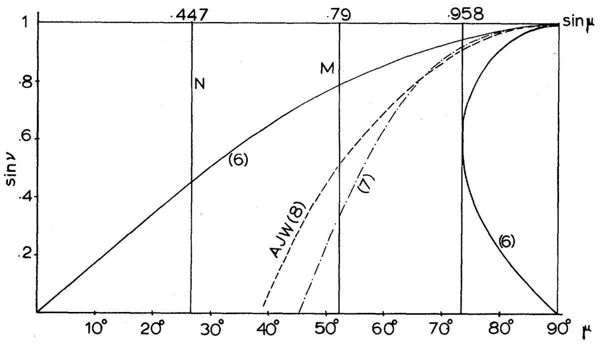

FIG. 2. Plots of all the existing uniqueness conditions: $\sin\nu$ as a function of $\sin\mu$. Vertical lines are $\sin\nu$-independent bounds. N and M refer to Newton's and Martin's bounds (Refs. 1 and 6). To the left of these values, 0.447 and 0.79, the solution is unique. (6), (7), and (8) refer to the equations given in the text. For the curves (7) and (8) the uniqueness regions are the domains above the curves. For the curve (6) the region of uniqueness is the entire domain of the picture except the half-top shaped area at the extreme right between the right branch of the curve (6) and the vertical line at $\mu = 90°$.

It is seen that for

$$\sin\mu < 0.9578$$

the condition (6) is satisfied regardless of the value of $\sin\nu$. This can be read off from Fig. 2 which shows the solution of Eq. (6), namely, $\sin\nu$ as an explicit function of $\sin\mu$. As can be seen, for $\sin\mu > 0.9578$ the curve has three branches. In this region the inequality (6) is still satisfied if $\sin\nu$ is less than the lowest branch or is between the two upper branches. For comparison I have also given my previous result,[3]

$$\sin\nu > \frac{\sin\mu - \cos\mu}{1 - \sin\mu\cos\mu}; \tag{7}$$

the result of Ref. 2,

$$\sin\nu > \sin\mu - \frac{\cos\mu}{(4\tan^2\mu - 1)^{1/2}}; \tag{8}$$

and the results of Ref. 1 and of Newton,[6]

$$\sin\mu < 0.79 \text{ and } \sin\mu < 1/\sqrt{5},$$

respectively. Figure 2 shows that except for the region at the extreme right of the figure the conjecture is proven.

This work was supported by the National Research Council of Canada.

---

[1] A. Martin, Nuovo Cimento A59, 131 (1969).

[2] D. Atkinson, P. W. Johnson, and R. L. Warnock, Commun. Math. Phys. 28, 133 (1972).

[3] I. A. Sakmar, J. Math. Phys. 19, 2124 (1978).

[4] I. A. Sakmar, J. Math. Phys. 20, 11 (1979).

[5] S. K. Chan and I. A. Sakmar, to be published.

[6] R. G. Newton, J. Math. Phys. 9, 2050 (1968).

Radiative-Emission Spectrum of 50-MeV Nuclear Excitations Populated by Proton Capture

L. G. Arnold
Department of Physics, The Ohio State University, Columbus, Ohio 43210
(Received 13 February 1979)

Qualitative features of the γ-emission spectrum from nuclear states in the 50-MeV excitation energy range to states in the giant-dipole-resonance region are described. Properties of the initial- and final-state structures are deduced.

In an exploratory study of proton radiative-capture reactions at the Indiana University Cyclotron Facility, Kovash *et al.*[1] observed intense primary radiation to excitations of the residual nucleus which were identified as stretched-configuration $\hbar\omega_0$ particle-hole states in closed-shell nuclei and the corresponding single-particle states in closed-shell-plus-one nuclei. The purpose of this Letter is to describe qualitative features of this radiation and the structure of the excitations involved. These features indicate that proton radiative-capture reactions for incident protons with energy greater than about 25 MeV may be used to map out the density of one-body states from the Fermi level to 60 MeV excitation, perhaps higher.

A schematic representation of the radiation observed[1] in closed-shell residual nuclei is shown in Fig. 1. Continuum states with excitation energy $E_x \gtrsim 2\hbar\omega_0$ are populated by capture of an incident proton. These states then decay by radiative transitions with energy E_γ to a broad structure in the $\hbar\omega_0$ excitation energy range which has a peak energy E_{xp}. For the cases studied thus far, E_{xp} is below E_{gd}, the peak energy of the giant dipole resonance; rather E_{xp} appears to be bracketed by the energies E_{sc} of the $T=0$ and 1 stretched configuration $\hbar\omega_0$ particle-hole states. The width and separation of these two states are much too small for them to be exclusively responsible for the entire structure. In this Letter, the identification of these states with E_{xp} in Ref. 1 is regarded as an aid to locating the most pronounced part of a much broader structure. The giant dipole width is also smaller than the width of the final-state structure. In fact the observed structure is broad enough to include the entire $\hbar\omega_0$ particle-hole state spectrum and not much broader.

The radiation observed in $^{11}B(p,\gamma)$ and a comparison spectrum for $^{12}C(p,\gamma)$ are shown in Fig. 2. A rough reproduction of the $^{11}B(p,\gamma)$ spectrum can be obtained by distributing the single-particle strengths inferred from $^{12}C(p,\gamma)$ over the discrete shell-model $\hbar\omega_0$ particle-hole states of ^{12}C with a $2J+1$ statistical weight factor. The peak of the broad structure seen in $^{11}B(p,\gamma)$ is dominated by the 3^- and 4^- states of the $1d_{5/2}(1p_{3/2})^{-1}$ configuration; its asymmetry is due to the 2^- and 3^- states of the $1d_{3/2}(1p_{3/2})^{-1}$ configuration. Actually, the shell-model level distribution predicts a second peak near 24 MeV. The empirical distribution of 2^- and 3^- levels[2] in this energy range of ^{12}C indicates that the second peak should be lower in energy where it would merge with the main peak at 19.2 MeV to yield the observed asymmetry. The possibility of resolving a second peak between 21 and 24 MeV may be worth considering in subsequent experiments.

A summary of characteristic energies for the excitations observed[1] in ^{12}C and ^{28}Si is given in Table I with related information on the giant dipole resonance and stretched-configuration states

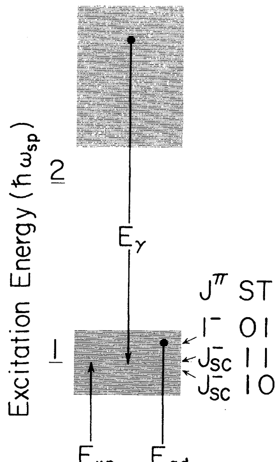

FIG. 1. Schematic representation of the radiative emission spectrum from nuclear states in the 50-MeV excitation energy range to states in or below the giant-dipole-resonance region of closed-shell nuclei. The energy scale is in units of the single-particle oscillator energy, $\hbar\omega_0 \approx 41/A^{1/3}$.

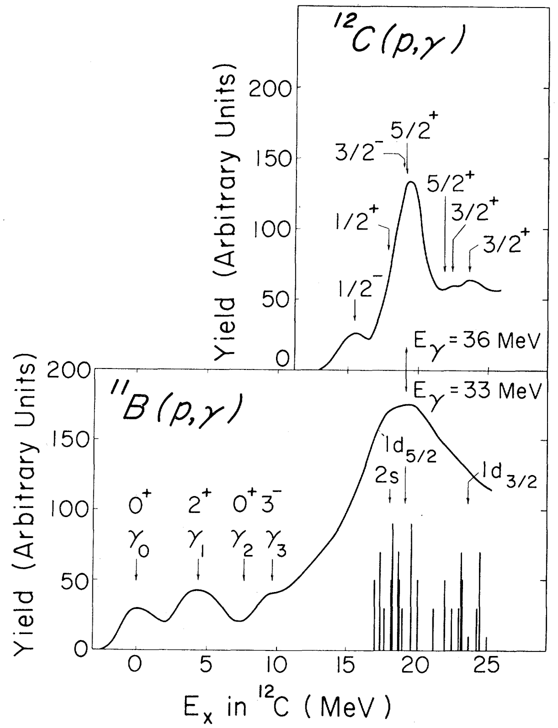

FIG. 2. γ emission spectra of ^{12}C and ^{13}N obtained from bombardment of ^{11}B and ^{12}C with 40-MeV protons. The solid lines represent the data reported in Ref. 1 plotted as a function of ^{12}C excitation energy. The ^{13}N spectrum is shifted in energy so that the s-d-shell single-particle states of this nucleus coincide with the centroid energies of the ^{12}C particle-hole states as determined by the 4^- stretched-configuration states. The spectra extend below zero excitation to show the experimental resolution associated with them. The line spectrum under the ^{12}C spectrum is representative of the shell-model $\hbar\omega_0$ particle-hole spectrum for $16 \lesssim E_x \lesssim 25$ MeV.

TABLE I. Characteristic energies associated with γ-emission spectra.

Energy[a]	^{12}C	^{28}Si	^{12}C[b]	^{28}Si[b]	K_{av}
$E_{x\mathrm{p}}$	19.2 ± 0.6	13.8 ± 0.6	44.0 ± 1.4	42.0 ± 1.8	43 ± 3
E_{gd}	22.6	20	52	61	
$E_{\mathrm{sc}}(T=0)$	18.27	11.58	41.8	35.2	38.5 ± 3.3
$E_{\mathrm{sc}}(T=1)$	19.57	14.36	44.8	43.7	44.2 ± 0.6
$E_{\mathrm{sc}}(\mathrm{av.})$	18.92	12.92	43.3	39.3	41.3 ± 2.0
	$1d_{5/2}(1p_{3/2})^{-1}$	$1f_{7/2}(1d_{5/2})^{-1}$	$1d_{5/2}(1p_{3/2})^{-1}$[b]	$1f_{7/2}(1d_{5/2})^{-1}$[b]	K_{av}
$E(j^{\mathrm{p}}, j^{\mathrm{h}}; J_{\mathrm{sc}})$[c]	20.17	13.88	46.2	42.2	44.2 ± 2.0
$\overline{E}(j^{\mathrm{p}}, j^{\mathrm{h}})$[c]	20.34	13.85	46.6	42.1	44.4 ± 2.1

[a] Reference 1 for $E_{x\mathrm{p}}$; other energies are from Ref. 2 for ^{12}C and Ref. 3 for ^{28}Si.

[b] $K = A^{1/3}E$.

[c] From particle-hole calculations of Ref. 4.

for these nuclei. There is a clear distinction between $E_{x\mathrm{p}}$ and E_{gd} in each case; E_{gd} is higher in energy than $E_{x\mathrm{p}}$ and the difference between them increases in going from ^{12}C to ^{28}Si. In each case, $E_{x\mathrm{p}}$ satisfies

$$E_{\mathrm{sc}}(T=0) < E_{x\mathrm{p}} \lesssim E_{\mathrm{sc}}(T=1); \tag{1}$$

$E_{x\mathrm{p}}$ is closer to $E_{\mathrm{sc}}(T=1)$ than $E_{\mathrm{sc}}(T=0)$ but this distinction is not nearly so pronounced as the difference between $E_{x\mathrm{p}}$ and E_{gd}. It is possible to identify $E_{x\mathrm{p}}$ with either $E_{\mathrm{sc}}(T=1)$ or $E_{\mathrm{sc}}(\mathrm{av.})$, the average energy of the stretched-configuration states. The most interesting feature of the structure observed in ^{12}C and ^{28}Si is the trend of $E_{x\mathrm{p}}$ with increasing mass number. The parameter $K_{x\mathrm{p}} = A^{1/3}E_{x\mathrm{p}}$ is approximately constant over a mass interval where $K_{\mathrm{gd}} = A^{1/3}E_{\mathrm{gd}}$ is known to increase significantly.[5] Further, the magnitude of $K_{x\mathrm{p}}$ agrees rather well with $K_0 = A^{1/3}\hbar\omega_0 \approx 41$ MeV. It follows that $E_{x\mathrm{p}}$ could be interpreted as the centroid energy of the dominant single-particle excitations, $1p_{3/2}$ to $1d_{5/2}$ in ^{12}C and $1d_{5/2}$ to $1f_{7/2}$ in ^{28}Si. The centroid energy of such excitations is the single-particle quantum of excitations[6]; thus,

$$E_{x\mathrm{p}} \approx \hbar\omega_0 \approx 41/A^{1/3}. \tag{2}$$

The energy scale of the schematic representation in Fig. 1 is based on this identification of $E_{x\mathrm{p}}$ with $\hbar\omega_0$. The previous association of $E_{x\mathrm{p}}$ with E_{sc} is equivalent in the sense that E_{sc} marks the approximate position of the centroid. The last three rows of Table I show that the equivalence of E_{sc} and the centroid energy is particularly good in ^{12}C and ^{28}Si where the dominant single-particle excitations are particle-hole states based on stretched $(j = l + \frac{1}{2})$ single-particle states for both particle and hole. The stretched-configuration particle-hole state, which has angular momentum $J_{\mathrm{sc}} = j_{\mathrm{sc}}{}^{\mathrm{p}} + j_{\mathrm{sc}}{}^{\mathrm{h}}$, is one of these states.

In the preceding remarks, the γ-emission spectra from states in the 50-MeV excitation range have been identified with the distribution of elementary particle-hole excitations of closed-shell nuclei and the corresponding single-particle states of closed-shell-plus-one nuclei. Population of low-lying multiple particle-hole excitations such as ^{12}C(7.66 MeV) is inhibited. Within the class of particle-hole excitations, there appears to be an absence of selective population of specific states other than would be expected from the $2J+1$ statistical weight factor. The observed spectra are a qualitative representation of the density of one-body states from the Fermi level to about 25 MeV excitation; they reveal the gross structure of the simplest nuclear excitations.

Having identified the dominant characteristic of the final-state structure, it becomes possible to describe features of the initial-state structure and the radiative transitions between them. Since the initial states are populated by capture of fast protons ($E_p \approx 40$ MeV), it is reasonable to expect, as noted by Kovash *et al.*,[1] that the important ones will be particle-hole or single-particle states. An alternative, that the initial-state structure is a giant resonance with well-defined L, S, J, and T quantum numbers, seems remote in comparison because of the unselective manner in which the final-state structure is populated by the emitted radiation. The initial-state structure considered here is giant in the same sense as the final-state structure; namely, a clumping of elementary excitations in the vicinity of $n\hbar\omega_0$ excitation energy.[7] Parity is the only good quantum number for giant structures of this type. The dominant multipolarity of radiative transitions between such structures can be determined from the Weisskopf single-particle estimates. These estimates, generally of limited value except for normalization purposes, are appropriate in the present context by definition. Electric dipole ($E1$) is the dominant mode of radiation. Since $E1$ radiation in self-conjugate nuclei such as ^{12}C and ^{28}Si is subject to spin and isospin selection rules[8] ($\Delta S = 0, \Delta T = 1$) that are most effective for single-particle transitions, separate arguments about the nature of the initial-state structure and emitted radiation reinforce each other once the final-state structure has been defined. As a result, the observed radiation is a composite of *all* allowed $E1$ transitions between $2\hbar\omega$ and $1\hbar\omega$ one-particle, one-hole states. The dominant characteristics of this composite are governed by the most important single-particle dipole transitions,[9] $1f_{7/2}(1p_{3/2})^{-1} \rightarrow 1d_{5/2}(1p_{3/2})^{-1}$ for ^{12}C and $1g_{9/2}(1d_{5/2})^{-1} \rightarrow 1f_{7/2}(1d_{5/2})^{-1}$ for ^{28}Si. These transitions correspond to the "second harmonic" giant dipole resonance suggested by Kovash *et al.*[1]

The $2J+1$ statistical weight factor is essential to this explanation of the emission spectra reported in Ref. 1. It has been used previously to explain the difference between (p,γ) cross sections for ground and excited states of the residual nucleus; for example, the ^{7}Li$(p,\gamma_1)^8$Be(2^+) cross section is about 5 times the ^{7}Li$(p,\gamma_0)^8$Be(0^+) for proton capture in the giant dipole resonance region.[10] As used, the $2J+1$ factor is a consequence of a sum rule analogous to the J-file sum rule[11] of atomic spectroscopy. The criterion of validity for use of this sum rule in the present context is that all initial states in the J-file sum be equally populated. This criterion is satisfied marginally for initial states in the giant-dipole-resonance region. In this region particle-hole states from a given (j^p, j^h) configuration are distributed over an energy interval that is comparable to or somewhat smaller than the intrinsic width of the unbound particle state. Population of these states by proton capture would depend on the distribution at each energy of the interval and would tend to be unequal. At 50 MeV excitation the width of an unbound particle state is much larger and the criterion should be satisfied quite well. It is worth noting that a J-file sum rule is satisfied exactly for optical-model initial states in which coupling to the spin of the target is neglected. This is a reasonable approximation for 40-MeV protons on light nuclei, but is suspect at the lower energies of the giant dipole region.

In summary, the important states populated by capture of protons in the region of 50 MeV excitation of the residual nucleus are expected to be optical single-particle or particle-hole states. Radiative transitions to states at lower energies are governed by a transition operator that is predominantly a one-body operator. As a result, the final states populated by the transition are optical single-particle or particle-hole states. The selectivity of this reaction process is one of separating simple and complex nuclear excitations in the lower energy range.

In thank S. L. Blatt, H. J. Hausman, and M. Kovash for useful discussions.

---

[1] M. A. Kovash, S. L. Blatt, R. N. Boyd, T. R. Donoghue, H. J. Hausman, and A. D. Bacher, Phys. Rev. Lett. 42, 700 (1979).

[2] F. Ajzenberg-Selove, Nucl. Phys. A248, 1 (1975).

[3] P. M. Endt and C. van der Leun, Nucl. Phys. A310, 1 (1978).

[4] T. W. Donnelly and G. E. Walker, Ann. Phys. (N.Y.) 60, 209 (1970).

[5] E. Hayward, in *Nuclear Structure and Electromagnetic Interactions*, edited by N. MacDonald (Plenum, New York, 1965), p. 141. See Fig. 12, p. 173.

[6] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. I, p. 209.

[7] The schematic representation of Fig. 1 shows the initial-state structure at an energy that is significantly above $2\hbar\omega_0$. This was done to take the energy dependence of the single-particle potential into account. Since the single-particle potential energy decreases as the energy increases, the centroid energy of two-quantum one-particle, one-hole excitations occurs at a higher energy than $2\hbar\omega_0$. The two-quantum excitation density of states could be mapped out by looking at transitions to a particular region of final-state structure as a function of proton bombarding energy. Experiments of this type, analogous to mapping out the giant dipole resonance from capture transitions to the ground state, are planned (S. L. Blatt, private communication).

[8] E. K. Warburton and J. Weneser, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969), p. 175.

[9] D. H. Wilkinson, Physica (Utrecht) 22, 1039 (1956).

[10] G. A. Fisher, P. Paul, F. Riess, and S. S. Hanna, Phys. Rev. C 14, 28 (1976).

[11] E. U. Condon and G. E. Shortley, *The Theory of Atomic Spectra* (Cambridge Univ. Press, London, 1935), pp. 238, 247, and 278. The J-file sum rule holds for any type of coupling.

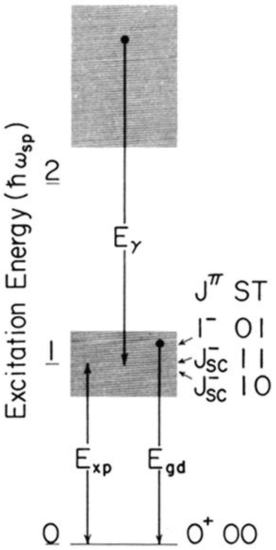

FIG. 1. Schematic representation of the radiative emission spectrum from nuclear states in the 50-MeV excitation energy range to states in or below the giant-dipole-resonance region of closed-shell nuclei. The energy scale is in units of the single-particle oscillator energy, $\hbar\omega_0 \approx 41/A^{1/3}$.