Lepton Mass Formula

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A recently proposed mass formula for the muon is extended to τ and other heavy leptons. It is postulated that a quantized magnetic self-energy of magnitude $\frac{3}{2}\alpha^{-1}n^4M_e$, where n is a new quantum number, be added to the rest mass of a lepton to get the next heavy lepton in the chain $e, \mu, \tau, \cdots$, with $n=1$ for μ, $n=2$ for τ, etc. I predict $M_\tau = 1786.08$ MeV, and for the next lepton $M_\delta = 10\,293.7$ MeV.

Recently I have suggested that the mass formula for the muon, $M_\mu = (\frac{3}{2}\alpha^{-1}+1)M_e$, can be derived on the basis of magnetic self-interaction of the electron.[1] The radiative effects give an anomalous magnetic moment to the electron which, when coupled to the self-field of the electron, implies an extra magnetic energy. Assuming that this energy of a charge in the field of a magnetic moment is quantized and is successively added to the rest energy, we arrive at the following set of mass values:

(i) M_e,

(ii) $M_\mu = M_e + M_{\text{magnetic}}(n=1)$,

(iii) $M_\tau = M_\mu + M_{\text{magnetic}}(n=2)$, etc.

The magnetic energy of a system consisting of a charge and a magnetic moment quantized according to Bohr-Sommerfeld procedure implies quantized energies

$$E_n = \lambda n^4,$$

where n is a principal quantum number.[2] Determining the proportionality constant λ from the muon-mass formula ($n=1$), we obtain

$$M_\tau = M_\mu + \tfrac{3}{2}\alpha^{-1}n^4M_e \quad (n=2)$$
$$= (M_e + \tfrac{3}{2}\alpha^{-1}1^4M_e) + \tfrac{3}{2}\alpha^{-1}2^4M_e$$
$$= 1786.08 \text{ MeV}.$$

The three very recent experimental determinations[3] of the τ mass agree with this value extremely well.

The next lepton with $n=3$ would have a mass

$$M_\delta = M_\tau + \tfrac{3}{2}\alpha^{-1}3^4M_e = 10\,293.7 \text{ MeV}.$$

It is possible that although the Bohr-Sommerfeld quantization is approximative, the final result might be exact as was the case in Bohr-Sommerfeld derivation of the Balmer formula. Therefore, the hypothesis which I advanced should be considered of a heuristic nature towards the development of a more complete theory. On the other hand, the occurrence of the fermion chain $e, \mu, \tau, \ldots$, is a novel phenomenon for which we have so far no theory in order to derive a mass formula from first principles: "We have no plausible precedent for, nor any theoretical understanding of this kind of superfluous replication of fundamental entities. Nor is any vision in sight wherein the various fermions may be regarded as composites of more elementary stuff. No problem is more basic than the problem of flavor, and it has been with us since the discovery of muons. Sadly, we are today no closer to a solution."[4]

[1] A. O. Barut, Phys. Lett. 73B, 310 (1978).

[2] Consider a charge moving in circular orbits in the field of a fixed magnetic dipole $\vec{\mu}$. The equations of Bohr-Sommerfeld quantization are

$$mv^2/r = e\mu v/r^3 \text{ and } mvr = n\hbar.$$

From the second of these equations we have $r = n\hbar/mv$, which we insert into the first equation to obtain $v = n^2\hbar^2/e\mu m$, or $v^2 = n^4\hbar^2/e^2\mu^2 m$. Hence quantized energies are proportional to n^4.

[3] R. Brandelik *et al.* [Phys. Lett. 73B, 109 (1978)] gave a value of $m_\tau = 1807 \pm 20$ MeV. W. Bacino *et al.* [Phys. Rev. Lett. 41, 13 (1978)] gave a value of $m_\tau = 1782^{+2}_{-7}$ MeV, now revised to $m_\tau = 1782^{+3}_{-4}$ MeV [see, e.g., G. J. Feldman, in *Proceedings of the Nineteenth International Conference on High Energy Physics, Tokyo, Japan, August 1978*, edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Physical Society of Japan, Tokyo, 1979), p. 786]. W. Bartel *et al.* [Phys. Lett. 77B, 331 (1978)] gave a value of $m_\tau = 1787^{+10}_{-18}$ MeV, now revised to $m_\tau = 1790^{+7}_{-10}$ MeV [see the rapporteur's talk by Feldman cited above].

[4] S. L. Glashow, Comments Nucl. Part. Phys. 8, 105 (1978).