, and drifts are generated by the applied field which are likewise relativistic. Additionally, there are only a few Debye lengths across the column, the Larmor radii are not much smaller than the column dimensions $[R_0/r_L(\text{ions}) \approx 3]$, and the number of superparticles per Debye cube is small $(n_{max}\lambda_D^3 \approx 5.4)$. However, the basic laws of physics have not been tampered with or replaced by intuition or approximations. If theories become available in which the parameters can be adjusted to match our simulation values, meaningful comparisons can, of course, be made.

We would like to emphasize that the observed phenomena are strictly three dimensional (3D) in character. Only a 3D code can provide adequate simulation. Likewise, since the self-magnetic

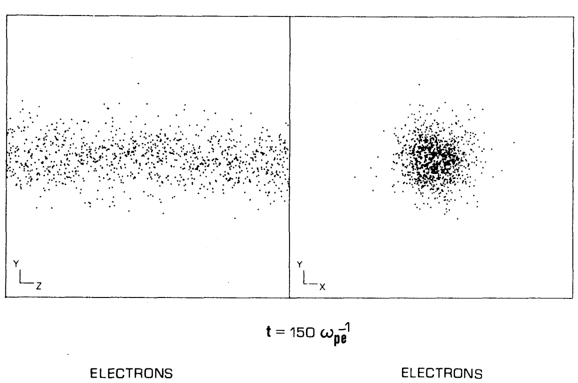

FIG. 5. Two views of the electron density distribution before the instability ($t = 150\omega_{pe}^{-1}$). On the left-hand side the view is perpendicular to the column axis. On the right-hand side the view is along the column axis.

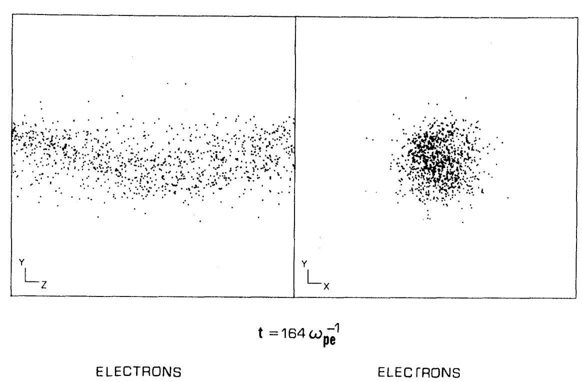

FIG. 6. Same as Fig. 5 but after the formation of the helix ($t = 164\omega_{pe}^{-1}$). Note in particular the density distribution in z. Both here and in Fig. 5, 5% of the electrons are plotted.

fields play an essential role, electrostatic models are insufficient. Since all three dimensions of the problem require roughly equivalent scale lengths, it is not possible to modify a 2D or $2\frac{1}{2}$D code by including a few modes in the third direction as is sufficient, for example, in the simulation of some tokomak properties.[3,4]

The principles of the simulation algorithms differ only slightly from those described by Buneman.[5] The use of Fourier transforms, quadratic spline interpolation [over a $(32)^3$ mesh], and careful charge shaping is retained, but the SPLASH code uses a different scheme for data management than that of C. Barnes described in Ref. 5. It is essential to keep most of the particle and field information on disk between time steps and care was taken to overlap input/output processes with computations. These techniques allow a simulation of this magnitude to be performed efficiently with the resources available through the NMFECC (National Magnetic Fusion Energy Computing Center) system. A more complete description of SPLASH, as well as the code itself, is available through the NMFECC on-line code-share facility LIBRIS, maintained at Lawrence Livermore Laboratory, University of California, Livermore, California 94550.

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Stability of Field-Reversed, Force-Free, Plasma Equilibria with Mass Flow

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The existence of a special set of cusp-shaped, stable, force-free plasma equilibria with finite pressure and flow velocity is demonstrated. They are confined by surface currents.

In a series of papers Woltjer[1] has formulated the stability of hydromagnetic equilibria in terms of a variational principle in which the energy is minimized while keeping a number of integrals of motion of the system constant. Thus, if a system is able to dissipate energy, without changing these integrals of motion, it will pass to a state of minimum energy compatible with these constraints.[1] More recently, Taylor[2] has employed this variational technique to show that the minimum-energy state for a straight pinch bounded by a rigid, infinitely conducting, cylindrical container is a force-free helical configuration in which the axial field is reversed on the outside. For plasma confinement, however, one must insist on a vacuum region that separates the fluid boundary from the wall. Therefore, in addition to the internal fluid perturbations, the possibility of surface deformations has to be entertained. These have been included in an elegant treatment by Rosenbluth and Bussac,[3] that evaluates the stability of a zero-pressure, force-free spheromak.[4] Since high-beta toroidal plasma confinement systems are highly desirable,[5] we seek stable configurations in which the plasma pressure is finite. There is no guarantee, however, that fluid kinetic energy of motion in such systems will be negligible.[6] Indeed experiments exist[7,8] and others are planned[9] in which this kinetic energy is comparable to the magnetic energy. In this Letter I demonstrate the existence of stable equilibria with finite flow velocity that have force-free fields in the interior but are confined by surface currents.

The hydromagnetic equations, in principle, possess an infinite number of integrals of motion, and to recover all possible states of motion one needs to consider all of them. However, one can obtain interesting states of minimum energy even by considering a reduced set.

I depart from Woltjer's treatment by considering the plasma in the two-fluid instead of the hydromagnetic approximation. I choose the following system integral constants: the flux invariant $K=\frac{1}{2}\int d^3x\vec{A}\cdot\vec{B}$ ($\vec{B}=\nabla\times\vec{A}$, $\vec{B}$ is the magnetic field),

