where we have forced the wave vector at T_N to be $q=\frac{1}{3}$. In this case (Fig. 1, dot-dashed line) there seems to be only *one* transition to a phase with $N=4$, and no new phases appear when N is increased, in contrast to the devil's-stairs behavior. However, by changing the parameters slightly, the devil's staircase immediately shows up again.

In conclusion, we have demonstrated that our model exhibits multiple phase transitions between commensurate phases when the temperature is varied, despite the fact that the model includes three temperature-independent parameters only, and we can increase the number of phases by refining the calculation. This is consistent with the devil's-stairs behavior. It would be interesting to investigate our model analytically to find out whether or not the devil's staircase is complete. We also suggest that accurate experiments be performed on periodic magnetic systems to search for the stepwise behavior.

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Textural Singularities and Frustration in Random-Anisotropy and Random-Field Models

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Spin models with random anisotropic fields and random magnetic fields are studied. It is shown that integral-index textural singularities of these fields act as disclination sources and participate in destroying long-range ferromagnetic order. Half-integral-index singularities give rise to frustration. The relation of these models to bond disordered spin-glasses is discussed.

Frustration is generally believed to play a fundamental role in determining the nature of spin-glass phases. The concept has, however, only been defined[1] and studied[2,3] for models where the frustrations are properties of the underlying bond structure. Recently spin models with random magnetic fields[3,4] and with random anisotropies[5,6] have been studied by several authors. The results show remarkable similarities to those found previously for bond-disordered spin-glasses. An interesting and related feature of these models is that long-range ferromagnetic order does not seem to exist below four dimensions.[4,6] Our purpose here is to show that one can indeed define frustration as an inherent property of the underlying (random) fields. Frustration defined in this way has properties very similar but not identical to those discussed by Toulouse[1] for the bond network. We find that it is closely related to, and in fact reflects, the topological properties of the textural singularities of these fields. We also find that other types of textural singularities, which do not induce frustration, may also play an important role in destroying long-range order.

It is convenient to start by discussing two-dimensional models, mainly because the singularities are simpler and easier to visualize. We first consider the two-dimensional analog of the Harris-Plischke-Zuckerman model,[5] i.e., a two-component ($n=2$) ferromagnet with random anisotropies in two dimensions:

$$H=-J\sum_{i,j}\vec{S}_i\cdot\vec{S}_j+K\sum_i\vec{S}_i\cdot(\vec{n}_i*\vec{n}_i-\tfrac{1}{2})\cdot\vec{S}_i, \qquad (1)$$

where the spins $\vec{S}_i$ are two-dimensional (2D) unit vectors, $\vec{n}_i=(\cos\varphi_i,\sin\varphi_i)$ is a two-component unit vector of random orientation and summations (i,j) are over the sites of a 2D lattice. For a continuum one would have an anisotropy field

$$A(\vec{r})=[\vec{n}*(\vec{r})\vec{n}(\vec{r})-\tfrac{1}{2}]. \qquad (2)$$

The singularities can be classified by an integral or semi-integral index m where

$$\oint\nabla\varphi\cdot d\vec{l}=2m\pi. \qquad (3)$$

The integral is on a closed loop around the singular point and $\varphi(\vec{r})$ is the angle determining the orientation of $\vec{n}(\vec{r})$. The analogous situation for nematics is discussed by de Gennes.[7] Frustration is implied by semi-integral values of m. The anisotropy field is rotated by an odd multiple of π in going around the singularity. As a result the spins cannot adjust to the anisotropy field by continuous deformations. Between any two points, continuous adjustment along paths passing on different sides of the singularity will lead to conflicting results. We illustrate this for the simplest cases ($m=\pm\frac{1}{2}$) in Figs. 1(a) and 1(b). Thus a singular point is frustrated if

$$\Phi=e^{2\pi mi}=-1. \qquad (4)$$

Frustrations cancel each other in pairs and can be "removed" by cuts along strings connecting them. An arbitrary path has frustration

$$\Phi=\prod\Phi_\nu=\exp(2\pi i\sum m_\nu), \qquad (5)$$

where ν is an index for the enclosed singularities. We note that there are many different types of frustrated points (for all semi-integral m) contrary to the bond model which has only one type of frustration. The frustration can always be canceled out between any two frustrated points. However, as we shall see, only points of equal and opposite m cancel each other completely. Otherwise one is left with a net effect equal to that of an integral point.

On a lattice the integrals [Eq. (2)] are, of course, not defined and the relationship between the $\vec{n}_i$ and the continuous field $\vec{n}(\vec{r})$ is not unique. We therefore replace the definition of Φ [Eq. (4)] by

$$\Phi=\begin{cases}\operatorname{sgn}\prod_{\text{bonds}}(\vec{n}_i\cdot\vec{n}_j)=\prod_{\text{bonds}}\operatorname{sgn}(\vec{n}_i\cdot\vec{n}_j) & \text{if all } \vec{n}_i\cdot\vec{n}_j\neq 0,\\ 1 & \text{if any } \vec{n}_i\cdot\vec{n}_j=0,\end{cases} \qquad (6)$$

where the product is over all bonds on a closed path on the lattice. Setting $\Phi=1$ whenever two neighboring $\vec{n}_i$ are orthogonal is arbitrary but convenient. With the definition (6) one has, for any path,

$$\Phi_{\text{path}}=\prod_{\text{plaquettes}}\Phi_\nu, \qquad (7)$$

as for bond frustration, where the product is over all elementary plaquetts enclosed. We also note that the (arbitrary) sign of $\vec{n}_i$ has no effect on the definition given. For any field $A(\vec{r})$ the definitions (4) and (6) become equivalent when the points i are sufficiently dense on the path. It is easy to see (e.g., by inspection) that Eq. (6) is correct form for an elementary plaquette. There are, of course, an infinite number of ways of constructing a continuous field $\vec{n}(\vec{r})$ equal to $\vec{n}_i$ at lattice sites i. With the above definition of Φ, we believe that it is always possible to construct such a continuous field with half-integral–index singularities at all frustrated plaquettes. We note that in some case, it may be possible to construct fields with only integral-index singularities even though there are frustrated plaquettes on the lattice.

It can thus be seen that the frustrations introduced by the anisotropy field have properties analogous to those generated by the interactions. The similarity becomes clearer when one considers the limit $K\to\infty$. This limit can be described in terms of the effective spin interactions

$$J_{ij}^{\text{eff}}=J\cos(\varphi_i-\varphi_j)=J\vec{n}_i\cdot\vec{n}_j \qquad (8)$$

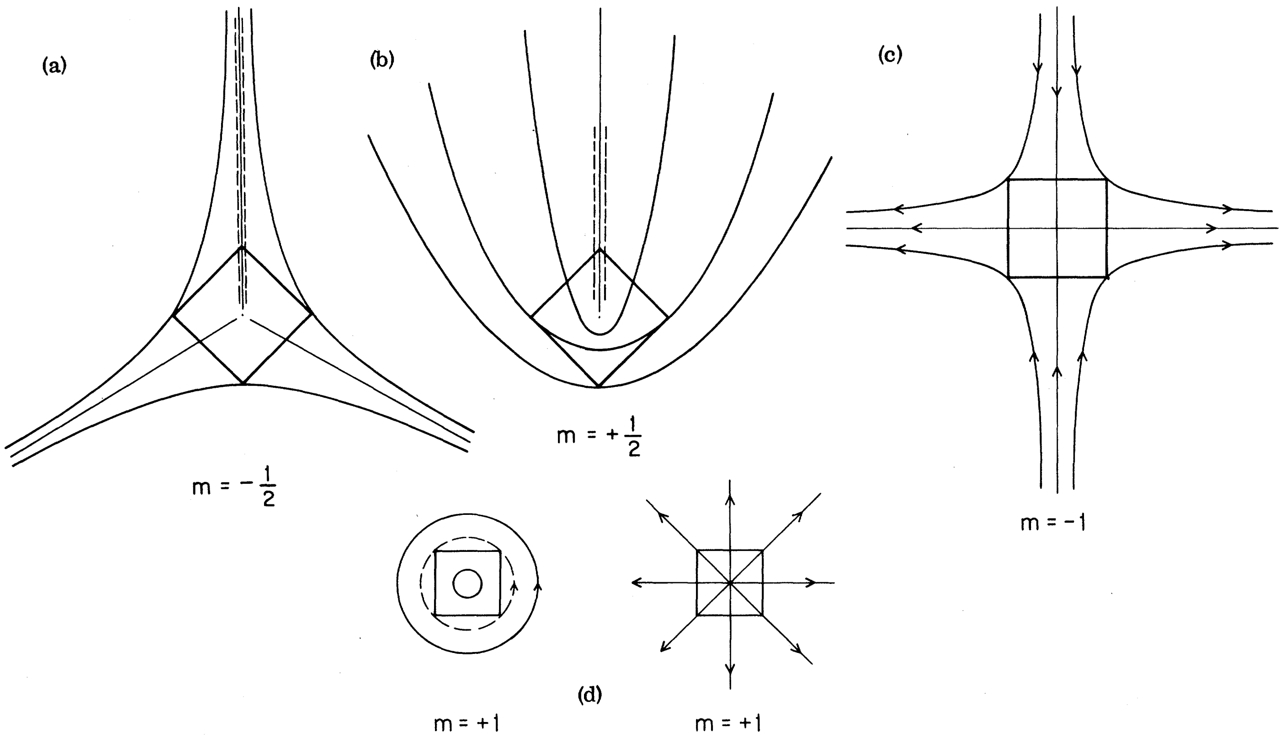

FIG. 1. Small-index singularities for the anisotropy field of a two-dimensional field in two dimensions. The lines indicate the orientation of the anisotropy fields and the arrows show representative minimum-energy arrangement for a ferromagnet. Squares are drawn in to illustrate how the singularity would manifest itself at the lattice points. Frustrated strings are drawn as double lines.

which are randomly distributed (but correlated).

We have seen that the anisotropy field has other types of singularities. The simplest ones which do not involve frustration ($m = \pm 1$) are shown in Figs. 1(c) and 1(d). It can be seen that one can deform a ferromagnet to adjust to these types of anisotropies and no frustrated cuts are required. Ferromagnetic order is, however, destroyed because the singularity is not consistent with any unique axis of magnetization. We first investigate the appearance of these singularities on a lattice. For an elementary plaquette with n cor-

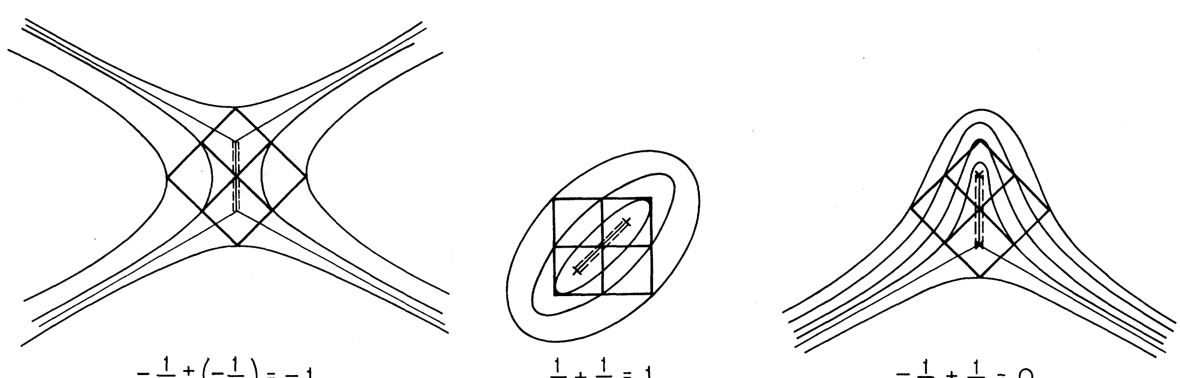

FIG. 2. Illustration of how two semi-integral–index points on neighboring squares add up. It can be seen that the pair of squares acts as a source (of strength $m \pm 1$) when the two singularities are of equal sign. When the signs are opposite, the effect of the pair is localized.

ners the maximum rotation angle of the magnetization induced by the anisotropies can never exceed

$$\Psi_{max} = \tfrac{1}{2}n\pi. \tag{9}$$

Thus for the triangular and square lattices (but not for the honeycomb lattice) any distribution of anisotropies can, therefore, be derived from a field $Q(r)$ which has only $m = \pm\frac{1}{2}$ singularities. They do, however, add up. Thus two $+\frac{1}{2}$ defects within a larger boundary will look like a $+1$ disclination source (Fig. 2). The disclination can only be eliminated by a disclination of opposite sign. Thus while frustration tends to be local one expects long-range disclination effects. Using the central limit theorem, one can argue that in two dimensions, the net strength of a random distribution of disclination sources inside a boundary increases linearly with the dimensions of the loop.

The generalization of these results is, in principle, straightforward. We consider first a three-dimensional spin with uniaxial anisotropy. The anisotropy field is isomorphic to a nematic liquid crystal. In two dimensions, the only topological singularity is a point[8] with $m = +\frac{1}{2}$. All points with semi-integral m become topologically equivalent and all integral points are unstable. Defects whose 2D projections have these structures still exist in the frozen anisotropy field but can no longer serve as sources of disclinations. (A 3D spin has no stable disclination points in two dimensions.) Thus the only sources of disorder are frustrations. In the infinite-K limit, the model becomes completely equivalent to a bond-disorder model of the Edwards-Anderson type[9] with a continuous distribution of magnitudes of J.

The 3D case is somewhat more complex because of the nature of the topoligical singularities. We discuss only the 3D spin with uniaxial anisotropy, and restrict ourselves to two types of singularities[8]: (a) lines with index $m = \frac{1}{2}$ derived from the 2D points; (b) points with index m, where m is a positive integer. These can serve as sources for point disclinations.

The lines should form closed loops. They manifest themselves on a lattice as arrays of the frustrated plaquettes through which they pass. The frustration can be removed by a cut on any covering surface bounded by the line. These are equivalent to the covering surfaces discussed by Toulouse[1] and by Kirkpatrick[2] for bond frustration.

We believe the closed loops can also act as sources for point disclinations with an index m depending on the way the line is closed. In three dimensions one can, however, also have lattice cells which display an $m = 1$ disclination point directly. For example, a cube with anisotropy axis along the body diagonals at all corners. Thus one certainly has disruption of long-range order by frustration and by disclination sources.

The situation with a random magnetic field is somewhat different. Since this is a vector field, only singularities with integral index are possible.[8] These act as sources for disclinations. Frustrations are only induced when the magnetic field itself is frustrated. This implies that not only the positions of the frustrated plaquettes but also the exact configuration of strings (or covering surfaces) are determined by the configuration of the underlying field. We note that this implies that one cannot have the ground-state disorder associated with different minimum-energy string (or covering surface) configurations which seems to be important for bond disorder.[1,2] This type of ground-state degeneracy is possible for the random-anisotropy model.

We would like to point out that our analysis suggests the possibility of two types of disordered phases for the random-anisotropy model as a function of the ratio

$$\kappa = K/J \tag{10}$$

[Eq. (1)]. When K is large, the anisotropy field dominates at all distance scales. Frustrations are therefore important and we expect spin-glass–like behavior. When K is small, it seems likely that the interaction will overpower the essentially local frustration effects. One then expects that long–range order is only destroyed by the accumulative effect of disclination sources. This presumably also implies a much weaker ground-state degeneracy. Thus a multicritical point for some critical value of K (K_c) is suggested. In the magnetic field case it is hard to envisage a phase transition when the random field becomes large. We believe this probably implies an upper critical value for the magnitude of the random magnetic field (H_c) above which there is no phase transition. It is, of course, possible that $H_c = 0$ and this may well be the case below four dimensions where it is known[4] that long-range ferromagnetic order is destroyed even by an infinitesimal random external field. We feel these possibilities should be investigated further.

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Search for Linear Polarization of the Cosmic Background Radiation

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We present preliminary measurements of the linear polarization of the cosmic microwave background (3 K blackbody) radiation. These ground-based measurements are made at 9-mm wavelength. We find no evidence for linear polarization, and set an upper limit for a polarized component of 0.8 mK with 95% confidence level. This implies that the present rate of expansion of the Universe is isotropic to 1 part in 10^6, assuming no reionization of the primordial plasma after recombination.

The observed cosmic microwave background radiation is generally thought to be a relic of the hot, dense, initial phase of the Universe. This radiation should then have characteristics which reflect its thermal origin and the geometry of the early Universe. The simple hot big-bang model predicts that this radiation has a blackbody (Planckian) spectrum, is unpolarized, exhibits the statistical properties of blackbody radiation,[1] and is isotropic in a reference frame comoving with the expansion of the Universe.

The spectrum of this radiation has been extensively investigated.[2] The best experiment to date is that of Woody and Richards.[3] Although their results qualitatively follow a blackbody spectrum, they report a 5σ deviation (σ is 1 standard deviation) from the best-fit blackbody curve. There have also been many measurements of the angular distribution of this radiation with recent reports of a (3.5 ± 0.6)-mK-amplitude first-order (cosine) anisotropy.[4,5] It is thought that this anisotropy is a result of the Doppler shift caused by the motion of the solar system relative to the cosmic radiation. If this interpretation is correct, the cosmic radiation shows an intrinsic anisotropy of less than 1 part in 3000 (1 mK). The polarization properties of the radiation have been largely unexplored although, in their original paper on the discovery of the background radiation, Penzias and Wilson assert that this radiation is "within the limits of our observations... unpolarized...".[6] Nanos[7] and Caderni *et al.*[8] have searched for evidence of linear polarization with negative results.

It is of great interest to measure the polarization because of its potential to detect and distinguish deviations from the simple big-bang model. Rees has shown that any intrinsic anisotropy in the cosmic radiation present at the time of decoupling or last scattering manifests itself as a

